DISTRIBUTED SOURCE LOCALIZATION VIA PROJECTION ONTO THE NEAREST LOCAL MINIMUM

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ABSTRACT

When addressing the energy–based source localization problem using wireless sensor networks, distributed localization method is necessary to reduce the energy and bandwidth consumption. In this paper, a novel distributed source localization method called projection onto the nearest local minimum (PONLM) is proposed, which can be carried out at each of active nodes with quite lightweight computation, in contrast to most existing centralized method. Simulation results show our method can yield much better performance than the previous methods.

Index Terms— Energy–based source localization, Wireless sensor networks, Distributed method, PONLM

1. INTRODUCTION

The problem of energy–based source localization in wireless sensor networks has recently received a great deal of attention [1–7]. In [1], X. Sheng et al. has addressed this problem asymptotically via maximum likelihood estimation (MLE) in a centralized manner, i.e., requiring the transmission of the full data set to a central point for processing. However, the centralized method is very costly for dense networks in terms of communication bandwidth and energy consumption, in contrast to distributed methods where the computation is performed at each of nodes. Therefore, several distributed methods have been proposed in [2–7], among which, [2], [5] are typical.

In [2], Rabbat and Nowak proposed a distributed source localization method based on incremental subgradient algorithm [8] where the estimation is calculated and passed from one node to the next. However, since the MLE problem associated with energy-based source localization is a highly nonconvex optimization problem with many local minima or saddle points, *the method mostly results in local minima and even divergence*, which has been shown in [5] via simulation. Recently, a new incremental optimization algorithm called normalized incremental subgradient (NIS) algorithm was proposed in [4], which can offer better convergence performance than the IS algorithm and work nicely especially when the number of active sensors is somewhat large (>10), however, demanding a decreasing stepsize to set.

A better method was proposed in [5], where the authors formulated the localization problem as a convex feasibility problem solved by finding a point in the intersection of some convex sets(i.e., some disks centered at the active sensor's location) using projection onto convex sets (POCS) method. The POCS method can be implemented in a distributed manner and guaranteed to converge to a limit point. However, it should be noted that the limit point may be far away from the true source location in two cases. The *first case* is that the intersection is too large, which mainly occurs when the source lies outside the convex hull of sensors. In this case, there are many points in the intersection which are far away from the true source location but may be taken as the estimation of the source location by the POCS method. The *second case* is that the intersection is empty, corresponding to the inconsistent convex flexibility problem which can be solved by steered sequential projection [9]. In this case, the method converges to a limit circle instead of a point and thus the estimation may be inferior.

In this paper, we propose a new energy-based distributed source localization method, which is based on a new concept called **p**rojection **o**nto the **n**earest **l**ocal **m**inimum (PONLM). Our proposed method differs significantly from the POCS method because our method attempts to find the point of intersection of some circles centered at the active sensor's location rather than a point in the intersection of some disks, which has been shown in section III. Extensive simulations have been conducted to compare the performance of two methods. The simulation results show that our method can yield much better performance than the POCS method.

2. PROBLEM STATEMENT

In the energy-based source localization problem, an acoustic source locates at an unknown location, $\theta \in \mathbb{R}^2$ (generalization to $\mathbb{R}^3$ is easy but not explored here), in the sensor field consisting of N sensor nodes, emits a signal that attenuates in space. At the same time, L sensors (named *active sensors*) detect the presence of the source and take signal energy measurements according to the energy decay model [1]:

$$e_i = \frac{A}{||\theta - p_i||^2} + w_i \quad i = 1, 2, \ldots, L \tag{1}$$

where A is signal strength the source emits, $p_i \in \mathbb{R}^2$ is the ith sensor's location which is known, e_i is the energy measurement the ith sensor takes, w_i is a zero-mean white Gaussian noise.

For simplicity, we suppose A is known. The case of unknown signal strength also can be disposed by generalizing our method to the energy-ratio nonlinear least square problem (see [1]).

The maximum likelihood estimation problem is formulated as follows:

$$\hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{L} \left(\frac{A}{||\theta - p_i||^2} - e_i \right)^2 \tag{2}$$

here, the objective function consists of L component functions, and each is nonconvex, thus the objective function is also nonconvex and has many local minima, the negative log of which has be shown in Fig. 1 of [5].

3. LOCALIZATION ALGORITHM

Consider the problem:

$$\min_{x \in \mathbf{R}^m} f(x) = \sum_{i=1}^{n} f_i(x) \tag{3}$$

where, component functions $f_i : \mathbf{R}^m \rightarrow \mathbf{R}, i = 1, 2, \cdots, n$ have common local minima, i.e., the global minima of f.

It has been mentioned in [4] that the incremental optimization method (e.g., IS [8] or NIS [4]) for problem (3) can theoretically avoid trapping into local minima by *alternately progressing to local minimum of each component function*, even if $f(x)$ is nonconvex. This is due to the fact only when reaching the global minimum of f with a suitable stepsize, the algorithm stagnates.

On the other hand, it can be observed that in problem (2), when in absence of measurement noise, component functions have a common local minimum, i.e., the solution of the problem. Moreover, all local minima of each component function of (2) lie on a circle. For example, the circle corresponding to the ith component function is defined by:

$$||\theta - p_i||^2 = \frac{A}{e_i} \tag{4}$$

Particularly, if given a point(not the center p_i), denoted by x, we can calculate the nearest local minimum, denoted by y, of the ith component function of (2) from x, by solving the following optimization problem

$$\begin{aligned} & y = \arg\min_{\theta} ||\theta - x||^2 \\ \text{s.t.} \quad & \left(\frac{A}{||\theta - p_i||^2} - e_i \right)^2 = 0 \text{ or } ||\theta - p_i||^2 = \frac{A}{e_i} \end{aligned} \tag{5}$$

A unique analytic solution can be obtained, which has the form

$$y = p_i + r_i \times \frac{x - p_i}{||x - p_i||} \tag{6}$$

where $r_i = \sqrt{\frac{A}{e_i}}$.

Based on the above analysis, we propose a new incremental source localization method called projection onto the nearest local minimum, where we take the nearest local minimum as the current estimation of θ. Our method for the noise–free case can be written in a subiterative framework as follows:

$$\begin{cases} \varphi_{i,k} = p_i + r_i \times \frac{\varphi_{i-1,k} - p_i}{||\varphi_{i-1,k} - p_i||}, \, i = 1, 2, \cdots, L \\ \theta_k = \varphi_{L,k} = \varphi_{0,k+1}, \, k = 1, 2, \cdots \end{cases} \tag{7}$$

Here, $\varphi_{i,k}$ denotes the nearest local minimum of the ith component function from $\varphi_{i-1,k}$, and the subscript ‘i’ is used to denote subiteration and ‘k’ to cycle (there are L subiterations in each cycle).

Below we present a proposition on the convergence of the PONLM method. We use the notation $\{\varphi_{i,k}\}_{k\rightarrow\infty}^{i=1\sim L}$ to denote the sequence generated by Eq. (7)

$$\cdots, \varphi_{1,k}, \varphi_{2,k}, \cdots, \varphi_{L,k}, \varphi_{1,k+1}, \varphi_{2,k+1}, \cdots, \varphi_{L,k+1}, \cdots$$

and $\{\varphi_{i,k}\}_{k\rightarrow\infty}$ to each subsequence

$$\cdots, \varphi_{i,k}, \varphi_{i,k+1}, \cdots$$

Proposition: *the sequence $\{\varphi_{i,k}\}_{k\rightarrow\infty}^{i=1\sim L}$ generated by Eq. (7) converges to the point of intersection of L circles if it converges.*

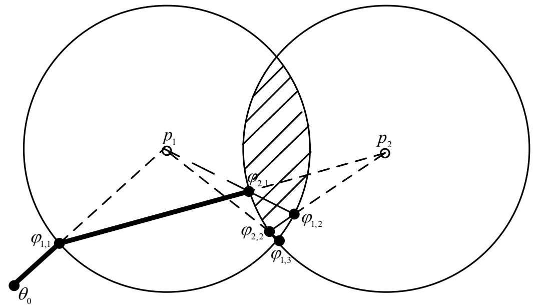

Fig. 1. An example comparison of convergence: POCS vs. PONLM.

Proof: Suppose the sequence $\{\varphi_{i,k}\}_{k\rightarrow\infty}^{i=1\sim L}$ converges to θ^*. Hence, it is readily shown that all subsequences $\{\varphi_{i,k}\}_{k\rightarrow\infty}$ also converges to θ^*, i.e., $\lim_{k\rightarrow\infty} \varphi_{i,k} = \theta^*$. Observe that

$$\varphi_{i,k} - \varphi_{i-1,k} = p_i + r_i \frac{\varphi_{i-1,k} - p_i}{||\varphi_{i-1,k} - p_i||} - \varphi_{i-1,k} \tag{8}$$

Hence,

$$||\varphi_{i,k} - \varphi_{i-1,k}|| = |r_i - ||\varphi_{i-1,k} - p_i||| \tag{9}$$

Take the limit of both sides in Eq. (9) as $k \rightarrow \infty$, then we have

$$||\theta^* - p_i|| = r_i, \quad i = 1, 2, \cdots, L \tag{10}$$

Eq. (10) means that θ^* is the point of intersection of L circles.■

Although it can not be guaranteed that the sequence $\{\varphi_{i,k}\}_{k\rightarrow\infty}^{i=1\sim L}$ must converge, we fortunately find from simulations that it can converge most of the time, whatever the algorithm initializes from.

Obviously, our method is to find the point of intersection of L circles, which differs from the POCS method [5] in that the solution obtained by the POCS method is a point in the intersection of L disks defined by $||\theta - p_i||^2 \leq \frac{A}{e_i}, i = 1, 2, \cdots, L$. For example, in Fig.1, both methods are initialized from the point θ_0. The thick line denotes the path $\overrightarrow{\theta_0\varphi_{1,1}\varphi_{2,1}}$ taken by the POCS method. One can find that the POCS method converges to $\varphi_{2,1}$, a point in the shadowed intersection of two disks, while our method takes the path $\overrightarrow{\theta_0\varphi_{1,1}\varphi_{2,1}\varphi_{1,2}\varphi_{2,2}\varphi_{1,3}}$ and converges to $\varphi_{1,3}$, the nearer point of intersection of two circles from θ_0. The further comparison between two methods is presented in Section 5.

Practically, component functions of (2) often have no common local minimum(i.e., L circles don't intersect at a point) when the measurement noise exists and thus the method demonstrated in (7) will not halt at any point. However, as the steered sequential projection method [9], our method is also suitable for the noise case with a decreasing stepsize α_k to guarantee its convergence to a point. Two phase implementation of the PONLM method is listed in Fig. 2. Note that the norm $k<20$ is employed to avoid some abnormal cases where the algorithm is “hesitating” in the vicinity of the true source location.

4. SIMULATION RESULTS

In this section, we have performed simulations in Matlab to compare the performance of our method with the POCS [5]. In our simulations, N(if not specified, N=2000) nodes are deployed uniformly at random within the region $[0, 100]m \times [0, 100]m$, taking measurements corrupted by zero–mean white Gaussian noise with variance

1. Initialization: θ_0 is arbitrary, $k = 0$
2. Phase one:
   DO
   $k = k + 1$
   $\varphi_{0,k} = \theta_{k-1}$
   FOR each i
   $$\varphi_{i,k} = p_i + r_i \times \frac{\varphi_{i-1,k} - p_i}{||\varphi_{i-1,k} - p_i||}$$
   $\theta_k = \varphi_{L,k}$
   WHILE $||\theta_k - \theta_{k-1}|| \geq 10^{-3}$ or $k < 20$
   $T1 = k$
3. Phase two:
   DO
   $k = k + 1$
   $\varphi_{0,k} = \theta_{k-1}$
   $\alpha_k = \frac{1}{k - T1 + 1}$
   FOR each i
   $$\varphi_{i,k} = (1 - \alpha_k)\varphi_{i-1,k} + \alpha_k \left[p_i + r_i \times \frac{\varphi_{i-1,k} - p_i}{||\varphi_{i-1,k} - p_i||} \right]$$
   $\theta_k = \varphi_{L,k}$
   WHILE $||\theta_k - \theta_{k-1}|| \geq 10^{-3}$

Fig. 2. Projection onto the Nearest Local Minimum algorithm.

equal to 1. The source is located at $\theta^* = [50, 50]$ and emits a signal with energy $A = 100$. Simultaneously, we suppose that sensors detect the presence of the source if their energy measurements are above 5, i.e., the minimum signal–to–noise rate is 7dB.

First, we show the convergence performance of two methods in two cases mentioned above, i.e., *when the intersection of disks is empty or large*, where the POCS method mostly converge to the point away from the true source while the PONLM method can work much better. The results of two methods initialized from multiple initial points on a grid are presented in Fig. 3, 4, 5 and 6, where the initial points are depicted by crosses, followed by a line which depicts the convergence path (consisting of θ_k only, e.g., $\overrightarrow{\theta_0\theta_1\theta_2}$) taken by the methods, and ends at the convergence point denoted by triangles; active sensors are denoted by empty dots and the source by a square; circles are plotted according to Eq.(4).

From simulations, we find that our method always converges to the vicinity of the true source when the source lies within the convex hull of sensors. However, when the source is outside the convex hull, the method may sometimes stagnate at a point away from the true source. An example is shown in Fig. 7, where the asterisks depicts the initial points which lead to not convergence to the vicinity of the true source. One can see that our method converges at most initial points. Also, it is observed that *at least one among four corner points* leads to convergence to the vicinity of the true source.

Then the overall performance of two methods is compared in terms of average errors through 10000 Monte Carlo simulations, where N is set from 100 to 2100 in 200 increments. The simulation results are presented in Fig. 8, where, "1stPhase" represents the average estimation error of the first phase of two methods. Note that the estimate of the first phase is obtained by averaging over L sensor's estimates of the T_1th cycle, i.e., $\sum_{i=1}^{L} \varphi_{i,T_1}$, which can be easily computed through a single communication cycle. As shown in the figure, the PONLM method is always better than the POCS method, especially when the number of active sensors is small. This is mainly due to the fact that the probability that the source is outside the convex hull of active sensors increases with the number of active sensors decreasing. It is also observed that, whether the POCS method or the PONLM method used, the estimation performance of the first phase is only somewhat lower than that of the second phase. However, it should be noted that the communication overhead of the first phase is much lower.

Additionally, we propose a better method called global PONLM without requiring a diminishing stepsize, motivated by the observation that among four corner points there is at least one rendering convergence to the vicinity of the true source. In this method, the final estimate is the best one among four estimates of the first phase corresponding to four initial points on the corner of the network, i.e., $[0, 0], [0, 100], [100, 0]$ and $[100, 100]$. Here, the best estimate means the one that minimizes the cost function (2). The average estimation performance is also shown in Fig. 8. From the figure, one can see that the global PONLM method significantly outperforms other methods, but only a single additional communication cycle is added for the cost function evaluation since the estimation process of four initial points and the evaluation process of four estimates can be done in parallel. Generally, the global PONLM method needs averagely less than 10 cycles in all.

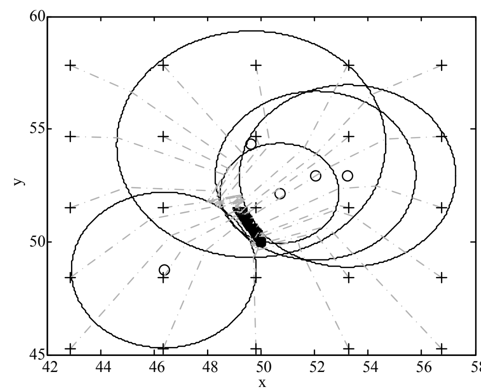

Fig. 3. Path taken by the POCS method when the intersection is empty.

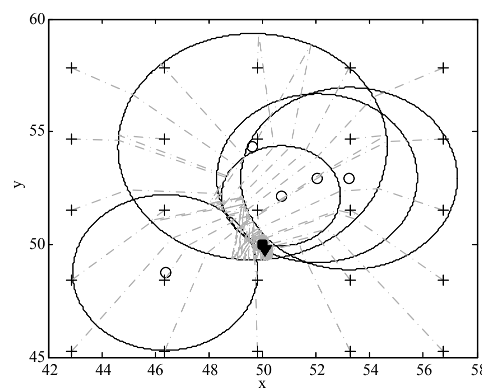

Fig. 4. Path taken by the PONLM method when the intersection is empty.

5. CONCLUSION

A lightweight distributed source localization method called PONLM is proposed in this paper. Simulation results show that the method

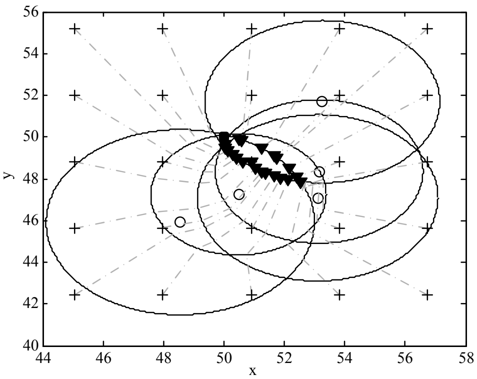

Fig. 5. Path taken by the POCS method when the intersection is large.

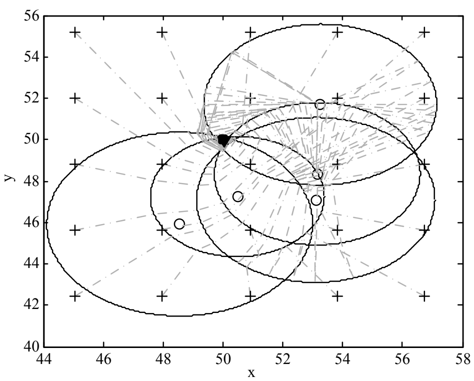

Fig. 6. Path taken by the PONLM method when the intersection is large.

mostly converges to the vicinity of the true source with only several cycles of communication averagely and yields much better performance than the POCS method. Furthermore, the concept of PONLM can be extended to deal with other problems where the nearest local minima of each component function in (3) can be obtained analytically or approximated by some simple optimization methods.

6. ACKNOWLEDGE

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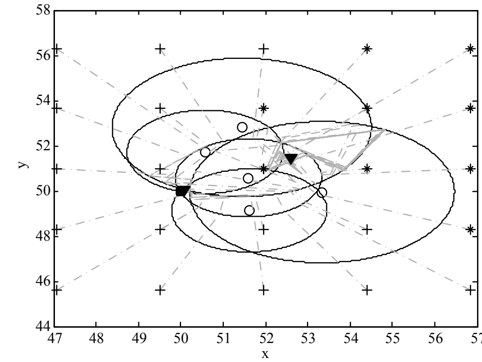

Fig. 7. The PONLM method may not converge globally when the source lies outside the convex hull of sensors.

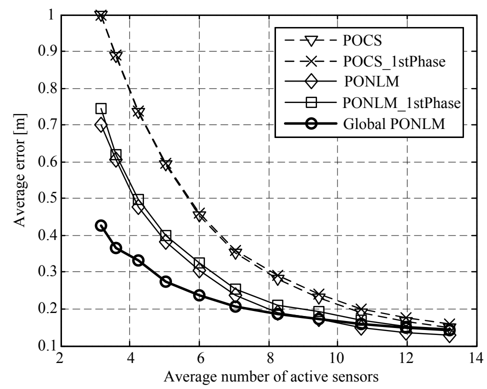

Fig. 8. Localization performance.

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