BAYESIAN NODE LOCALISATION IN WIRELESS SENSOR NETWORKS

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ABSTRACT

Node localisation in wireless sensor networks is a difficult problem due to the large number of parameters to be estimated and the nonlinear relationship between the measurements and the parameters. Assuming the presence of a number of anchor nodes with known positions and a centralised architecture, a Bayesian algorithm for node localisation in wireless sensor networks is proposed. The algorithm is a refinement of an existing importance sampling method referred to as progressive correction. A simulation analysis shows that, with only a few anchor nodes, the proposed method is capable of accurately localising a large number of nodes.

Keywords: Sensor networks, Localisation, Bayes procedures.

1. INTRODUCTION

Networks of large numbers of sensors are finding increased use due to the availability of low cost, low power miniature devices capable of sensing, computation and communication [9]. A prerequisite for use of a sensor network is knowledge of the sensor, or node, positions. In a randomly deployed *ad hoc* network this knowledge is not generally available since fitting each node with a positioning device, such as a global positioning system (GPS), is too costly. Instead, it is customary to fit a small number of sensors with a GPS and then use these anchor nodes as the basis of a procedure for determining the positions of the remaining nodes. This will be the approach taken here.

The information required to locate the nodes is obtained by generating measurements between neighbouring nodes. These measurements could be the strength or time of arrival of a signal transmitted from one node to another [1, 9]. Alternatively, the radio inteferometric positioning system (RIPS) uses the phase difference between signals simultaneously transmitted at a pair of nodes and received at another pair of nodes to compute a sum of range differences between the four nodes [5]. In our examples we use RIPS measurements because, in practice, they combine accuracy with a large range.

It is assumed here that all measurements are sent to a central node for processing. Node localisation from these measurements is a difficult parameter estimation problem. Since the measurements are typically nonlinear functions of the node locations, maximum likelihood estimates can be found only by numerical optimisation [6]. Reasonably accurate initialisation is required to prevent convergence to a local maximum.

In this paper the node localisation problem is approached in a Bayesian framework. In this framework the optimal estimator, in the mean square error sense, is the posterior mean. Since the posterior distribution, and hence the posterior mean, cannot be found exactly, an approximation is necessary. This is a common problem in Bayesian estimation and many numerical methods of approximation have been proposed [4, 10]. Importance sampling, in which random samples are drawn from an importance density and then weighted, will be used here. The difficulty of applying importance sampling to this problem is that the dimension of the sampling space is typically very large and the prior PDF for the parameters is much more diffuse than the likelihood. These difficulties are addressed by the use of progressive correction. Progressive correction is a multi-stage procedure in which samples are obtained from a series of intermediate distributions which become progressively closer to the posterior distribution [7]. The idea is that the intermediate distributions used in the early stages should be simpler to obtain samples from than the true posterior distribution. Annealed importance sampling, in which samples from the importance density are drawn via a specially constructed Markov chain, is a similar idea [8].

The progressive correction procedure as originally proposed in [7] in the context of particle filtering is not suitable for estimating the large number of parameters which typically arise in the node localisation problem. A generalisation of the procedure is proposed which enables significantly more accurate node localisation. Numerical simulations will be presented which demonstrate that the proposed algorithm is able to accurately locate large numbers of nodes with moderate computational expense.

The paper is organised as follows. A mathematical model of the problem is given in Section 2. The exact Bayesian solution is discussed in Section 3 and approximation via progressive correction is developed in Section 4. A performance analysis is given in Section 5.

2. MODELLING

Assume the network has K anchor nodes with the kth node having position $\boldsymbol{y}_k \in \mathbb{R}^2$. The number of nodes with unknown position is denoted as M with the mth such node having position $\boldsymbol{x}_m \in \mathbb{R}^2$. Let $\boldsymbol{y} = [\boldsymbol{y}_1', \ldots, \boldsymbol{y}_K']'$ and $\boldsymbol{x} = [\boldsymbol{x}_1', \ldots, \boldsymbol{x}_M']'$ denote the collections of known and unknown node positions, respectively. These nodes are to be localized from a vector of T measurements $\boldsymbol{d} = [d_1, \ldots, d_T]'$. The measurements could be distances between nodes or the distance differences given by RIPS. Assuming that the measurements are subject to additive Gaussian noise which is independent for each measurement, the likelihood of the parameter $\boldsymbol{x}$ given the measurements $\boldsymbol{d}$ can be written as

$$l(\boldsymbol{d}|\boldsymbol{x}) = \prod_{t=1}^{T} N(d_t; h_t(\boldsymbol{x}, \boldsymbol{y}), \sigma^2), \tag{1}$$

where $N(\boldsymbol{z}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$ is the Gaussian probability density function (PDF) with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ evaluated at $\boldsymbol{z}$. The function h_t is determined by the quantity being measured and the nodes used in the generation of the tth measurement. For example, the performance analysis of Section 5 uses RIPS measurements.

In this case T is even and we define the measurement function $\mu_t : \{1,\ldots,4\} \to \{1,\ldots,K+M\}$, $t = 1,\ldots,T/2$ which determines the nodes used to produce the $(2t-1)$th and $2t$th measurements. Let

$$\boldsymbol{z}_{t,j} = \begin{cases} \boldsymbol{y}_{\mu_t(j)}, & \mu_t(j) \leq K, \\ \boldsymbol{x}_{\mu_t(j)-K}, & \text{otherwise}, \end{cases} \tag{2}$$

and $\rho_t(i,j) = \|\boldsymbol{z}_{t,i} - \boldsymbol{z}_{t,j}\|$ where $\|\cdot\|$ is the Euclidean norm. According to this notation, for $t = 1,\ldots,T/2$ [5],

$$h_{2t-1}(\boldsymbol{x},\boldsymbol{y}) = \rho_t(1,4) - \rho_t(2,4) + \rho_t(2,3) - \rho_t(1,3), \tag{3}$$
$$h_{2t}(\boldsymbol{x},\boldsymbol{y}) = \rho_t(1,4) - \rho_t(3,4) + \rho_t(2,3) - \rho_t(1,2). \tag{4}$$

3. BAYESIAN ESTIMATION OF THE NODE LOCATIONS

The Bayesian approach assumes that the vector parameter of interest is a random variable for which a prior distribution is available. Thus, assume that $\boldsymbol{x} \sim \pi_0$. The key quantity in Bayesian estimation is the distribution of the random paramater conditional on the measurements, i.e., the posterior distribution. Applying Bayes' rule gives the posterior PDF as

$$\pi(\boldsymbol{x}) \propto l(\boldsymbol{d}|\boldsymbol{x})\pi_0(\boldsymbol{x}). \tag{5}$$

The minimum mean square error (MMSE) estimator of $\boldsymbol{x}$ is the posterior mean,

$$\hat{\boldsymbol{x}} = \mathsf{E}(\boldsymbol{x}|\boldsymbol{d}) = \int \boldsymbol{x}\,\pi(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}. \tag{6}$$

The posterior PDF π, and hence the posterior mean, cannot be found exactly, so an approximation is required. Approximation of (6) via importance sampling involves drawing samples of the parameter vector $\boldsymbol{x}$ from an importance density q and approximating the integral (6) by a weighted sum of the samples [10],

$$\hat{\boldsymbol{x}} \approx \sum_{i=1}^{n} w^i \boldsymbol{x}^i, \tag{7}$$

where, for $i = 1,\ldots,n$, $\boldsymbol{x}^i \sim q$ and $w^i = C\,l(\boldsymbol{d}|\boldsymbol{x}^i)\pi_0(\boldsymbol{x}^i)/q(\boldsymbol{x}^i)$ with C such that the weights sum to one.

Sampling from any importance density which satisfies certain regularity conditions will enable exact computation of the posterior mean as the sample size $n \to \infty$. However, obtaining an accurate approximation with as few samples as possible requires careful selection of the importance density. The simplest candidate is the prior, i.e., $q = \pi_0$. The weights are then $w^i = C\,l(\boldsymbol{d}|\boldsymbol{x}^i)$. Accurate approximation of the posterior mean requires several samples to attract a significant weighting. Thus several samples drawn from the prior need to be in regions of high likelihood. The probability of this happening is small if there is a large number of parameters and/or the prior distribution of the parameters is diffuse. Both situations are likely in the node localisation problem. This makes the prior an unsuitable choice of importance density.

4. IMPORTANCE SAMPLING VIA PROGRESSIVE CORRECTION

Progressive correction is implemented in S stages using likelihoods at each stage which become progressively closer to the true likelihood. Let l_s, $s = 1,\ldots,S$ denote the likelihood used for the sth stage with $l_S = l$. The likelihood at the sth stage takes the form $l_s(\boldsymbol{d}|\boldsymbol{x}) = l(\boldsymbol{d}|\boldsymbol{x})^{\Gamma_s}$ where $\Gamma_s = \sum_{j=1}^{s}\gamma_j$, $\gamma_j \in (0,1]$ and $\Gamma_S = 1$. The likelihood used for $s < S$ is broader than the true likelihood, particularly in the earlier stages, making it more probable that samples drawn from the diffuse prior will have a high likelihood. In the later stages the likelihood approximation sharpens so that the samples gradually concentrate in the area of the parameter space suggested by the true likelihood.

Let $\pi_s(\boldsymbol{x}) \propto l_s(\boldsymbol{d}|\boldsymbol{x})\pi_0(\boldsymbol{x})$, $s = 1,\ldots,S$ be the posterior PDF of the node locations according to the sth likelihood. Note that $\pi_S = \pi$ is the posterior PDF under the true likelihood. Progressive correction works by successivley drawing samples from π_1, π_2 and so on up to π_S. Consider the sth stage of the procedure and assume that the approximation

$$\pi_{s-1}(\boldsymbol{x}) \approx \sum_{i=1}^{n} w^{s-1,i} g_{s-1}(\boldsymbol{x} - \boldsymbol{x}^{s-1,i}), \tag{8}$$

is available. In (8) g_{s-1} is a kernel density [11]. It is desired to draw samples from the sth intermediate distribution with PDF,

$$\begin{aligned}\pi_s(\boldsymbol{x}) &\propto l_s(\boldsymbol{x}|\boldsymbol{d})\pi_0(\boldsymbol{x}) = l(\boldsymbol{d}|\boldsymbol{x})^{\gamma_s} l_{s-1}(\boldsymbol{d}|\boldsymbol{x})\pi_0(\boldsymbol{x}) \\ &\propto l(\boldsymbol{d}|\boldsymbol{x})^{\gamma_s}\pi_{s-1}(\boldsymbol{x}).\end{aligned} \tag{9}$$

Substituting (8) into (9) gives the approximation

$$\pi_s(\boldsymbol{x}) \approx C \cdot l(\boldsymbol{d}|\boldsymbol{x})^{\gamma_s} \sum_{i=1}^{n} w^{s-1,i} g_{s-1}(\boldsymbol{x} - \boldsymbol{x}^{s-1,i}) \tag{10}$$

where C is a normalising constant. The mixture density (10) can be re-written as, for $i = 1,\ldots,n$,

$$\pi_s(\boldsymbol{x}, i) \approx C \cdot w^{s-1,i} l(\boldsymbol{d}|\boldsymbol{x})^{\gamma_s} g_{s-1}(\boldsymbol{x} - \boldsymbol{x}^{s-1,i}) \tag{11}$$

where i is an index on the mixture. Sampling from (11) involves drawing a parameter vector and a mixture index. It is proposed to draw mixture indices and parameter samples from an importance density q_s of the form

$$q_s(\boldsymbol{x}, i) = \psi^{s,i} f^{s,i}(\boldsymbol{x}). \tag{12}$$

where $\psi^{s,1},\ldots,\psi^{s,n}$ sum to one and $f^{s,i}$ is a PDF. Sampling is performed by drawing mixture indices from the distribution $q_s(i) = \psi^{s,i}$ and then drawing parameters conditional on the mixture indices using $q_s(\boldsymbol{x}|i) = f^{s,i}(\boldsymbol{x})$. This procedure is performed sequentially for $s = 1,\ldots,S$ to obtain samples from the posterior PDF π which are used to approximate the posterior mean. The complete procedure is summarised in Table 1. In this paper the kernels $g_0,\ldots,g_{S-1}$ are Gaussian PDFs with zero mean and covariance matrix selected as suggested in [11, Chapter 4]. Two possibilities for q_s will be considered in the following subsections.

4.1. Blind progressive correction (B-PC)

The progressive correction of [7] involves setting $q_s = \pi_{s-1}$, so that $\psi^{s,i} = w^{s-1,i}$ and $f^{s,i}(\boldsymbol{x}) = g_{s-1}(\boldsymbol{x} - \boldsymbol{x}^{s-1,i})$. The weights in Step 3(d) can be found as $w^{s,i} = C\,l(\boldsymbol{d}|\boldsymbol{x}^{s,i})^{\gamma_s}$. This procedure will be referred to as blind progressive correction (B-PC) since samples from the $(s-1)$th stage are selected without considering the refined likelihood.

4.2. Measurement-directed progressive correction (MD-PC)

The second progressive correction variant seeks to use only those samples which are favoured by the refined likelihood. This approach is motivated by the success of measurement-directed sampling techniques in particle filtering [3].The kernel g_{s-1} is Gaussian with zero-mean and covariance matrix $\boldsymbol{\Sigma}_{s-1}$, i.e., $g_{s-1}(\boldsymbol{x}) =$

Table 1: Importance sampling with progressive correction.

1. Select $\gamma_1, \ldots, \gamma_S$.
2. For $i = 1, \ldots, n$, draw $\boldsymbol{x}^{0,i} \sim \pi_0$ and set $w^{0,i} = 1/n$.
3. For $s = 1, \ldots, S$:
   (a) Compute the selection weights $\psi^{s,1}, \ldots, \psi^{s,n}$.
   (b) Select indices $j^{s,1}, \ldots, j^{s,n}$ such that $\mathsf{P}(j^{s,i} = k) = \psi^{s,k}$.
   (c) For $i = 1, \ldots, n$, draw $\boldsymbol{x}^{s,i} \sim f^{s,j^{s,i}}$.
   (d) For $i = 1, \ldots, n$, compute the weights

$$w^{s,i} = C \frac{w^{s-1,j^{s,i}} l(\boldsymbol{d}|\boldsymbol{x}^{s,i})^{\gamma_s} g_{s-1}(\boldsymbol{x}^{s,i} - \boldsymbol{x}^{s-1,j^{s,i}})}{q_s(\boldsymbol{x}^{s,i}, j^{s,i})}$$

4. Compute the parameter estimate $\hat{\boldsymbol{x}} = \sum_{i=1}^{n} w^{S,i} \boldsymbol{x}^{S,i}$.

$N(\boldsymbol{x}; 0, \boldsymbol{\Sigma}_{s-1})$. Let $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}) = [h_1(\boldsymbol{x}, \boldsymbol{y}), \ldots, h_T(\boldsymbol{x}, \boldsymbol{y})]'$. The linearised likelihood of $\boldsymbol{x}$ about the point $\hat{\boldsymbol{x}}$ given the measurements $\boldsymbol{d}$ is

$$\hat{l}(\boldsymbol{d}|\boldsymbol{x}, \hat{\boldsymbol{x}}) = N(\boldsymbol{d}; \boldsymbol{h}(\hat{\boldsymbol{x}}, \boldsymbol{y}) + \boldsymbol{H}(\hat{\boldsymbol{x}})(\boldsymbol{x} - \hat{\boldsymbol{x}}), \sigma^2 \boldsymbol{I}_T) \tag{13}$$

where $\boldsymbol{H}(\hat{\boldsymbol{x}}) = \nabla_{\boldsymbol{x}'} \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y})|_{\boldsymbol{x}=\hat{\boldsymbol{x}}}$. The weights and parameter sampling densities are

$$\begin{aligned} \psi^{s,i} &\propto w^{s-1,i} \int \hat{l}(\boldsymbol{d}|\boldsymbol{x}, \boldsymbol{x}^{s-1,i})^{\gamma_s} g_{s-1}(\boldsymbol{x} - \boldsymbol{x}^{s-1,i}) \, \mathrm{d}\boldsymbol{x} \\ &= D \cdot w^{s-1,i} N(\boldsymbol{d}; \hat{\boldsymbol{d}}^{s,i}, \boldsymbol{S}^{s,i}) \qquad (14) \\ f^{s,i}(\boldsymbol{x}) &= \frac{\hat{l}(\boldsymbol{d}|\boldsymbol{x}, \boldsymbol{x}^{s-1,i})^{\gamma_s} g_{s-1}(\boldsymbol{x} - \boldsymbol{x}^{s-1,i})}{\int \hat{l}(\boldsymbol{d}|\boldsymbol{\xi}, \boldsymbol{x}^{s-1,i})^{\gamma_s} g_{s-1}(\boldsymbol{\xi} - \boldsymbol{x}^{s-1,i}) \, \mathrm{d}\boldsymbol{\xi}} \\ &= N(\boldsymbol{x}; \hat{\boldsymbol{x}}^{s,i}, \boldsymbol{P}^{s,i}) \qquad (15) \end{aligned}$$

where D is a normalising constant and

$$\hat{\boldsymbol{d}}^{s,i} = \boldsymbol{h}(\boldsymbol{x}^{s-1,i}, \boldsymbol{y}), \tag{16}$$

$$\boldsymbol{S}^{s,i} = \boldsymbol{H}(\boldsymbol{x}^{s-1,i}) \boldsymbol{\Sigma}_{s-1} \boldsymbol{H}(\boldsymbol{x}^{s-1,i})' + \sigma^2 \boldsymbol{I}_T / \gamma_s, \tag{17}$$

$$\hat{\boldsymbol{x}}^{s,i} = \boldsymbol{x}^{s-1,i} + \boldsymbol{K}^{s,i}(\boldsymbol{d} - \hat{\boldsymbol{d}}^{s,i}), \tag{18}$$

$$\boldsymbol{P}^{s,i} = \boldsymbol{\Sigma}_{s-1} - \boldsymbol{K}^{s,i} \boldsymbol{H}(\boldsymbol{x}^{s-1,i}) \boldsymbol{\Sigma}_{s-1}, \tag{19}$$

with $\boldsymbol{K}^{s,i} = \boldsymbol{\Sigma}_{s-1} \boldsymbol{H}(\boldsymbol{x}^{s-1,i})' (\boldsymbol{S}^{s,i})^{-1}$. The use of (14) and (15) will be referred to as measurement-directed progressive correction (MD-PC). The sample weights in Step 3(d) of Table 1 are

$$w^{s,i} = C \, [l(\boldsymbol{d}|\boldsymbol{x}^{s,i}) / \hat{l}(\boldsymbol{d}|\boldsymbol{x}^{s,i}, \boldsymbol{x}^{s-1,j^{s,i}})]^{\gamma_s} \tag{20}$$

4.3. Example

The operation of the progressive correction schemes is demonstrated below using a simple example with $K = 3$ anchor nodes and $M = 4$ unknown nodes. Node localisation is performed using $T = 8$ measurements generated via RIPS. B-PC and MD-PC are applied with $n = 100$ samples and $S = 6$ stages are used. Samples from the intermediate posterior distributions π_s, $s = 0, 2, 4, 6$ are shown in Figures 1 and 2 for B-PC and MD-PC, respectively. The samples generated by B-PC do not provide an accurate characterisation of the posterior distribution and the resulting location estimates are poor. By contrast, the samples generated by MD-PC gradually congregate about the true node positions as the likelihood sharpens. At the expense of additional computations, B-PC could be improved by increasing S and/or n.

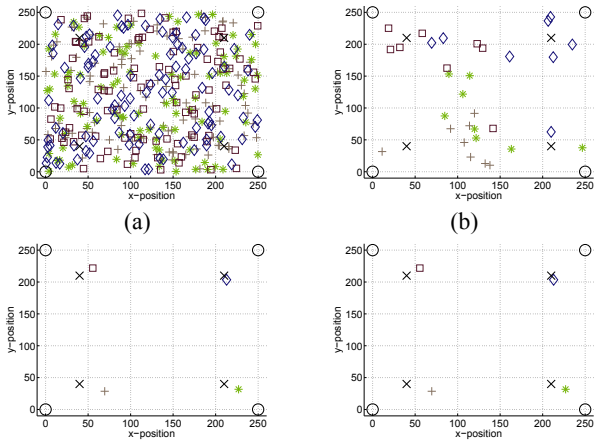

Figure 1: Localisation using B-PC. Samples from π_s are shown for (a) $s = 0$, (b) $s = 2$, (c) $s = 4$ and (d) $s = 6$.

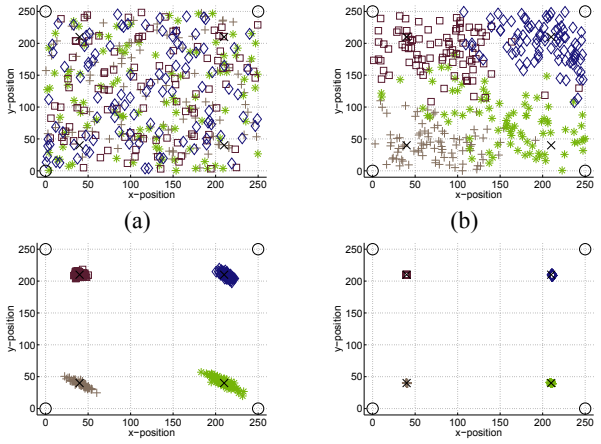

Figure 2: Localisation using MD-PC. Samples from π_s are shown for (a) $s = 0$, (b) $s = 2$, (c) $s = 4$ and (d) $s = 6$.

4.4. Discussion

The performance of the progressive correction schemes depends somewhat on the number S of steps and selection of the expansion factors $\gamma_1, \ldots, \gamma_S$. The adaptive scheme proposed in [7] can be used to select both the number and size of the corrections.

A natural prior density for the node localisation problem can be obtained by considering the connectivity of the unknown nodes

with the anchor nodes. Let V_k, $k = 1, \ldots, K$ denote the region of space within communication range of the kth anchor node. For $m = 1, \ldots, M$, let $C_m = \{k \in \{1, \ldots, K\} | \boldsymbol{x}_m \in V_k\}$ denote the set of anchor nodes which can communicate with the mth unknown node and $\bar{C}_m = \{1, \ldots, K\} - C_m$. Let U_A denote the uniform PDF over the region A. Then the prior PDF for the mth unknown node is U_{W_m} where $W_m = \bigcap_{k \in C_m} V_k - \bigcup_{j \in \bar{C}_m} V_j$.

5. PERFORMANCE ANALYSIS

A performance analysis has been conducted using Monte Carlo simulations. The basic scenario consists of four anchor nodes placed at the vertices of the square bounding the region $[0, 250]^2$ and unknown nodes randomly distributed in the region $[10, 240]^2$. Measurements are generated according to the RIPS, as described in Section 2. The variance of the additive noise in the RIPS measurements is set to $\sigma = 0.1\text{m}^2$. The Cramér-Rao bound (CRB), derived under the assumption that the node locations are deterministic parameters, is used as a benchmark.

In principle, RIPS measurements can be generated between all quartets of nodes which are within communication range of each other. This is impractical in large networks. The following is a brief summary of the method we use to select quartets which are used to generated RIPS measurements. First, the nodes are grouped into cliques such that all nodes in a clique can communicate. Second, three nodes in each clique are selected as pseudo-anchors. Third, measurements are generated between all node quartets which are composed of nodes in the same clique and include the selected pseudo-anchors. The procedure is similar to that of [2] but the method used here forms a larger number of cliques and consequently generates a larger number of measurements.

We first compare B-PC with MD-PC for $M = 10$ and $M = 20$ unknown nodes. The communication range of the motes is set to 220m to ensure idenitifiablity of the relatively small number of unknown nodes. Algorithm performance is measured by the RMS position error averaged over the nodes using 100 Monte Carlo realisations. The results for both progressive correction schemes are given in Table 2. When MD-PC is used a sample size of 200 is sufficient to give performance close to the CRB for $M = 10$ and $M = 20$. B-PC does not provide accuracy close to the CRB for either value of M even when 2000 samples are used.

Table 2: RMS position error of the Bayesian estimator computed with B-PC and MD-PC.

M	B-PC				MD-PC	CRB
	200	500	1000	2000	200	
10	5.91	3.63	2.71	1.62	0.52	0.54
20	3.91	3.05	2.69	2.31	0.35	0.30

In the second set of experiments we consider larger numbers of unknown nodes and a smaller communication range. Results are given only for MD-PC as B-PC does not provide useful estimates for any reasonable sample size in these scenarios. RMS position errors averaged over 100 realisations are shown in Table 3 for $M = 50$ and 100 and a communication range of 150m. Even with relatively small samples sizes, the algorithm performance is close to the CRB for both values of M.

Table 3: RMS position error of the Bayesian estimator computed with MD-PC.

M	Sample size			CRB
	200	500	1000	
50	0.73	0.60	0.55	0.49
100	0.55	0.51	0.45	0.40

6. CONCLUSIONS

A Bayesian algorithm for localisation of sensors in a sensor network has been proposed. Assuming a centralised architecture, an importance sampling technique was used to approximate the optimal Bayesian estimator. The proposed algorithm is a refinement of an existing method referred to as progressive correction. Numerical simulations showed that the refinement offers considerable improvement in performance and enables accurate localisation of a large number of nodes. Ongoing work includes adapting the algorithm for distributed estimation and examining identifiability issues.

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