Energy-Constrained MMSE Decentralized Estimation via Partial Sensor Noise Variance Knowledge†

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***Abstract*-This paper studies the energy-constrained MMSE decentralized estimation problem with the best-linear-unbiased-estimator fusion rule, under the assumptions that i) each sensor can only send a quantized version of its raw measurement to the fusion center (FC), and ii) exact knowledge of the sensor noise variance is unknown at the FC but only an associated statistical description is available. The problem setup relies on maximizing the reciprocal of the MSE averaged with respect to the prescribed noise variance distribution. While the considered design metric is shown to be highly nonlinear in the local sensor transmit energy (or bit loads), we leverage several analytic approximation relations to derive a associated tractable lower bound; through maximizing this bound a closed-form solution is then obtained. Our analytical results reveal that sensors with bad link quality are shut off to conserve energy, whereas the energy allocated to those active nodes is proportional to the individual channel gain. Simulation results are used to illustrate the performance of the proposed scheme.**

***Index Terms:* Decentralized estimation; Sensor networks; Energy efficiency; Quantization; Convex optimization.**

I. INTRODUCTION

Low energy/power cost is a critical concern for various application-specific designs of sensor networks [15], [16]. In the decentralized estimation scenario, wherein each sensor can transmit only a compressed version of its raw measurement to the fusion center (FC) owing to bandwidth and power limitations, several energy-efficient estimation schemes have been reported in the literature [1], [7], [10], [11], [13], [14]. Since the transmission energy is proportional to the message length [2], [13], all these works are formulated within a quantization bit assignment setup, with the optimal bit load determined via the knowledge of *instantaneous* local sensor noise characteristics, e.g., the noise variance if the fusion rule follows the best-linear-unbiased- estimator (BLUE) principle [5, chap. 6]. To maintain the estimation performance against the variation of sensing conditions, repeated update of the noise profile is needed: this inevitably incurs more training overhead and hence extra energy consumption. The design of distributed estimation algorithms independent of the instantaneous noise parameters remains an open problem [13, p-419]. Relying on partial noise variance knowledge in the form of the statistical distribution, the problem of minimizing total transmission energy under an allowable average distortion level (measured in terms of a mean-square-error (MSE) based criterion averaged with respect to some prescribed statistical distribution) is recently considered in [11].

This paper complements the study of [11] by addressing the counterpart problem: how to find the optimal bit load which minimizes the average distortion under a fixed total energy budget. The main contribution of the current work can be summarized as follows: (i) while the design metric, in the form of the reciprocal of the MSE averaged with respect to the distribution, is shown in [11] to be highly nonlinear in the sensor bit load, we leverage several analytic approximation relations to derive an associated tractable low bound, (ii) by maximizing this lower bound the problem can be further formulated in the form of convex optimization which yields a closed-form solution. Our analytic results reveal that, toward utmost estimation accuracy under a limited energy budget, sensors with bad link quality should be shut off, and energy allocated to those active nodes should be proportional to the individual channel gain; a similar energy conservation policy is also found in the previous works [7], [11], [13]. Numerical simulation evidences the effectiveness of the proposed scheme: it outperforms the uniform allocation strategy in an energy-limited environment.

II. SYSTEM SCENARIO

Consider a wireless sensor network, in which N spatially deployed sensors cooperate with a FC for estimating an unknown deterministic parameter θ. The local observation at the ith node is

$$x_i = \theta + n_i,\ \ 1 \le i \le N, \tag{2.1}$$

where n_i is a zero-mean measurement noise with variance σ_i^2. Due to bandwidth and power limitations each sensor quantizes its observation into a b_i-bit message, and then transmits this locally processed data to the FC to generate a final estimate of θ. In this paper the uniform quantization scheme with nearest- rounding [9], is adopted; the quantized message at the ith sensor can thus be modeled as

$$m_i = x_i + q_i,\ \ 1 \le i \le N, \tag{2.2}$$

where q_i is the quantization error uniformly distributed with zero mean and variance $\sigma_{q_i}^2 = R^2/(12\cdot 4^{b_i})$ [9], where $[-R/2, R/2]$ is the available signal amplitude range common to all sensors. The adopted quantizer model (2.2) and the uniform quantization error assumption, though being valid only

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when the number of quantization bits is sufficiently large [9], are widely used in the literature due to analytical tractability. Assuming that the channel link between the ith sensor and the FC is corrupted by a zero-mean additive noise v_i with variance σ_v^2 The received data from all sensor outputs can thus be expressed in a vector form as[a]

$$[y_1 \cdots y_N]^T = [1 \cdots 1]^T \theta + \underbrace{[n_1 \cdots n_N]^T}_{:=\mathbf{n}} + \underbrace{[q_1 \cdots q_N]^T}_{:=\mathbf{q}} + \underbrace{[v_1 \cdots v_N]^T}_{:=\mathbf{v}}, \quad (2.3)$$

where $(\cdot)^T$ denotes the transpose. This paper focuses on linear fusion rules for parameter recovery. More specifically, by assuming that the noise components $\{\mathbf{n}, \mathbf{q}, \mathbf{v}\}$ in (2.3) are mutually independent and the respective samples n_i's, q_i's, and v_i's are also independent across sensors, the parameter θ is retrieved via the BLUE [5, p-138] scheme via

$$\hat{\theta} = \left(\sum_{i=1}^{N} \frac{y_i}{\sigma_i^2 + \sigma_v^2 + \beta 4^{-b_i}} \right) \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2 + \sigma_v^2 + \beta 4^{-b_i}} \right)^{-1}; \quad (2.4)$$

the incurred MSE is thus [5, p-138]

$$E\left|\hat{\theta} - \theta\right|^2 = \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2 + \sigma_v^2 + \beta 4^{-b_i}} \right)^{-1}, \quad \beta := R^2/12. \quad (2.5)$$

A commonly used statistical description for sensing noise variance is [7], [13]:

$$\sigma_i^2 = \delta + \alpha z_i, \quad 1 \le i \le N, \quad (2.6)$$

where δ models the network-wide noise variance threshold, α controls the underlying variation from the nominal minimum, and $z_i \sim \chi_1^2$ are i.i.d. central Chi-Square distributed random variables each with degrees-of-freedom equal to one [6, p-24]. The proposed energy-constrained MMSE decentralized estimation scheme is based on the noise variance model (2.6) and is discussed next.

III. MAIN RESULTS

A. Problem Setup

We assume that the ith sensor sends the b_i-bit message m_i by using QAM with a constellation size 2^{b_i}. The consumed energy is thus [2], [13],

$$E_i = w_i\left(2^{b_i} - 1\right) \quad \text{for some } w_i, \ 1 \le i \le N; \quad (3.1)$$

the energy density w_i is defined as [2]

$$w_i := \rho d_i^{\kappa_i} \cdot \ln\left(2/P_b\right), \quad (3.2)$$

in which ρ is a constant depending on the noise profile, d_i is the distance between the ith node and the FC, κ_i is the ith path loss exponent, and P_b is the target bit error rate assumed common to all sensor-to-FC links. With (2.5) and (3.1), the energy allocated to the ith sensor is thus determined by the number of quantization bits b_i. For a fixed set of sensing noise variances σ_i^2's, the problem of MMSE decentralized estimation, under an allowable total energy budget E_T, can be formulated as

$$\text{Minimize } \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2 + \sigma_v^2 + \beta 4^{-b_i}} \right)^{-1}, \ \text{s.t. } \sum_{i=1}^{N} w_i\left(2^{b_i} - 1\right) \le E_T,$$
$$\text{and } b_i \in \mathbb{Z}_0^+, \ 1 \le i \le N, \quad (3.3)$$

or equivalently,

$$\text{Maximize } \sum_{i=1}^{N} \frac{1}{\sigma_i^2 + \sigma_v^2 + \beta 4^{-b_i}}, \text{ s.t. } \sum_{i=1}^{N} w_i\left(2^{b_i} - 1\right) \le E_T$$
$$\text{and } b_i \in \mathbb{Z}_0^+, \ 1 \le i \le N, \quad (3.4)$$

where $\mathbb{Z}_0^+$ denotes the set of all nonnegative integers. To obtain a universal solution irrespective of instantaneous noise conditions, we will consider the following optimization problem, in which the equivalent distortion cost function in (3.4) is instead averaged with respect to the noise variance statistic characterized in (2.6):

$$\text{Maximize } \int_{\mathbf{z}} \sum_{i=1}^{N} \frac{1}{\tilde{\delta} + \alpha z_i + \beta 4^{-b_i}} p(\mathbf{z}) \, d\mathbf{z},$$
$$\text{s.t. } \sum_{i=1}^{N} w_i\left(2^{b_i} - 1\right) \le E_T, \ b_i \in \mathbb{Z}_0^+, \ 1 \le i \le N, \quad (3.5)$$

where $\tilde{\delta} := \delta + \sigma_v^2$ and $\mathbf{z} := [z_1 \cdots z_N]^T$ with $p(\mathbf{z})$ denoting the associated distribution. To solve (3.5), the first step is to find an analytic expression of the equivalent mean MSE metric. Since $z_i \sim \chi_1^2$ is i.i.d. and $p_{\chi_1^2}(z) = \frac{1}{\sqrt{2\pi z}} \exp(-z/2) u(z)$ [6, p-24], where $u(z)$ denotes the unit step function, it can be shown that (see [12] for a proof)

$$\int_{\mathbf{z}} \sum_{i=1}^{N} \frac{1}{\tilde{\delta} + \alpha z_i + \beta 4^{-b_i}} p(\mathbf{z}) \, d\mathbf{z} = \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{N} \int_0^\infty \frac{e^{-z_i/2}}{\left(\alpha z_i + \tilde{\delta} + \beta 4^{-b_i}\right)\sqrt{z_i}} \, dz_i$$
$$= \frac{2\pi \cdot e^{(\tilde{\delta} + \beta 4^{-b_i})/2\alpha} \cdot Q\left(\sqrt{\left(\tilde{\delta} + \beta 4^{-b_i}\right)/\alpha}\right)}{\sqrt{\alpha\left(\tilde{\delta} + \beta 4^{-b_i}\right)}}, \quad (3.6)$$

where $Q(x) := \int_x^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \, dt$ is the Gaussian tail function. Based on (3.6), problem (3.5) can be equivalently rewritten as

$$\text{Maximize } \sqrt{2\pi} \cdot \sum_{i=1}^{N} \frac{e^{(\tilde{\delta} + \beta 4^{-b_i})/2\alpha} \cdot Q\left(\sqrt{\left(\tilde{\delta} + \beta 4^{-b_i}\right)/\alpha}\right)}{\sqrt{\alpha\left(\tilde{\delta} + \beta 4^{-b_i}\right)}},$$
$$\text{under } \sum_{i=1}^{N} w_i\left(2^{b_i} - 1\right) \le E_T, \text{ and } b_i \in \mathbb{Z}_0^+, \ \forall\, i. \quad (3.7)$$

The optimization problem (3.7) appears rather formidable to tackle because the cost function is highly nonlinear in b_i. In what follows we will propose an alternative formulation which is more tractable and can yield an analytic solution.

B. Alternative Formulation

The proposed approach is grounded on the following approximation to $Q(\cdot)$ function [8, p-115]:

$$Q(x) \approx \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-x^2/2}}{(1 - \pi^{-1})x + \pi^{-1}\sqrt{x^2 + 2\pi}} \right]; \quad (3.8)$$

the approximation (3.8) is quite accurate since the peak relative error is less than 1.2% for $x \ge 0$, and is almost identical to zero whenever $x \ge 5$. Based on (3.8) together with some straightforward manipulations, the cost function in (3.7) can be well approximated by

a. As in [1], [7], and [13] we assume orthogonal channel access among all the sensor-to-fusion links, which can be realized via, e.g., TDMA or CDMA with orthogonal spreading.

$$\sqrt{2\pi}\cdot\sum_{i=1}^{N}\frac{e^{(\tilde{\delta}+\beta 4^{-b_i})/2\alpha}\cdot Q\left(\sqrt{\left(\tilde{\delta}+\beta 4^{-b_i}\right)/\alpha}\right)}{\sqrt{\alpha\left(\tilde{\delta}+\beta 4^{-b_i}\right)}}$$

$$\approx\sum_{i=1}^{N}\frac{1}{(1-\pi^{-1})\left(\tilde{\delta}+\beta 4^{-b_i}\right)+\pi^{-1}\sqrt{\left(\tilde{\delta}+\beta 4^{-b_i}\right)^2+2\pi\alpha\left(\tilde{\delta}+\beta 4^{-b_i}\right)}}. \tag{3.9}$$

The main advantage of the approximation (3.9) is that it can lead to an associated lower bound in a more tractable form; through otherwise maximizing this lower bound we can eventually obtain a closed-form solution. More specifically, it can be shown that (see [12])

$$\sum_{i=1}^{N}\frac{1}{(1-\pi^{-1})\left(\tilde{\delta}+\beta 4^{-b_i}\right)+\pi^{-1}\sqrt{\left(\tilde{\delta}+\beta 4^{-b_i}\right)^2+2\pi\alpha\left(\tilde{\delta}+\beta 4^{-b_i}\right)}}$$

$$\geq\sum_{i=1}^{N}\frac{1}{(1-\pi^{-1})\left(\tilde{\delta}+\beta 4^{-b_i}\right)+\pi^{-1}\left[\left(\tilde{\delta}+\beta 4^{-b_i}\right)+\pi\alpha\right]}$$

$$=\sum_{i=1}^{N}\frac{1}{\left(\tilde{\delta}+\beta 4^{-b_i}\right)+\alpha}=\sum_{i=1}^{N}\frac{4^{b_i}}{\beta+(\alpha+\tilde{\delta})4^{b_i}}. \tag{3.10}$$

Based on (3.10) we will instead focus on maximizing the lower bound, and thus reformulate the optimization problem as

$$\text{Maximize}\ \sum_{i=1}^{N}\frac{4^{b_i}}{\beta+(\alpha+\tilde{\delta})4^{b_i}},\ \text{s.t.}\ \sum_{i=1}^{N}w_i\left(2^{b_i}-1\right)\leq E_T,\ \text{and}$$

$$b_i\in\mathbb{Z}_0^+,\ 1\leq i\leq N. \tag{3.11}$$

To facilitate analysis we first observe that, since $b_i\in\mathbb{Z}_0^+$, it follows $\sum_{i=1}^{N}w_i\left(2^{b_i}-1\right)\leq\sum_{i=1}^{N}w_i\left(4^{b_i}-1\right)$: this implies we can replace the total energy constraint in (3.11) by the following one without violating the overall energy budget requirement:

$$\sum_{i=1}^{N}w_i\left(4^{b_i}-1\right)\leq E_T. \tag{3.12}$$

With the aid of (3.12) and by performing a change of variable with $B_i:=4^{b_i}-1$, the optimization problem becomes

$$\text{Maximize}\ \sum_{i=1}^{N}\frac{B_i+1}{(\alpha+\beta+\tilde{\delta})+(\alpha+\tilde{\delta})B_i},$$

$$\text{subject to}\ \sum_{i=1}^{N}w_iB_i\leq E_T,\ \text{and}\ B_i\geq 0,\ 1\leq i\leq N. \tag{3.13}$$

In (3.13), the intermediate variable B_i is relaxed to be a nonnegative real number so as to render the problem tractable; once the optimal real-valued B_i (and hence b_i) is computed, the associated bit loads can be obtained through upper integer rounding, as in [7], [11], [13]. The major advantage of the alternative problem formulation (3.13) is that it admits the form of convex optimization and can moreover lead to a closed-form solution, as is shown next.

C. Optimal Solution

Based on the standard Lagrainge techniques, the optimal solution to (3.13) can be obtained as follows (see [12] for detailed proof). Let us assume $w_1\geq w_2\geq\cdots\geq w_N$ without loss of generality, and define the function

$$f(K):=\frac{E_T\left(1+\dfrac{\beta}{\alpha+\tilde{\delta}}\right)^{-1}+\sum_{j=K}^{N}w_j}{\sqrt{w_K}\sum_{j=K}^{N}\sqrt{w_j}},\quad 1\leq K\leq N. \tag{3.14}$$

Let $1\leq K_1\leq N$ be the unique integer such that $f(K_1-1)<1$ and $f(K_1)\geq 1$; if $f(K)\geq 1$ for all $1\leq K\leq N$, then simply set $K_1=1$ (the existence and uniqueness of such K_1 when otherwise is shown in [12]). Then the optimal solution pair $\left(\lambda^{opt},B_i^{opt}\right)$ is given by

$$\sqrt{\lambda^{opt}}=\frac{\sqrt{\beta}}{\alpha+\tilde{\delta}}\left(\sum_{j=K_1}^{N}\sqrt{w_j}\right)\left(E_T+\left(1+\frac{\beta}{\alpha+\tilde{\delta}}\right)\sum_{j=K_1}^{N}w_j\right)^{-1}, \tag{3.15}$$

and

$$B_i^{opt}=\begin{cases}0, & 1\leq i\leq K_1-1,\\ \dfrac{1}{\alpha+\tilde{\delta}}\sqrt{\dfrac{\beta}{\lambda^{opt}w_i}}-\left(1+\dfrac{\beta}{\alpha+\tilde{\delta}}\right), & K_1\leq i\leq N.\end{cases} \tag{3.16}$$

With $B_i=4^{b_i}-1$ and $\tilde{\delta}=\delta+\sigma_v^2$, the optimal bit load b_i^{opt} can be directly obtained from (3.16); the resultant average distortion level is thus (cf. (3.7))

$$\overline{MSE}=\left(\sqrt{2\pi}\cdot\sum_{i=K_1}^{N}\frac{e^{(\delta+\sigma_v^2+\beta 4^{-b_i^{opt}})/2\alpha}\cdot Q\left(\sqrt{\left(\delta+\sigma_v^2+\beta 4^{-b_i^{opt}}\right)/\alpha}\right)}{\sqrt{\alpha\left(\delta+\sigma_v^2+\beta 4^{-b_i^{opt}}\right)}}\right)^{-1} \tag{3.17}$$

IV. DISCUSSIONS AND SIMULATION

1. We note that the minimal achievable average MSE is attained whenever all the raw sensor measurements with infinite-precision are available to the FC (i.e., the case when $b_i=\infty$, $1\leq i\leq N$). Hence, by setting $b_i=\infty$ in the mean MSE formula specified in (3.7), we have the following performance bound

$$MSE_{\min}=\left[Ne^{(\delta+\sigma_v^2)/2\alpha}Q\left(\sqrt{(\delta+\sigma_v^2)/\alpha}\right)\sqrt{\frac{2\pi}{\alpha(\delta+\sigma_v^2)}}\right]^{-1}. \tag{4.1}$$

   Formula (4.1) reveals the impacts of the noise model parameters α and δ on the estimation performance. Specifically, it is easy to see from (4.1) that the minimal MSE increases with α: this implies the estimation accuracy degrades as the sensing environment becomes more and more inhomogeneous. Furthermore, it can be checked that $MSE_{\min}$ also increases with the minimal noise power threshold δ. This is reasonable since a large δ implies poor measurement quality of *all* sensor data, and hence a less accurate parameter estimate. We note that, although these facts are inferred based on the idealized distortion measure (4.1), a similar tendency is also observed for $\overline{MSE}$ in (3.17) attained with sensor data quantization (see the numerical results below).
2. Recall from (3.2) that the energy density factor w_i is proportional to the path loss gain d_i^{κ} (assuming $\kappa_i=\kappa$ throughout all links). Large values of w_i, therefore, correspond to sensors deployed far away from the FC (with large d_i), usually with poor background channel gains. In light of this point, the proposed optimal solution (3.16) is intuitively attractive: sensors associated with the (K_1-1) th largest w_i 's are turned off to conserve energy. We note that a similar energy conservation strategy via shutting off sensors alone poor channel links is also found

in [7], [11], [13]. Also, we further note from (3.16) that, for those active nodes, the assigned message length is inversely proportional to $\sqrt{w_i}$: this is intuitively reasonable since sensors with better link conditions should be allocated with more bits (energy) to improve the estimation accuracy.

3. We compare the simulated performance of the proposed solution (3.16) against the uniform energy allocation scheme with bit load determined through

$$w_i\left(2^{b_i}-1\right)=E_T/N\,,\quad 1\le i\le N\,. \tag{4.2}$$

In each run we simply choose $w_i = d_i^{\kappa}$ with $\kappa = 2$, and d_i's are uniformly drawn from the interval $[1,10]$ as in [13]. In the following experiments we set the number of sensors to be $N = 200$, link noise $\sigma_v^2 = 0.05$, and consider three different levels of total energy: $E_T = \gamma \sum_{i=1}^{N} w_i$ with $\gamma = 0.25,\ 1,\ 3$, which respectively correspond to the low, medium, and high energy regimes. With fixed $\delta = 2$, Figure 1-(a) shows the computed mean MSE as α varies from 0 to 8, whereas Figure 1-(b) depicts the MSE for fixed $\alpha = 2$ and $0.5 \le \delta \le 8$. The results show that, as expected, the estimation accuracy improves as E_T increases. Also, the proposed solution (3.16) outperforms (4.2), especially when E_T is small; it is thus more effective in an energy-limited environment. We finally note that the simulated MSE increases with both α and δ: this coincides with the asserted facts in the previous discussions. Figure 2 further depicts the histogram of the computed bit loads; it appears that a large fraction of the active nodes are assigned with one or two quantization bits (hence with BPSK or 4-QAM modulations adopted).

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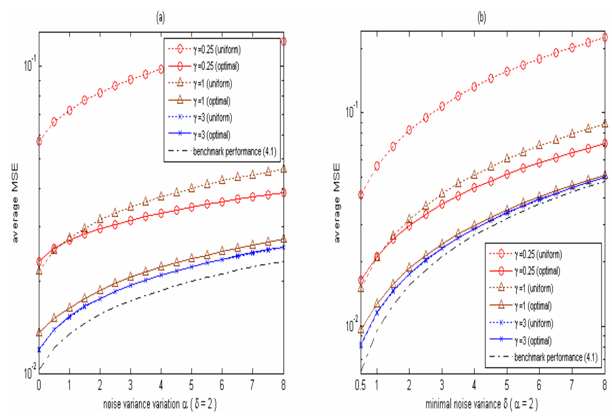

Figure 1. Performance comparison of the proposed solution (3.16) with the uniform allocation scheme (4.2).

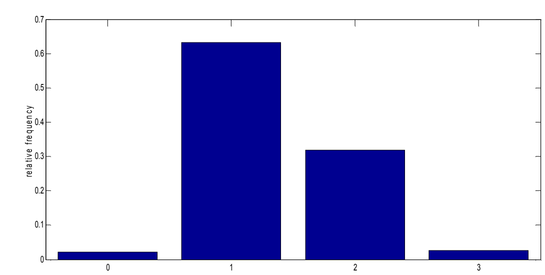

Figure 2. Histogram of the quantization bits.