A SENSOR SELECTION APPROACH FOR TARGET TRACKING IN SENSOR NETWORKS WITH QUANTIZED MEASUREMENTS

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ABSTRACT

This paper extends our earlier work on sensor selection [1]. We are now focusing on a more challenging problem of how to effectively utilize quantized sensor data for target tracking in sensor networks by considering sensor selection problems with quantized data. A subset of sensors are dynamically selected to optimize the tracking performance. The one-step-look-ahead *posterior* Cramér-Rao Lower Bound (CRLB) on the state estimation error is proposed as the sensor selection criterion. Particle filtering method is employed to compute the *posterior* CRLB, as well as to estimate the target state. Simulation results show that the proposed *posterior* CRLB based method outperforms the one based on information theoretic measures.

***Index Terms*—** Target Tracking, Sensor Networks, Particle Filters, Quantization, *posterior* CRLB

1. INTRODUCTION

Our previous work [1] proposed a method for selecting the optimal set of sensors to participate in target tracking in sensor networks. In [1], it was assumed that the information provided by each sensor was complete and perfect. Thus the communication cost was high requiring substantial sensor node energy consumption. However, in real systems, sensor data are communicated over bandlimited channels. So it is critical to consider issues of transmission of quantized data from the sensors to the fusion center and eventually the fusion of such quantized data.

There exist some publications on tracking in sensor networks based on quantized sensor measurements. In [2], the authors design a new framework for target tracking using quantized data transmitted over noisy channel between sensors and fusion center. In [3], the authors propose a delta-modulation based intelligent quantizer for measurement fusion. Each sensor dynamically designs the quantizer according to the updated target state estimate sent from the fusion center. In this paper, we focus on the problem of selecting a subset of sensors to maximize the tracking performance with quantized sensor measurements. It is assumed that each sensor employs the same uniform quantization scheme. There exist many sensor selection algorithms. Among them information driven [4] method is the most popular one. The main idea is to select the sensors that can provide the most useful information, which is measured by entropy or mutual information. However, if we select more than one sensor, the information-theoretic measure is decoupled for each sensor, implies that the total information measure is the sum of each individual senor's information measure. Hence, the information-theoretic measure based method is a greedy method in that it chooses a set of sensors with the highest stand-alone information measures, rather than a set of sensors which collectively give the most information regarding the target state. In this paper, we propose a *posterior* CRLB based sensor selection approach. There are two main motivating reasons for our approach. First, the tracking accuracy in terms of the MSE is bounded below by CRLB [5]. This lower bound gives an indication of performance limits, so it can be used as a criterion for sensor selection. Second, the *posterior* CRLB based method chooses a set of sensors which collectively minimize the PCRLB on the estimation errors.

2. SYSTEM MODELS

2.1. Target Motion Model

We consider a single target moving in a 2-D Cartesian coordinate plane according to a dynamic white noise acceleration model [6]:

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{v}_k \tag{1}$$

where constant parameter $\mathbf{F}$ models the state kinematics, the target state at time k is defined as $\mathbf{x}_k = [x_k\ \dot{x}_k\ y_k\ \dot{y}_k]^T$, x_k and y_k denote the target position, $\dot{x}_k$ and $\dot{y}_k$ denote the velocities. $\mathbf{v}_k$ is white Gaussian noise with covariance matrix $\mathbf{Q}$.

2.2. Sensor Measurement Model

In this paper, we assume that a large number of homogeneous bearing only sensors are randomly deployed in a two dimensional Cartesian coordinate plane. There exists a fusion center that is responsible for collecting information from each sensor and providing the estimate of the target state. The fusion center has knowledge about the individual sensors, such as their positions, measurement accuracy and quantization scheme. At each time, only a small number of sensors are activated to perform the sensing task and provide their quantized measurements to the fusion center. The measurement model is given by

$$\theta_k^j = h(\mathbf{x}_k) + \mathbf{w}_k^j = tan^{-1}\left(\frac{y_k - y^{s_j}}{x_k - x^{s_j}}\right) + \mathbf{w}_k^j \tag{2}$$

$$\mathbf{z}_k^j = \mathcal{Q}(\theta_k^j \bmod 2\pi) \tag{3}$$

where θ_k^j is the original measurement from sensor j with additive white Gaussian noise $\mathbf{w}_k^j$, whose variance is parameterized as $\mathbf{R}$. x^{s_j} and y^{s_j} represent the corresponding coordinates of sensor j. The remainder after θ_k^j is divided by 2π is sent to the quantizer. $\mathcal{Q}$ is a m-bit uniform quantizer on $(-\pi, \pi)$.

3. POSTERIOR CRLB WITH QUANTIZED MEASUREMENTS

Let $\hat{\mathbf{x}}_k$ be an unbiased estimator of the state vector $\mathbf{x}_k$. The covariance of $\mathbf{x}_k$ is bounded below by the recursive PCRLB, which is defined to be the inverse of the Fisher Information Matrix (FIM) J_k

$$P_k = E\{[\hat{\mathbf{x}}_k - \mathbf{x}_k][\hat{\mathbf{x}}_k - \mathbf{x}_k]^T\} \geq J_k^{-1} \tag{4}$$

$$J_k = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \log p(\mathbf{x}_k, \mathbf{z}_k)\} \tag{5}$$

where J_k^{-1} is the *posterior* CRLB matrix and $\Delta_\Psi^\Theta = \nabla_\Psi \nabla_\Theta^T$. ∇ is the first-order partial derivative operator defined as

$$\nabla_{\mathbf{x}} = \left[\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_r}\right]^T \tag{6}$$

In [5], Tichavske et al. provide an elegant recursive equation to calculate the sequential FIM J_k

$$J_{k+1} = D_k^{22} - D_k^{21}(J_k + D_k^{11})^{-1} D_k^{12} \tag{7}$$

where

$$D_k^{11} = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \text{log} p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} \tag{8}$$

$$D_k^{12} = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_{k+1}} \text{log} p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} \tag{9}$$

$$D_k^{21} = E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_k} \text{log} p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} = (D_k^{12})^T \tag{10}$$

$$\begin{aligned} D_k^{22} &= E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_{k+1}} \text{log} p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} + \\ &\quad E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_{k+1}} \text{log} p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})\} \\ &= D_k^{22,a} + D_k^{22,b} \end{aligned} \tag{11}$$

Note that all the above expectations are taken with respect to the joint probability density function (PDF) $p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1})$, where $\mathbf{X}^{k+1} \triangleq \mathbf{x}_{0:k+1}$ and $\mathbf{Z}^{k+1} \triangleq \mathbf{z}_{1:k+1}$ denote all the states and observations up to time $k+1$.

The recursion of Equation(7) starts from an initial FIM J_0, which can be calculated from the *a priori* PDF $p(x_0)$.

$$J_0 = E_{p(\mathbf{x}_0)}\{-\Delta_{\mathbf{x}_0}^{\mathbf{x}_0} \text{log} p(\mathbf{x}_0)\} \tag{12}$$

For the linear motion model and nonlinear measurement model adopted in this paper, the Equations (8)∼(11) become

$$D_k^{11} = \mathbf{F}^T \mathbf{Q}^{-1} \mathbf{F} \tag{13}$$

$$D_k^{12} = (D_k^{12})^T = -\mathbf{F}^T \mathbf{Q}^{-1} \tag{14}$$

$$D_k^{22} = \mathbf{Q}^{-1} + D_k^{22,b} \tag{15}$$

In general, it would be a difficult task to calculate $D_k^{22,b}$, as well as dynamic estimation with the quantized measurements. But particle filter techniques [7] provide us a powerful tool to circumvent the above difficulties. Each PDF in the expectations can be represented as a set of samples with associated weights. With the system models described in Section 2 and only one sensor j activated at any moment , $D_k^{22,b}$ can be expressed as:

$$D_k^{22,b} = \begin{bmatrix} M_{1,1}^k & 0 & M_{1,3}^k & 0 \\ 0 & 0 & 0 & 0 \\ M_{3,1}^k & 0 & M_{3,3}^k & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

where

$$M_{1,1}^k = -E_{p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1})}\left[\frac{\partial^2 \log p(\mathbf{z}_{k+1}^j|\mathbf{x}_{k+1})}{\partial x_{k+1}^2}\right] \tag{17}$$

$$M_{1,3}^k = M_{3,1}^k = -E_{p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1})}\left[\frac{\partial^2 \log p(\mathbf{z}_{k+1}^j|\mathbf{x}_{k+1})}{\partial x_{k+1} \partial y_{k+1}}\right] \tag{18}$$

$$M_{3,3}^k = -E_{p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1})}\left[\frac{\partial^2 \log p(\mathbf{z}_{k+1}^j|\mathbf{x}_{k+1})}{\partial y_{k+1}^2}\right] \tag{19}$$

Using the properties of a first-order Markovian system, the joint PDF can be expressed as the following iterative equation

$$p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1}) = p(\mathbf{X}^k, \mathbf{Z}^k) \cdot p(\mathbf{x}_{k+1}|\mathbf{x}_k) \cdot p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1}) \tag{20}$$

With this property, (17) can be simplified as

$$M_{1,1}^k = E_{p(\mathbf{x}_{k+1}|\mathbf{x}_k) p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})}\left\{\frac{\left[\frac{\partial p(\mathbf{z}_k^j|\mathbf{x}_{k+1})}{\partial x_{k+1}}\right]^2}{p^2(\mathbf{z}_{k+1}^j|\mathbf{x}_{k+1})}\right\} \tag{21}$$

Simplification of Equations (18) and (19) may be carried out in a similar manner

$$M_{1,3}^{k} = E_{p(\mathbf{x}_{k+1}|\mathbf{x}_k)p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})}\left[\frac{\frac{\partial p(\mathbf{z}_{k+1}^{j}|\mathbf{x}_{k+1})}{\partial x_{k+1}}\frac{\partial p(\mathbf{z}_{k+1}^{j}|\mathbf{x}_{k+1})}{\partial y_{k+1}}}{p^2(\mathbf{z}_{k+1}^{j}|\mathbf{x}_{k+1})}\right] \tag{22}$$

$$M_{3,3}^{k} = E_{p(\mathbf{x}_{k+1}|\mathbf{x}_k)p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})}\left\{\frac{\left[\frac{\partial p(\mathbf{z}_{k+1}^{j}|\mathbf{x}_{k+1})}{\partial y_{k+1}}\right]^2}{p^2(\mathbf{z}_{k+1}^{j}|\mathbf{x}_{k+1})}\right\} \tag{23}$$

When the periodicity of bearings around 2π is taken into account, the likelihood function for each quantization level l can be found by

$$\begin{aligned} Pr\{\mathbf{z}_{k+1}^{j} = l|\mathbf{x}_{k+1}\} = \sum_{n=-\infty}^{\infty} & Pr\{(l-1)\eta + 2n\pi \\ & < \tan^{-1}\frac{\Delta y_{k+1}^{S_j}}{\Delta x_{k+1}^{S_j}} + \mathbf{w}_{k+1}^{j} < l\eta + 2n\pi\} \end{aligned} \tag{24}$$

$$\begin{aligned} Pr(\mathbf{z}_{k+1}^{j} = l|\mathbf{x}_{k+1}) = \sum_{n=-\infty}^{\infty} & \left\{\Phi\left(\frac{l\eta + \beta_{k+1,n}^{S_j}}{\sigma}\right) - \right. \\ & \left. \Phi\left(\frac{(l-1)\eta + \beta_{k+1,n}^{S_j}}{\sigma}\right)\right\} \end{aligned} \tag{25}$$

where $\Delta y_{k+1}^{S_j} \triangleq y_{k+1} - y^{S_j}$, and $\Delta x_{k+1}^{S_j} \triangleq x_{k+1} - x^{S_j}$, $\beta_{k+1,n}^{S_j} = 2n\pi - \tan^{-1}\frac{\Delta y_{k+1}^{S_j}}{\Delta x_{k+1}^{S_j}}$, $l = -L/2+1, -L/2+2, \dots, L/2$, and $L = 2^m$. $\eta = 2\pi/L$, σ is the standard deviation of the measurement noise, Φ is a cumulative Gaussian distribution with mean 0 and variance 1.

The partial derivatives in above equations can be found by

$$\frac{\partial p(\mathbf{z}_{k+1}^{j}|\mathbf{x}_{k+1})}{\partial x_{k+1}} = \frac{\Delta y_{k+1}^{S_j}\sum_{n=-\infty}^{\infty}\gamma(k+1,n,l,S_j)}{\sqrt{2\pi}\sigma\left[(\Delta x_{k+1}^{S_j})^2 + (\Delta y_{k+1}^{S_j})^2\right]} \tag{26}$$

$$\frac{\partial p(\mathbf{z}_{k+1}^{j}|\mathbf{x}_{k+1})}{\partial y_{k+1}} = \frac{-\Delta x_{k+1}^{S_j}\sum_{n=-\infty}^{\infty}\gamma(k+1,n,l,S_j)}{\sqrt{2\pi}\sigma\left[(\Delta x_{k+1}^{S_j})^2 + (\Delta y_{k+1}^{S_j})^2\right]} \tag{27}$$

where

$$\gamma(k+1,n,l,S_j) \triangleq e^{-\frac{l\eta + \beta_{k+1,n}^{S_j}}{\sigma}} - e^{-\frac{(l-1)\eta + \beta_{k+1,n}^{S_j}}{\sigma}}$$

Due to quantization, the likelihood function $p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})$ becomes a probability mass function and the PDF $p(\mathbf{x}_{k+1}|\mathbf{x}_k)$ can be represented approximately by propagating the samples $\{\mathbf{x}_k^{(i)}\}$ from time k to $k+1$ according to the particle filter theory.

$$p(\mathbf{x}_{k+1}|\mathbf{x}_k) \approx \sum_{i=1}^{N}\omega_k^{(i)} \cdot \delta_{\mathbf{x}_{k+1}^{(i)}}(\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^{(i)}) \tag{28}$$

Therefore the integrals due to expectation can be converted into summation and further can be evaluated approximately by particle filters only if we know the current PDF $p(\mathbf{x}_k|\mathbf{Z}^k)$, which can be easily derived by particle filter theory and represented approximately by the following equation

$$p(\mathbf{x}_k|\mathbf{Z}^k) \approx \sum_{i=1}^{N}\omega_k^{(i)} \cdot \delta_{\mathbf{x}_k^{(i)}}(\mathbf{x}_k - \mathbf{x}_k^{(i)}) \tag{29}$$

where N is the number of particles.

Till now, we have shown how to compute the FIM for the case of one activated sensor. With multiple sensors activated at each moment, the total FIM is just the sum of each individual FIM under the assumption that the measurements taken by sensors are independent of each other, which is usually considered to be true. The *posterior* CRLB can be obtained by simply taking the inverse of the FIM.

4. POSTERIOR CRLB BASED SENSOR SELECTION

The *posterior* CRLB gives an indication of performance bounds, and no unbiased estimators can outperform it in terms of MSE. Usually people are more concerned with the target position. So we choose the summation of the position bound along each axis as the cost function for time $k+1$

$$\mathcal{C}_{k+1} = J_{k+1}^{-1}(1,1) + J_{k+1}^{-1}(3,3) \tag{30}$$

where the $J_{k+1}^{-1}(1,1)$ and $J_{k+1}^{-1}(3,3)$ are the bounds on the MSE corresponding to x_{k+1} and y_{k+1} respectively. Assume we choose a subset consisting of L_s^k sensors from the total L_s candidates on every tracking snapshot at time k, where L_s^k can change over time. Those sensors that collectively minimize the above cost function will be activated at the next time $k+1$. In this paper, we use the optimal enumerative search method to determine the optimal combination of sensors, which minimizes the cost function.

$$L_s^{k+1,*} \triangleq \underset{L_s^{k+1}\subset\mathcal{S}}{\text{argmin}}\, \mathcal{C}_{k+1}(L_s^{k+1}) \tag{31}$$

where $\mathcal{S}$ denotes the set containing all the sensors.

5. EXPERIMENTAL RESULTS

In this section, we evaluate the performance of the proposed algorithm by tracking a single target moving through a 500×500 field, where 20 bearing-only sensors are randomly deployed. The measurement noise variance is set to $R = 0.1$. F and Q are chosen as follows respectively

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, Q = q\begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} & 0 & 0 \\ \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \frac{T^3}{3} & \frac{T^2}{2} \\ 0 & 0 & \frac{T^2}{2} & T \end{bmatrix} \tag{32}$$

where $T = 2$ is the time interval between two consecutive sampling points, and $q = 0.1$ is the process noise factor.

For comparison purposes, we also implement two other methods with quantized measurements: 1) Nearest neighbor: the sensors that have the closest distance to the predicted position of the target will be selected, and 2) Information driven: the sensors that have the minimal expected posterior entropies will be selected. For each simulation, the number of particles is $N = 200$, and 50 Monte Carlo runs are preformed.

Tracking results by selecting two sensors at any moment are shown in Figure 1. A coarse quantization of measurements with $m = 4$ is adopted. Figure 2 and Figure 3 show the MSEs of the estimate of target position in two dimensional coordinates respectively. From the simulation, we can see that our proposed method achieves more accurate tracking results.

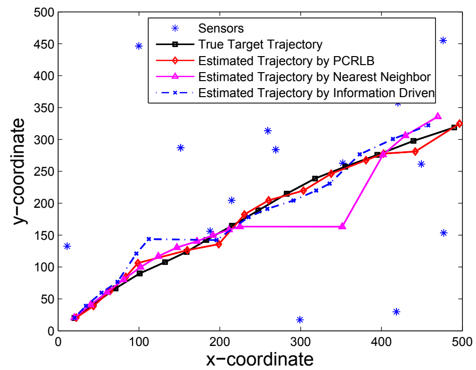

Fig. 1. True and the estimated target trajectories using different sensor selection methods.

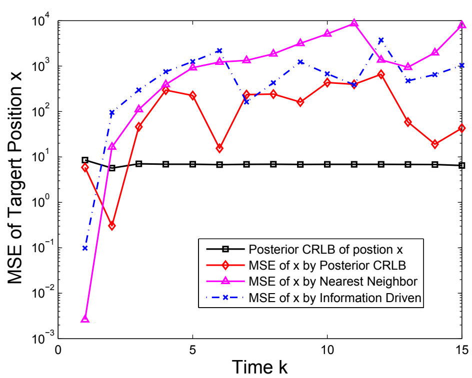

Fig. 2. Comparison of *posterior* CRLB with MSEs for the x-coordinate using different sensor selection methods.

6. CONCLUSIONS

In this paper, we considered a sensor selection problem for tracking a single target in sensor networks with quantized measurements. The one-step look ahead *posterior* CRLB is approximated recursively by using a particle filter that is used

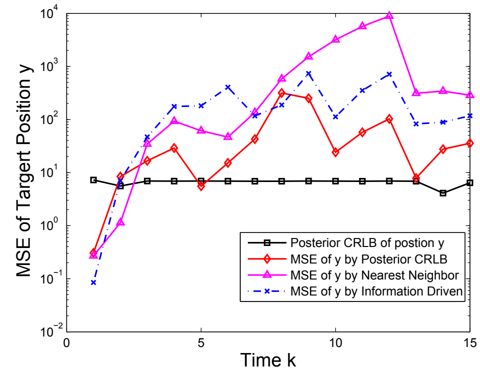

Fig. 3. Comparison of *posterior* CRLB with MSEs for the y-coordinate using different sensor selection methods.

for dynamic estimation as well. Sensors, that collectively minimize the cost function established on *posterior* CRLB, are activated, while other sensors remain in the idle state to save energy. We compared our method with the one based on the information-theoretic measure. Simulation results demonstrate the significantly improved performance of our approach.

7. REFERENCES

[1] L. Zuo, R. Niu, and P. K. Varshney, "Posterior CRLB based sensor selection for target tracking in sensor networks," Proc. ICASSP, Apr. 2007, vol. 2, pp. 1041–1044.

[2] O. Ozdemir, R. Niu, and P.K. Varshney, "Channel aware particle filtering for tracking in sensor networks," Proc. of the Thirty-Ninth Annual Asilomar Conference on Signals, Systems, and Computers, Oct. 2006.

[3] Y. Ruan, P. Willett, and A. Marrs, "Fusion of quantized measurements via particle filtering," Proceedings of the 2003 Aerospace Conference, Mar 2003, pp. 1967–1978.

[4] F. Zhao, J. Shin, and J. Reich, "Information-driven dynamic sensor collaboration," IEEE Signal Processing Magazine, 2002, vol. 19, pp. 61–72.

[5] P. Tichavsky, C. H. Muravchik, and A. Nehorai, "Posterior Cramér-Rao bounds for discrete-time nonlinear filtering," IEEE Trans. on Signal Processing, May 1998, pp. 1386–1396.

[6] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation*, John Wiley and Sons, 2001.

[7] A. Doucet, N. de Freitas, and N. Gordon, *Sequential Monte Carlo Methods in Practice*, Springer-Verlag, 2001.