MAXIMUM LIKELIHOOD ESTIMATION OF CLOCK PARAMETERS FOR SYNCHRONIZATION OF WIRELESS SENSOR NETWORKS

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ABSTRACT

Clock synchronization plays a fundamental role in the operation of wireless sensor networks. This paper derives the Maximum Likelihood Estimator (MLE) of clock phase offset and skew between two nodes that assume a two-way timing message exchange mechanism to achieve synchronization. The MLE is derived under quite general conditions assuming an exponential distribution for the variable network delays and the presence of an unknown deterministic component due to the fixed network delays.

***Index Terms*—** Synchronization, Maximum Likelihood Estimation.

1. INTRODUCTION

Today WSNs are increasingly becoming the target of active research due to the several applications they offer in military and civilian domains [1]. Clock synchronization in WSNs is important for various tasks such as coordinated sleep and wakeup modes for energy conservation, object tracking, data fusion, etc. The protocols proposed for synchronizing clocks over the network depend on some kind of timing message exchange among the nodes. A popular two-way timing message exchange mechanism (depicted in Fig. 1) has been advocated by several important synchronization protocols (NTP, TPSN, see e.g., [2], [3] and the references cited herein). However, the existing contributions concentrated only on estimating the clock phase offset within the context of the two-way timing message exchange mechanism. This paper derives the MLEs of clock phase offset and skew for the above-mentioned two-way message exchange mechanism. Thus, the inferred MLEs do no require extra observations to recover the additional skew information. Clock skew estimation not only results in more accurate synchronization but it also helps in restricting the synchronization error within a certain bound for a longer period of time. Finally, an algorithm which describes the complete steps for the ML-estimator is presented as well.

Fig. 1 shows Node A sending a synchronization message to Node B with its current timestamp $T_{1,k}$ which is recorded at $T_{2,k}$ by Node B according to its current time. Node B then

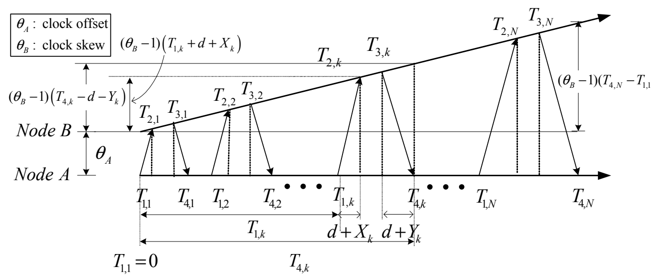

Fig. 1. Model for two way timing message exchange

sends at time $T_{3,k}$ a synchronization message to Node A containing $T_{2,k}$ and $T_{3,k}$. Node A timestamps the reception time of this message sent by Node B as $T_{4,k}$. Hence, at the end of N such message exchanges, Node A has a set of timestamps $\{T_{1,k}, T_{2,k}, T_{3,k}, T_{4,k}\}$, $k = 1, \cdots, N$, where $T_{1,1}$ is considered to be the reference time. Therefore, the timing message exchange mechanism is captured by the model

$$T_{2,k} = (T_{1,k} + d + X_k)\,\theta_B + \theta_A, \quad (1)$$
$$T_{3,k} = (T_{4,k} - d - Y_k)\,\theta_B + \theta_A, \quad (2)$$

where θ_A and θ_B are the clock offset and skew, respectively, of Node B with respect to Node A, d stands for the *unknown* deterministic portion of delay, e.g., the sum of transmission, propagation and reception times, etc., and X_k and Y_k are variable portions of delay assumed to be independent and identically distributed random variables that assume an exponential distribution with the same mean α. A complete justification of modeling random delays as exponential can be found in [4] and [5].

2. ESTIMATING CLOCK OFFSET AND SKEW

From (1) and (2), the likelihood function is given by

$$L(d, \theta_B, \theta_A) = \alpha^{-2N} . e^{-\frac{1}{\alpha}\sum_{k=1}^{N}\left\{\frac{T_{2,k}-T_{3,k}}{\theta_B} - (T_{1,k}-T_{4,k}) - 2d\right\}} . \prod_{k=1}^{N} I\left[\frac{T_{2,k}-\theta_A}{\theta_B} - T_{1,k} \geq d; \quad T_{4,k} - \frac{T_{3,k}-\theta_A}{\theta_B} \geq d\right],$$

where $d > 0$ (unknown fixed portion of time delay) and $\theta_B > 0$ (clocks can not stop or move backward). In maximizing the likelihood for this model over the set $\Theta = \{(d, \theta_A, \theta_B) : d > 0, -\infty < \theta_A < \infty, \theta_B > 0\}$, the geometry of the region, where the likelihood function is not zero, needs to be considered. First for simplicity, assume that θ_A is known, say 0 without any loss of generality. The general case, when θ_A is unknown, is discussed later in the paper. Now the likelihood function can be maximized by making its argument $\phi = \sum_{k=1}^{N}[(T_{2,k} - T_{3,k})/\theta_B - (T_{1,k} - T_{4,k}) - 2d]$ as small as possible. Considering the constraints in the likelihood function shown in Fig. 2,

$$d \leq \frac{T_{2,k}}{\theta_B} - T_{1,k}, \quad k = 1, \cdots, N, \tag{3}$$

$$d \leq T_{4,k} - \frac{T_{3,k}}{\theta_B}, \quad k = 1, \cdots, N, \tag{4}$$

the MLE $(\hat{d}, \hat{\theta}_B)$ can be derived through the following theorems.

Theorem 1: *MLE $\hat{d}$ lies on either $(T_{2,k}/\theta_B - T_{1,k})_{(1)}$ or $(T_{4,k} - T_{3,k}/\theta_B)_{(1)}$, i.e., on the boundary of the support region. The subscript (1) denotes the minimum order statistic.*

Proof: This can be proved by contradiction. Let us assume that the $\hat{d}$ does not lie on the boundary, but somewhere else inside the support region. Then for some minimizing $\hat{\theta}_B$, ϕ can be further decreased by increasing $\hat{d}$ to the top of the allowable region (which coincides with one of the the above mentioned curves) for the same $\hat{\theta}_B$, hence a contradiction.

Theorem 2: *MLE $\hat{d}$ lies either on the uppermost vertex formed by the intersection of the curves $(T_{2,k}/\theta_B - T_{1,k})_{(1)}$ and $(T_{4,k} - T_{3,k}/\theta_B)_{(1)}$ (shown as point A in Fig. 2) or on one of the vertices formed by the intersection of the curves $(T_{4,k} - T_{3,k}/\theta_B)$, $k = 1, \cdots, N$ (shown as points B, C, etc. in Fig. 2).*

Proof: From Theorem 1, it is known that $\hat{d}$ lies somewhere on the boundary of the support region. Notice further that in order to minimize ϕ, it is necessary to select d as large as possible and θ_B as small as possible.

Let $i = \arg\min_{1 \leq k \leq N} (T_{2,k}/\theta_B - T_{1,k})$ and $j = \arg\min_{1 \leq k \leq N} (T_{4,k} - T_{3,k}/\theta_B)$ correspond to the maximum d (i.e., point A in Fig. 2) and suppose that $\hat{d}$ lies on $(T_{2,i}/\theta_B - T_{1,i})$, then ϕ can be written as

$$\phi = \sum_{k=1}^{N} \left[\frac{T_{2,k} - T_{3,k}}{\theta_B} - (T_{1,k} - T_{4,k}) - 2\left(\frac{T_{2,i}}{\theta_B} - T_{1,i} \right) \right],$$
$$= \sum_{k=1}^{N} \left[\frac{1}{\theta_B} (T_{2,k} - T_{3,k} - 2T_{2,i}) - (T_{1,k} - T_{4,k} - 2T_{1,i}) \right].$$

Since the term $\sum_{k=1}^{N} (T_{2,k} - T_{3,k} - 2T_{2,i})$ is always negative, ϕ can be minimized by taking θ_B as small as possible on $(T_{2,i}/\theta_B - T_{1,i})$. Hence, $\hat{d}$ and $\hat{\theta}_B$ are *equal to* or *less than* the intersection of the curves $(T_{2,i}/\theta_B - T_{1,i})$ and $(T_{4,j} - T_{3,j}/\theta_B)$, which yields

$$\hat{d} = \frac{T_{2,i}T_{4,j} - T_{1,i}T_{3,j}}{T_{2,i} + T_{3,j}}, \tag{5}$$

$$\hat{\theta}_B = \frac{T_{2,i} + T_{3,j}}{T_{1,i} + T_{4,j}}. \tag{6}$$

Now suppose that $\hat{d}$ lies on $(T_{4,j} - T_{3,j}/\theta_B)$, then ϕ can be written as

$$\phi = \sum_{k=1}^{N} \left[\frac{T_{2,k} - T_{3,k}}{\theta_B} - (T_{1,k} - T_{4,k}) - 2\left(T_{4,j} - \frac{T_{3,j}}{\theta_B} \right) \right],$$
$$= \sum_{k=1}^{N} \left[\frac{1}{\theta_B} (T_{2,k} - T_{3,k} + 2T_{3,j}) - (T_{1,k} - T_{4,k} + 2T_{4,j}) \right].$$

From this equation, it is clear that ϕ can be minimized by taking the largest possible θ_B if $\sum_{k=1}^{N} (T_{2,k} - T_{3,k} + 2T_{3,j})$ is positive and by taking the smallest possible θ_B if $\sum_{k=1}^{N} (T_{2,k} - T_{3,k} + 2T_{3,j})$ is negative as depicted by Fig. 2. Therefore, for $2NT_{3,j} > \sum_{k=1}^{N}(T_{3,k} - T_{2,k})$, the MLE is given by (5) and (6), and for $2NT_{3,j} < \sum_{k=1}^{N}(T_{3,k} - T_{2,k})$, MLE is given by the intersection of the curves $(T_{4,m} - T_{3,m}/\theta_B)$ and $(T_{4,n} - T_{3,n}/\theta_B)$ (denoting the intersections of the curves $d = T_{4,k} - T_{3,k}/\theta_B$ and $d = T_{4,l} - T_{3,l}/\theta_B$ as $d^{k,l}$), where

$$(m, n) = \arg\max_{k,l}[d^{k,l} \mid 2NT_{3,k} < \sum_{r=1}^{N} (T_{3,r} - T_{2,r}); \quad 2NT_{3,l} > \sum_{r=1}^{N} (T_{3,r} - T_{2,r})].$$

In Fig. 2, this vertex is represented by the point B, and the

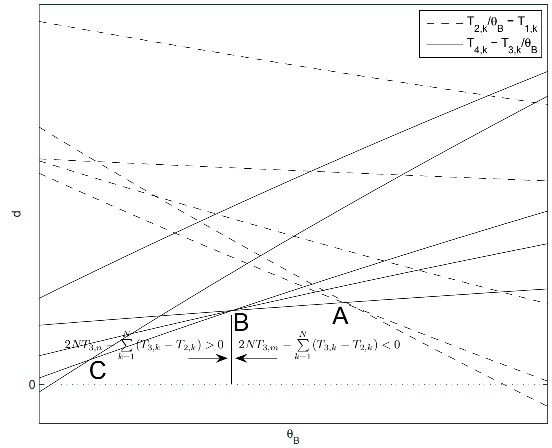

Fig. 2. Support region of the likelihood function

MLE $(\hat{d}, \hat{\theta}_B)$ in this case is given by

$$\hat{d} = T_{4,m} - \frac{T_{3,m}(T_{4,m} - T_{4,n})}{T_{3,m} - T_{3,n}}, \quad (7)$$

$$\hat{\theta}_B = \frac{T_{3,m} - T_{3,n}}{T_{4,m} - T_{4,n}}, \quad (8)$$

Theorem 3: *To the left of the point where the curves $(T_{2,k}/\theta_B - T_{1,k})_{(1)}$ and $(T_{4,k} - T_{3,k}/\theta_B)_{(1)}$ intersect (i.e., point A in Fig. 2), the boundary of the support region is formed by the curves $(T_{4,k} - T_{3,k}/\theta_B)$, $k = 1, \cdots, N$ in such a way that as θ_B increases, a curve $(T_{4,m} - T_{3,m}/\theta_B)$ forms the new boundary of the support region after intersecting the curve $(T_{4,n} - T_{3,n}/\theta_B)$ if and only if $m < n$.*

Proof: The curve $(T_{4,N} - T_{3,N}/\theta_B)$ starts as the most negative for small θ_B and ends up as the largest positive asymptotically approaching $T_{4,N}$ as θ_B increases. Similarly, the curve $(T_{4,1} - T_{3,1}/\theta_B)$ starts as the least negative for small θ_B and ends up as the smallest positive asymptotically approaching $T_{4,1}$ as θ_B increases. All the curves $(T_{4,k} - T_{3,k}/\theta_B)$, $k = 1, \cdots, N$, are arranged in descending order for small θ_B and in ascending order for large θ_B and they intersect each other somewhere around the true value of θ_B. Since the slope of each of them is $T_{3,k}/\theta_B^2$, the slope of the curve with index m is lesser than the slope of the curve with index n if $m < n$. Therefore, as θ_B increases, a curve can form the new boundary of the support region by intersecting another curve only if its index is lower than the previous one.

Theorem 4: *MLE $(\hat{d}, \hat{\theta}_B)$, whether (5), (6) or (7), (8), is unique.*

Proof: Note that the likelihood function is continuous on the boundary of the support region because different curves intersect each other on the vertices due to which there will be no jumps in ϕ and subsequently in the likelihood function. Considering the fact that $2NT_{3,j} > \sum_{k=1}^{N}(T_{3,k} - T_{2,k})$ for $j = N$, let $q = \arg\max_j \{T_{3,j} | 2NT_{3,j} < \sum_{k=1}^{N}(T_{3,k} - T_{2,k})\}$. Then it must also be true that $2NT_{3,j} < \sum_{k=1}^{N}(T_{3,k} - T_{2,k})$ $\forall\ j < q$, i.e., for $j = 1, \cdots, q-1$ and $2NT_{3,j} > \sum_{k=1}^{N}(T_{3,k} - T_{2,k})$ $\forall\ j > q$, i.e., for $j = q+1, \cdots, N$. There will always be just one change, if any, in the sign of this term from positive to negative. Therefore, ϕ can be minimized by making θ_B as large as possible on $(T_{4,q+1} - T_{3,q+1}/\theta_B)$ and as small as possible on $(T_{4,q} - T_{3,q}/\theta_B)$ (or on $(T_{2,i}/\theta_B - T_{1,i})$ if there is no such q).

This fact, combined with Theorem 3, proves that the intersection of the curves forming the MLE is always unique.

Next, we turn our attention to the general case when θ_A is unknown. Now the 3-D region over which the likelihood function is nonzero is shown in Fig. 3.

Within the constraint $d > 0$, $(T_{2,k} - \theta_A)/\theta_B - T_{1,k}$ are monotonically decreasing functions of θ_A and θ_B $\forall\ k$, and $T_{4,k} - (T_{3,k} - \theta_A)/\theta_B$ are monotonically increasing functions of θ_A and θ_B $\forall\ k$. It is clear from Fig. 3 that the support region for the likelihood function is similar in shape to a dome if we look at it standing on (θ_A, θ_B) plane. Theorem 1 asserts that the MLE $(\hat{d}, \hat{\theta}_A, \hat{\theta}_B)$ should lie somewhere on the ceiling of this dome. The line on (θ_A, θ_B) plane, on which the intersections of the surfaces lie for $k, l = 1, \cdots, N$, is given by

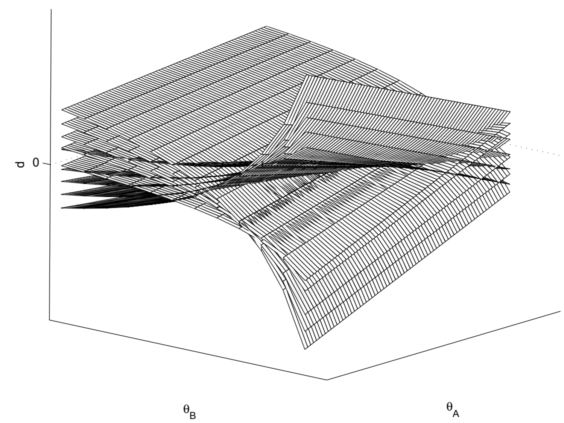

Fig. 3. d as a function of θ_A and θ_B.

$$\theta_A = \frac{1}{2}\left[(T_{2,k} + T_{3,l}) - \theta_B(T_{1,k} + T_{4,l})\right], \quad (9)$$

Although d is a function of both θ_A and θ_B, it can be written as a function of either θ_A only or θ_B only by utilizing this linear relationship between these two parameters, e.g., Fig. 4 shows the *imaginary* 2-D region where d is drawn as a function of θ_B only. This is actually a 3-D plot, but the points on the bottom θ_A axis are replaced with (9). Over this line (9), d is given by

$$d \leq \frac{1}{2}\left[\frac{T_{2,k} - T_{3,l}}{\theta_B} + (T_{4,l} - T_{1,k})\right]. \quad (10)$$

Consider the set of N^2 curves given in (10) and plotted in Fig. 4. Since the signs of $T_{2,k} - T_{3,l}$ and $T_{4,l} - T_{1,k}$ are always opposite, $N(N-1)/2$ of these curves have positive numerator in the term involving θ_B and negative constant term, while the remaining $N(N+1)/2$ have negative numerator in the term involving θ_B and positive constant term. Based on this observation, (10) can be written in the form of two sets of inequalities such that $(T_{2,k} - T_{3,l}) > 0$ for one set and $(T_{2,k} - T_{3,l}) < 0$ for the other as shown in Fig. 4. Then the current scenario assumes quite a similar form to the set of constraints (3) and (4). Theorems $1, 2, 3$ and 4 are then similarly true for these sets of inequalities and the MLEs can be derived by following a similar procedure. Let us denote $\left[(T_{2,k} - T_{3,l})/\theta_B + (T_{4,l} - T_{1,k})\right]/2_{(1)} | (T_{2,k} - T_{3,l}) > 0$ as

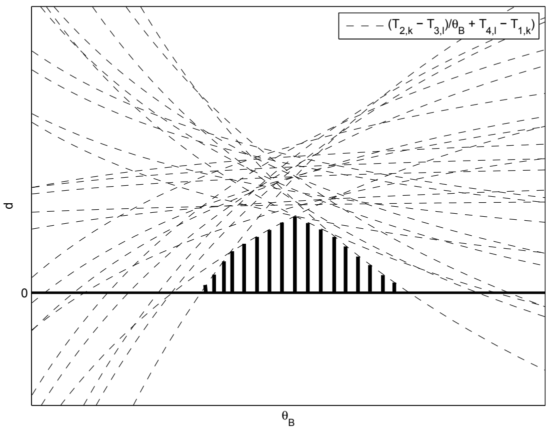

Fig. 4. d as a function of θ_B only.

$[(T_{2,i}-T_{3,j})/\theta_B+(T_{4,j}-T_{1,i})]/2$ and $[(T_{2,k}-T_{3,l})/\theta_B+(T_{4,l}-T_{1,k})]/2_{(1)}|(T_{2,k}{-}T_{3,l})<0$ as $[(T_{2,m}-T_{3,n})/\theta_B+(T_{4,n}-T_{1,m})]/2$. Then if $\sum_{k=1}^{N}[T_{2,k}-T_{3,k}-(T_{2,m}-T_{3,n})]$ is positive, the MLE $(\hat{d},\hat{\theta}_B)$ is intersection of this curve with the one discussed above, i.e.,

$$\hat{d}=\frac{1}{2}\left[\frac{(T_{2,i}-T_{3,j})\left[(T_{1,i}-T_{1,m})+(T_{4,n}-T_{4,j})\right]}{(T_{2,i}-T_{2,m})+(T_{3,n}-T_{3,j})}+(T_{4,j}-T_{1,i})\right],\qquad \hat{\theta}_B=\frac{(T_{2,i}-T_{2,m})+(T_{3,n}-T_{3,j})}{(T_{1,i}-T_{1,m})+(T_{4,n}-T_{4,j})}.$$

Otherwise, if $\sum_{k=1}^{N}[T_{2,k}-T_{3,k}-(T_{2,m}-T_{3,n})]$ is negative, then the MLE is the intersection of the curves $[(T_{2,p}-T_{3,q})/\theta_B+(T_{4,q}-T_{1,p})]/2$ and $[(T_{2,r}-T_{3,s})/\theta_B+(T_{4,s}-T_{1,r})]/2$ (denoting the intersections of the curves in (10) as $d^{k,l,m,n}$, $\forall\,(k,l,m,n)$), where

$$(p,q,r,s)=\arg\max_{k,l,m,n}\{d^{k,l,m,n}\ \mid\ N(T_{3,l}-T_{2,k})<\sum_{k=1}^{N}(T_{3,k}-T_{2,k});\ \ N(T_{3,n}-T_{2,m})>\sum_{k=1}^{N}(T_{3,k}-T_{2,k})\}.$$

Hence, here the MLE $(\hat{d},\hat{\theta}_B)$ is

$$\hat{d}=\frac{1}{2}\left[\frac{(T_{2,p}-T_{3,q})\left[(T_{1,p}-T_{1,r})+(T_{4,s}-T_{4,q})\right]}{(T_{2,p}-T_{2,r})+(T_{3,s}-T_{3,q})}+(T_{4,q}-T_{1,p})\right],\qquad \hat{\theta}_B=\frac{(T_{2,p}-T_{2,r})+(T_{3,s}-T_{3,q})}{(T_{1,p}-T_{1,r})+(T_{4,s}-T_{4,q})}.$$

Finally, (9) can then be utilized to find $\hat{\theta}_A$. The complete procedure for finding the MLE is described in Algorithm 1. It starts from the curve for which $(T_{2,m}-T_{3,n})$ is minimum, i.e., $(T_{2,1}-T_{3,N})$ and then compares its intersections with other curves. It keeps on replacing this curve with the one giving the next minimum $d^{k,l,m,n}$ within the constraints until the MLE is found according to the above procedure.

Algorithm 1 Finding $\hat{\theta}_A$, $\hat{\theta}_B$ and $\hat{d}$ for d and θ_A unknown

$(m,n)=(1,N)$;
LABEL:
Find $d^{k,l,m,n}=\frac{1}{2}\left[\frac{(T_{2,k}-T_{3,l})((T_{1,k}-T_{1,m})+(T_{4,n}-T_{4,l}))}{(T_{2,k}-T_{2,m})+(T_{3,n}-T_{3,l})}+(T_{4,l}-T_{1,k})\right]$; $\theta_B^{k,l,m,n}=\frac{(T_{2,k}-T_{2,m})+(T_{3,n}-T_{3,l})}{(T_{1,k}-T_{1,m})+(T_{4,n}-T_{4,l})}$; $\forall\,(k,l)\neq(m,n)$;
$(p,q)=\arg\min_{k,l}\{d^{k,l,m,n}\}$
if $T_{2,p}-T_{3,q}>0$ **then**
  $\hat{d}=d^{p,q,m,n}$; $\hat{\theta}_B=\theta_B^{p,q,m,n}$; $\hat{\theta}_A=\frac{1}{2}\left[(T_{2,p}+T_{3,q})-\hat{\theta}_B(T_{1,p}+T_{4,q})\right]$;
else
  if $N(T_{2,p}-T_{3,q})>\sum_{k=1}^{N}(T_{3,k}-T_{2,k})$ **then**
    $\hat{d}=d^{p,q,m,n}$; $\hat{\theta}_B=\theta_B^{p,q,m,n}$; $\hat{\theta}_A=\frac{1}{2}\left[(T_{2,p}+T_{3,q})-\hat{\theta}_B(T_{1,p}+T_{4,q})\right]$;
  else
    Remove (m,n) curve;
    $(m,n)=(p,q)$;
    goto LABEL;
  end if
end if

3. CONCLUSION

This paper derives the Maximum Likelihood Estimates for clock offset and skew for a two-way timing message exchange mechanism under an exponential delay model in a Wireless Sensor Network. The problem has been solved assuming that the knowledge of the deterministic portion of delays is not available at the nodes.

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