DESIGN AND EVALUATION OF V-SHAPED ARRAYS FOR 2-D DOA ESTIMATION

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ABSTRACT

A new method for optimum design of V-shaped arrays is presented for azimuth and elevation angle estimation simultaneously. The design criterion is based on the Cramer-Rao Bound (CRB) for joint estimation where the coupling effect between the azimuth and elevation direction of arrival (DOA) angles is taken into account. The design method finds an optimum angle between the linear sub-arrays of the V-array. The computation of the optimum angle is simple due to the monotonic characteristics of the best and worst performance levels of CRB. The proposed method can be used to obtain directional arrays with significantly better DOA performance compared to the circular arrays. Furthermore V-array angle can be chosen for isotropic angle performance which is better than the circular arrays. The modeling error for the sensor positions is also investigated. It is shown that both V-array and circular array have similar robustness for the position errors.

***Index Terms*—** Array signal processing, 2-D direction of arrival estimation, Planar arrays, V-shaped array, Isotropic arrays

1. INTRODUCTION

Design of optimum array geometry for the best DOA estimation performance is an important problem. This problem is investigated in previous works for the most general parameter settings. In these works, CRB for a fixed elevation angle is used as the cost function and multiple sources are considered [1]. It turns out that optimum array geometry for DOA estimation depends on many parameters including the number of sources and their DOA's, number of sensors, etc [1]. Furthermore it is not easy to find a single optimum geometry since the cost function changes depending on the number of sources and DOA's.

In this work, we select the array geometry to be a V-shaped one and optimize the angle of the V-shaped array geometry. V-shaped arrays have certain advantages. It is possible to design V-shaped array with isotropic response such that the DOA accuracy is uniform for all directions. Furthermore, since there are two linear sub-arrays, it is suitable for forward-backward spatial smoothing when there are multipath signals. Also fast algorithms can be applied for each sub-array and the results can be combined as in [2]. V-shaped arrays are not fully investigated in the literature. In [3], V-shaped arrays are considered with certain limitations. Coupling between the azimuth and elevation DOA estimation is ignored and V-angle for isotropic response is given for infinitely many sensors. Furthermore the design method in [3] finds a V-angles interval which results better performance than a circular array. This limits the application of the technique and the directional characteristic of the V-shaped array is not fully exploited. In our case, an optimum V-angle is found such that the DOA performance is the best for a given angular sector.

In this work, we consider the coupling effect between the azimuth and elevation angle estimation. The optimization of the V-angle is done by defining a cost function over the CRB which takes into account the coupling effect of azimuth and elevation DOAs. The proposed design considers two regions, namely, focused and unfocused region. The limits of the regions determine the angular accuracy and the performance of the V-array. It is shown that the optimum V-angle can be found easily with only a limited search due to the monotonic characteristics of the cost function for the worst and best levels specified in the design parameters of the regions.

The proposed approach is used to find the V-angles for isotropic performance when the number of sensor is finite. A table is presented for this purpose. Circular and V-shaped arrays are compared when there is modeling errors for the sensor positions. It is shown that both V-array and circular array have similar robustness for the position errors.

2. PROBLEM FORMULATION

2.1. Data Model

Consider an array of M sensors located at the positions [x_m, y_m], m = 1,…,M. $\Theta = [\phi, \theta]$ is the source DOA angles, where ϕ and θ are the azimuth and elevation angles respectively (Figure 1). There are L source signals where L<M. If the sensors are identical and far-field assumption is made, sensor output vector **y**(t) is given as:

$$\mathbf{y}(t) = \mathbf{A}(\Theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

It is assumed that the noise, **n**(t), is zero-mean, white and Gaussian with variance σ^2. It is also uncorrelated with the source signals. $\mathbf{A}(\Theta)_{M\times L}$ is the steering matrix. Under these assumptions, output covariance matrix is obtained as

$$E\{\mathbf{y}(t)\mathbf{y}(t)^H\} = \mathbf{R} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (2)$$

where $(.)^H$ denotes the conjugate transpose of a matrix.

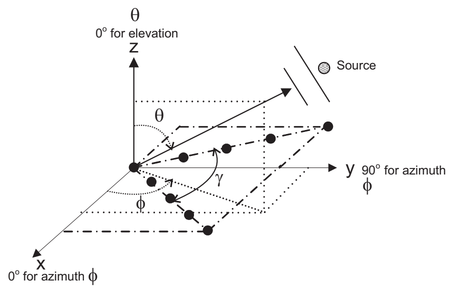


Fig. 1. Coordinate system for 2-D (azimuth and elevation) angle estimation and V-shaped array.

2.2. Cramer-Rao Bound for 2-D DOA Estimation

CRB shows the ultimate performance of an unbiased estimate for a given array geometry. When 2-D DOA estimation is considered, there is statistical coupling between the azimuth and elevation DOA performances in general. The existence of coupling depends on array geometry. Some of the array geometries are uncoupled, such as, circular arrays. V-shaped arrays show coupling effects and therefore coupling should be taken into account for the CRB. The proposed V-shaped array design method uses the CRB as the cost function. Therefore, a review of the coupling effect for the CRB is considered in this section. The Cramer-Rao inequality for estimating multiple parameters is given as,

$$var(\hat{\theta}_m) \geq [\mathbf{F}^{-1}]_{mm}, \quad (3)$$

where the mn^{th} element of the Fisher information matrix **F** is given by [4]

$$\mathbf{F}_{mn} = N.tr\{\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial p_m}\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial p_n}\}, \quad (4)$$

N is the number of samples in time. For 2-D (azimuth and elevation) angle estimation, the unknown parameter vector is defined by $\mathbf{p} = [\phi\ ,\ \theta]$, where ϕ is the azimuth and θ is the elevation angle. Fisher information matrix (FIM) is given by

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\phi\phi} & \mathbf{F}_{\phi\theta} \\ \mathbf{F}_{\theta\phi} & \mathbf{F}_{\theta\theta} \end{bmatrix} \quad (5)$$

where

$$\mathbf{F}_{\phi\phi} = 2NRe\{(\mathbf{R}_s\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\mathbf{R}_s) \times (\dot{\mathbf{A}}_\phi^H\mathbf{P}_A^\perp\mathbf{R}^{-1}\dot{\mathbf{A}}_\phi)^T\} \quad (6)$$

$$\mathbf{F}_{\phi\theta} = 2NRe\{(\mathbf{R}_s\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\mathbf{R}_s) \times (\dot{\mathbf{A}}_\phi^H\mathbf{P}_A^\perp\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta)^T\} \quad (7)$$

$\mathbf{F}_{\theta\theta}$, can be written similar to (6), ($\mathbf{F}_{\phi\theta} = \mathbf{F}_{\theta\phi}$),

$$\mathbf{P}_A^\perp = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H, \quad (8)$$

$$\dot{\mathbf{A}}_\phi = \sum_{n=1}^{L}\frac{\partial \mathbf{A}}{\partial \phi_n}. \quad (9)$$

The derivation details of these expressions can be found in [4]. If the off-diagonal terms of the $\mathbf{F}_{\phi\theta}$ are zero, the estimates of azimuth and elevation are uncoupled. But in general for some array geometries, this off-diagonal term, $\mathbf{F}_{\phi\theta}$, is nonzero and this reduces the DOA accuracy. For 2-D angle estimation, the CRB defined in [5] takes the coupling effect into account. The CRB for the azimuth and elevation angles are given as,

$$CRB_\phi = \frac{1}{\mathbf{F}_{\phi\phi}}\left[\frac{1}{1-\rho^2}\right] \quad (10)$$

$$CRB_\theta = \frac{1}{\mathbf{F}_{\theta\theta}}\left[\frac{1}{1-\rho^2}\right] \quad (11)$$

where

$$0 \leq \rho^2 = \frac{\mathbf{F}_{\phi\theta}^2}{\mathbf{F}_{\phi\phi}\mathbf{F}_{\theta\theta}} \leq 1. \quad (12)$$

If ρ^2=1, the estimates are said to be perfectly coupled. If $\rho^2 \neq 0$, uncertainty in one parameter degrades the other parameter estimates [5], [6].

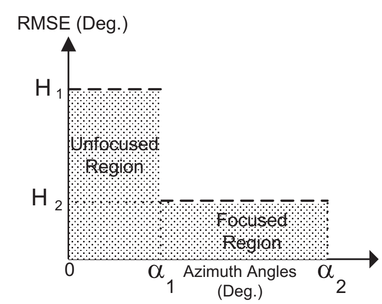
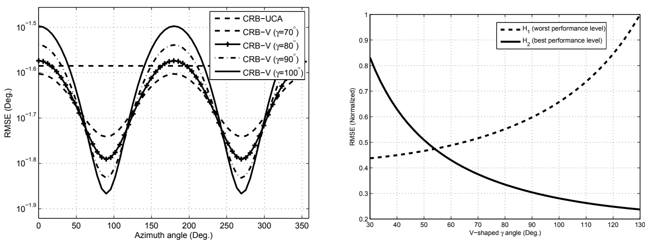


Fig. 2. The design regions and parameters for V-shaped array geometry.

3. DIRECTIONAL V-SHAPED ARRAY DESIGN

3.1. Design Procedure

V-shaped array and the coordinate axes are shown in Figure 1. The design procedure finds the optimum γ^0 angle to obtain the best DOA performance for the azimuth while the coupling effect of the elevation is taken into account. Note that if the coupling effect is not taken into account, design of V-array can converge to a degenerate case, such as, a linear array. In the design procedure, two regions are specified as shown in Figure 2. Focused region is the angular sector where the best



(a) (b)

Fig. 3. (a) The V-shaped array's azimuth CRB for different γ angles. (b) The best and worst performance levels of the azimuth CRB versus V-angle, γ.

possible DOA accuracy is desired. Unfocused region is an angular sector where a DOA accuracy below a certain level, H_1, is acceptable. When the above points are considered, the design parameters are the azimuth angles, α_1 and α_2 which determine the focused region limits and the worst performance level H_1 in the unfocused region. Therefore the target is to find γ^0 given the parameters α_1, α_2 and H_1. In Figure 3a, azimuth CRB (10) is shown for different V-angles, γ. The best (H_2) and worst (H_1) levels are easily seen from this figure. In Figure 3b, H_1 and H_2 levels are plotted with respect to γ. As it is seen H_1-γ, and H_2-γ curves are monotonic. The proposed design method has the following steps:

- **Step 1:** H_1, α_1, and α_2 values are specified (Figure 2).
- **Step 2:** From Figure 3b, γ angle (γ_1) corresponding to H_1 is found.
- **Step 3:** CRB expression in (10) is evaluated for α_1 and α_2 azimuth angles for the given V-angle, γ_k, namely CRB(α_1, γ_k) and CRB(α_2, γ_k). The cost for γ_k is e(k) = max$\{$CRB(α_1, γ_k), CRB(α_2, γ_k)$\}$.
- **Step 4:** Decrease γ_k angle by Δ, (γ_{k+1} = γ_k - Δ), and repeat step 3 for k = 2, ..., K. (Δ is the step size)
- **Step 5:** Find the minimum e(k) and the corresponding γ_k angle as the optimum V-angle, γ^0

$$\gamma^0 = \arg\min_{\gamma_k}\{e(k)\}. \tag{13}$$

3.2. Isotropic V-Shaped Arrays

In Figure 3b, H_1 and H_2 levels intersect at the $\gamma_{isotropic}$ angle and at this angle, V-shaped array gives the same DOA performance for all directions. This special angle is changes with the number of sensors. The proposed design method can be used to find these angles for finite number of sensors easily. In order to find the isotropic angle, it is sufficient to set α_1=0° and α_2=90°. As it is seen from the Figure 3a, the best and worst levels are observed with 180° period. If the proposed design technique is applied as it is described above, isotropic response angles are obtained as in Table 1.

Table 1. Isotropic V-shaped array angles for the specified number of sensors.

Number of Sensors	$\gamma^o_{isotropic}$ (degree)
3	60^o
5	55.77113^o
7	54.50407^o
9	53.96812^o
11	53.69345^o
13	53.53443^o
15	53.43426^o
17	53.36715^o
19	53.32001^o
21	53.28564^o

4. SIMULATIONS

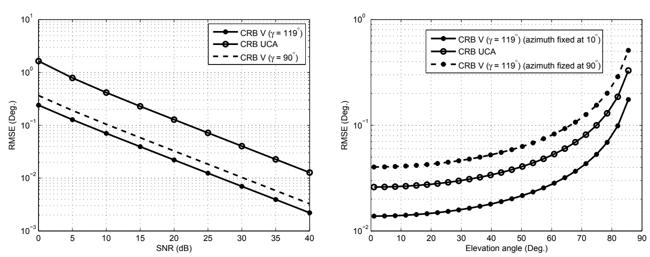
In this part, we consider the directional and isotropic V-shaped arrays in order to show the characteristics of the V-array for these two different cases. We also compare the DOA performance with the uniform circular array (UCA). Furthermore the effect of sensor position error is investigated for both V and circular arrays by using the MUSIC algorithm.

In the directional case, sources are assumed to be localized in a narrow angular sector. There are M=9 sensors and the number of snapshots N=256. There are two sources at ϕ_1=81° and ϕ_2=98° degrees. The design parameters are chosen as α_1=80^o, α_2=100^o and the H_1 = 0.5°, step size $\Delta = 1°$. If the design procedure is applied for these parameters, the best DOA performance is obtained for $\gamma^0 = 119°$. The azimuth CRB of the designed V-shaped, and the UCA array is given in Figure 4a. The performance for the elevation angle is shown in Figure 4b. As it is seen from the figure, for different azimuth angles, elevation performance changes for the directional V-array but not for the circular array. If the design procedure is used to find the γ angle for isotropic performance for M=9, $\gamma_{isotropic} = 53.96812°$. The performance of V and circular arrays for this case is shown in Figure 5. As it is seen from this figure, V-array shows better performance than circular array.

The position error, p_e, also considered for the V and circular arrays. p_e is taken with respect to the intersensor distance, d=$\lambda/2$ (λ is the wavelength). Therefore % 1 position error corresponds to $\frac{|p_e|}{d} = 0.01$. Error displacement is on a circle with radius $|p_e|$ and the circle center is at the true sensor position. In this case, both the directional and isotropic V-arrays are considered for different percentage of sensor position errors. Figure 6a shows the results for directional array while Figure 6b shows the performance for the isotropic case. From these two figures, we see that the both the V and circular arrays have similar robustness for the position errors. When we consider the above results, it can concluded that V-arrays have some important advantages compared to other array geometries.

5. CONCLUSIONS

In this paper, a design method for V-shaped array is proposed. The proposed method finds the optimum V-shaped γ angle for the specified design parameters. When the sources are in an angular sector, directional V-array performs significantly better compared to other array geometries. We have compared the V-arrays with circular, X, Y and Δ shaped arrays. It turns out that V-shaped arrays have the best performance for the same number of sensors and inter-element distance. The proposed method can be used to design V-arrays with isotropic angular performance. In this case, the performance of the V-array is better than the circular array for the same number of sensors and inter-element distances. V-arrays are also robust to position errors. They show similar characteristics like the circular arrays for position errors.



(a) (b)

Fig. 4. (a) The azimuth CRB of designed V-shaped array and UCA versus SNR for M=9 sensors 2 sources at $\phi_1 = 81°$, $\phi_2 = 98°$ and elevation angles are fixed to $\theta = 90°$. (b)The elevation CRB for designed V-shaped array and UCA with different azimuth angles.

6. REFERENCES

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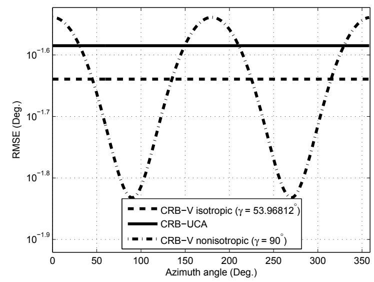
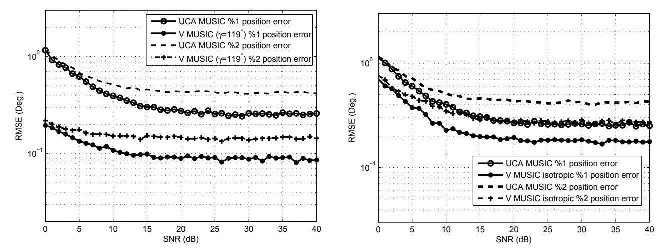


Fig. 5. The azimuth CRB of isotropic, directional V-shaped arrays and UCA for all azimuth angles when M=9 sensors and elevation angles are fixed to $\theta = 90°$.



(a) (b)

Fig. 6. These simulations are done for M=9 sensors 2 sources at $\phi_1 = 81°$, $\phi_2 = 98°$ and elevation angles are fixed to $\theta = 90°$, N=256 snaphots and with 1000 iterations. (a) The MUSIC algorithm's DOA performance for directional V-shaped array and UCA versus SNR for different position error percentages. (b) The MUSIC algorithm's DOA performance for isotropic V-shaped array and UCA versus SNR for different position error percentages.

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