MODE SIGN ESTIMATION TO IMPROVE SOURCE DEPTH ESTIMATION

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ABSTRACT

This paper describes source depth estimation in shallow water environments using a Horizontal Line Array of hydrophones on the sea bottom. A method based on modal propagation, using mode amplitude modulus, has been proposed by Nicolas [1]. As knowledge of the sign of mode amplitudes can improve source depth estimation, we present a estimator of these signs and integrate them in the source depth estimation method. Results on real data are shown.

***Index Terms*—** Underwater acoustics, source depth estimation, mode sign estimation, matched-mode processing.

1. INTRODUCTION

Passive source localisation has been a topic of considerable recent attention in underwater acoustics. In shallow water environments, many methods have been developed to estimate source depth based on Matched-Field Processing (MFP) [2, 3] and Matched-Mode Processing (MMP) [4, 5]. MFP performs a correlation (Bartlett processor) between the acoustic field recorded on the array and replica fields generated by a propagation model. The main drawback of MFP is its sensitivity to environmental mismatch [6] due to the use of the entire acoustic field in the correlation process. To avoid this problem, methods based on modal propagation (MMP) have been developed. MMP typically estimates source depth using modes amplitudes extracted from real and simulated data recorded on a Vertical Line Array (VLA) of hydrophones [4].

For practical respects : easiness of deployment, stability of the array, the use of a Horizontal Line Array (HLA) can be more adapted to practical applications. Consequently, we developed in a previous work, a MMP using a HLA placed on the sea bottom to estimate the source depth using mode amplitude modulus [1]. Besides, we also shown that the knowledge of the sign of mode amplitudes improves source depth estimation. As a result, we propose in this paper, a method based on an estimator developed by Le Touzé [7] to estimate mode amplitude sign. This information is used with mode amplitude modulus to improve source depth estimation by MMP.

After a presentation of propagation and MMP on a HLA, mode amplitude sign estimator is presented. This method is applied on real data. Source depth is estimated and compared with results obtained using only mode amplitude modulus.

2. MODES AND MATCHED-MODE PROCESSING

2.1. Normal modes in a oceanic waveguide

As propagation phenomena are almost identical for complex waveguides, normal mode theory is presented for a perfect waveguide made of a homogeneous layer of fluid (celerity V, density ρ) between perfectly reflecting boundaries at depth 0 (surface) and D (sea bottom)[8].

Considering an omnidirectional point source at ($r = 0$, $z = z_s$) radiating a broadband signal $s(t)$, the pressure field $p(r, z, t)$, measured at $M(r, z)$ is expressed, using a Fourier transform, by [8]:

$$p(r, z, t) = \int_f s(f)p(r, z, f) \exp(2\pi \jmath ft)df \qquad (1)$$

with f the frequency and $s(f)$ the source spectrum. At long range, according to normal mode theory, $p(r, z, f)$ can be expanded into a sum of depth-dependent normal modes m:

$$p(r, z, f) \propto \sum_m \psi_m(z_s)\psi_m(z)\frac{\exp(\jmath k_{rm}r)}{\sqrt{k_{rm}r}} \qquad (2)$$

with:

- k_{rm} and k_{zm}, respectivelly the horizontal and vertical component of the wavenumber $k = \sqrt{k_{zm}^2 + k_{rm}^2} = 2\pi f/V$,
- $\psi_m(z_s)$=$\sqrt{2/D}\sin(k_{zm}z_s)$ the mode amplitude at source depth z_s, called "mode amplitude" in the following,
- $\psi_m(z)$ the mode amplitude at receiver depth, equal to ± 1 for a receiver on the sea bottom.

Information on the source depth is contained in mode amplitudes $\psi_m(z_s)$. We will develop signal processing methods to extract these mode amplitudes and finally estimate the source depth. To perform this extraction we use the signals recorded by a HLA placed on the sea bottom (at depth D).

$f - k$ transform:

We introduce the frequency-wavenumber transform ($f - k$), which is the modulus of the 2D Fourier transform of a section ($p(r, z, t), r = r_1..r_I, t = t_1..t_T$) in time t and distance r:

$$P(k_r, z, f) = \left| \int_t \int_r p(r, z, t)e^{-2\jmath\pi(ft - k_r r)}\, dtdr \right| \qquad (3)$$

For an antenna at long range, placed on the sea bottom, and in a perfect waveguide, the theoretical $f - k$ transform is [1]:

$$P(k_r, f) \approx B \sum_{m=1}^{+\infty} |\psi_m(z_s)|\, \delta(k_r - k_{rm}(f)) \qquad (4)$$

with B a constant. For each mode m, the energy is located on the dispersion curve $k_r = k_{rm}(f)$ of the mode. Moreover, amplitude along the curve is proportional to mode amplitude modulus. This property gives the possibility to extract information about mode amplitudes (their modulus) using the $f-k$ transform. The proposed method to estimate mode amplitude modulus, based on filtering in the $f-k$ domain, is described in detail in [1].

Mode phase in the frequency domain:
More information on mode amplitudes can be found using the phase of the pressure signal $s(f)p(r,z,f)$ recorded on the receiver for each frequency f. The phase of a given mode m extracted from this signal is given (using equation 2) by:

$$\begin{aligned}\Phi_m(f) \quad = \quad & \pi/4 + \phi_s(f) + 2\pi f t_0 + \phi(\psi_m(z_s)) \\ & + \phi(\psi_m(z)) + k_{rm}(f)r\end{aligned} \tag{5}$$

where

- $\phi_s(f)$ is source phase at frequency f,
- $2\pi f t_0$ is due to the time delay t_0 of the recorded signal,
- $k_{rm}(f)r$ expresses propagation in the water layer between source and receiver.
- $\phi(\psi_m(z_s))$ is linked to the sign of mode function at source depth. It is equal to 0 if $\psi_m(z_s) > 0$ and to π if $\psi_m(z_s) < 0$:
$$\phi(\psi_m(z_s)) = \pi\delta_{sign(-1)\,,sign(\psi_m(z_s))} \tag{6}$$
- $\phi(\psi_m(z))$ is linked to the sign of mode function at receiver depth.

Equation 5 shows that mode phase contains information on mode amplitude sign. Estimation of mode amplitude sign using mode phase is possible only if the mode m has been filtered (to access its own phase). Details on the sign estimation method are given in section 3.

To conclude on modal propagation, we can note the dependence of mode amplitudes on source depth. That leads us to extract information on mode amplitudes and to use it to estimate source depth. This information : modulus and sign of mode amplitudes is obtained using respectively $f-k$ transform and mode phase in the frequency domain.

2.2. Source depth estimation by MMP on a HLA

To estimate source depth using mode amplitudes, Matched Mode Processing (MMP) compares a set of mode amplitudes extracted from real data to a set of theoretical mode amplitudes obtained with a model, using a contrast function.

As discussed in section 1, MMP is generally performed on a VLA. For practical respects, we proposed a method using a HLA based on mode amplitude modulus estimation in the $f-k$ domain (detailed description is given in [1]). The different steps of the estimation method are:

- V velocity correction of the recorded signals, which consists in time shifting each recorded signal so that the direct wave impinges all the sensors at the same time (to avoid aliasing in the $f-k$ domain),
- Simulation of the acoustic field recorded on a HLA, for a point source at each depth in the guide, using a finite-difference algorithm or parabolic equations (PE) algorithm (this point will be discussed in section 4),
- Building masks of the modes in the $f-k$ domain, using propagation characterisation in waveguides,
- Mode amplitude modulus extraction, using masks in the $f-k$ domain, on real data and on simulations,
- Maximisation of the contrast function which compares measured and simulated mode amplitude modulus.

This method gave satisfactory results but we showed that the estimation is not always correct due to the presence of ambiguous peaks in the contrast function [1]. These peaks can become smaller using, not only mode amplitude modulus, but mode amplitudes with their sign [1]. As a result, we propose a method, based on an estimator developed by Le Touz, to estimate mode amplitude sign and then use the previous method (with mode amplitudes instead of mode amplitudes modulus) to estimate source depth.

3. MODE SIGN ESTIMATION

3.1. Initial estimator

To build the initial estimator, we consider the difference between two mode phases where m and n are mode numbers:

$$\begin{aligned}\Delta\Phi(m,n,f) &= \Phi_m(f) - \Phi_n(f) \\ &= \Delta\phi(m,n,z_s) + \Delta\phi(m,n,z) + r\Delta k_r(m,n,f)\end{aligned} \tag{7}$$

with

- $\Delta k_r(m,n,f) = (k_{rm}(f) - k_{rn}(f))$,
- $\Delta\phi(m,n,u)$, the difference between two mode amplitude phases at depth u:
$$\begin{aligned}\Delta\phi(m,n,u) &= \phi(\psi_m(u)) - \phi(\psi_n(u)) \\ &= \pi\delta_{sign(\psi_m(u)),sign(\psi_n(u))} \pmod{2\,\pi}\end{aligned} \tag{8}$$

		sign of $\psi_n(u)$	
		+	-
sign of $\psi_m(u)$	+	0	π
	-	π	0

Fig. 1. Values of $\Delta\phi(m,n,u)$

Figure 1 summarises the values of $\Delta\phi(m,n,u)$. In the case of receivers on the sea bottom ($z = D$) and for two consecutive modes, $\Delta\phi(m,n,z) = \pi \pmod{2\,\pi}$. In the following, for the sake of simplicity, we will consider this case but estimator is also built for two modes that are not consecutive as long as mode numbers are known. The difference between two mode phases is then:

$$\Delta\Phi(m,n,f) = \Delta\phi(m,n,z_s) + \pi + r\Delta k_r(m,n,f) \pmod{2\pi} \tag{9}$$

Using modes extracted from the recorded signal, we measure, for each frequency f_p the quantity $\Delta\Phi_{exp}(m,n,f_p)$, where

exp means experimental. Then, the cost function can be built:

$$B(r, \Delta\phi(m, n, z_s)) = \sum_{f_p} \Big| d(\Delta\Phi_{exp}(m, n, f_p), \Delta\Phi(m, n, f_p)) \Big|^2 \quad (10)$$

The distance function d between phases ϕ_1 and ϕ_2, used in this cost function, is defined by :

$$d(\phi_1, \phi_2) = arg(\exp(\jmath(\phi_1 - \phi_2))) \quad (11)$$

where the function argument arg is restricted to the basic interval $]-\pi, \pi]$. The estimator "sign/distance" which estimates the distance source/receiver and the sign difference between two consecutive mode amplitudes is:

$$\{\hat{r}, \widehat{\Delta\phi}(m, n, z_s)\} = \underset{r,\, \Delta\phi(m,n,z_s)=\{0,\pi\}}{\text{argmin}} B(r, \Delta\phi(m, n, z_s)) \quad (12)$$

where $\widehat{\Delta\phi}(m, n, z_s) = 0$ (resp. π) means that mode amplitudes of modes m and n have same (resp. opposite) signs. We note $\hat{S}_{rel}(m, n, z_s)$ the sign difference between mode amplitudes (for two modes m and n) and call it "sign difference" between two modes m and n in the following. We have:

$$\begin{array}{lcl} \widehat{\Delta\phi}(m, n, z_s) = 0 & \longrightarrow & \hat{S}_{rel}(m, n, z_s) = + \\ \widehat{\Delta\phi}(m, n, z_s) = \pi & \longrightarrow & \hat{S}_{rel}(m, n, z_s) = - \end{array} \quad (13)$$

3.2. Implementation

We discuss different points of the implementation:

- To use this "sign/distance" estimator, modes must be extracted from the recorded signals. Filtering is made using masks in the $f-k$ domain where modes are separated. As this transformation is invertible we get the filtered modes on each receiver of the HLA.
- To implement the estimator, we need the wavenumbers $k_{rm}(f)$. They are estimated on the $f-k$ transform (by finding the maximum of each mode at each frequency)
- The number of frequency f_p used in the estimation is given by the frequency band of modes m and n.
- In our case, distance source/receiver r is approximatively known and we want to estimate mode amplitude signs. As a result, we use the estimator with a search bound of 100 m around the known distance r which reduces the number of ambiguous peaks of the estimator.

3.3. Adaptation

Initial estimator has to be adapted to our specific objective. The initial estimator provides, using two modes m and n extracted from a unique signal, the sign difference between these two modes. In our case, we will take advantage of all the signals $(p(r_i, z, t), i = 1..I)$ recorded on the HLA and we will estimate the absolute sign of each mode amplitude $\hat{S}_{abs}(m, z_s)$ (called "mode sign" in the following), and not the sign difference between two modes. This estimation is possible as mode 1 is always positive : $\hat{S}_{abs}(1, z_s) = +$.

The different steps of mode sign estimation are presented for a mode m. For each recorded signal i on the HLA, we estimate $\hat{S}^i_{rel}(m, n, z_s)$ using modes n ($n < m$ because sign of modes with a number n inferior to m are known). Moreover modes n have to share a frequency band with mode m. In practical applications, to estimate mode m, we will use modes $n = m-1$ and $n = m-2$. Using two modes to estimate a mode sign will limitate error propagation from a mode to the other. Then $\hat{S}_{rel}(m, n, z_s)$ is the sign which has been the most often found in all the $\hat{S}^i_{rel}(m, n, z_s)$.

At this point we have estimated $\hat{S}_{rel}(m, m-1, z_s)$ and $\hat{S}_{rel}(m, m-2, z_s)$ (we can note that $\hat{S}_{abs}(m-1, z_s)$ and $\hat{S}_{abs}(m-2, z_s)$ are already known). Which sign difference should then be used to estimate $\hat{S}_{abs}(m, z_s)$? To answer this question, let us consider the case of a mode $m-1$ with a small energy. Its sign estimation can be false (which will not be a problem in source depth estimation as $\psi_m(z_s)$ will have a small contribution in the contrast function) and can affect the estimation of mode m sign: this mode $m-1$ should not be used to estimate mode m sign. In this case we saw that $\hat{S}^i_{rel}(m, m-1, z_s)$ is non constant for the different recorded signals i. As a result, to avoid this error propagation, we choose, between modes $m-1$ and $m-2$, the mode for which sign difference estimation$\hat{S}^i_{rel}$ was the most constant on the different recorded signals i. In the example, mode m sign estimation would then be:

$$\hat{S}_{abs}(m, z_s) = \hat{S}_{rel}(m, m-2, z_s)\hat{S}_{abs}(m-2, z_s) \quad (14)$$

Sign estimation is made, in increasing order, for each mode and mode sign is integrated in the contrast function to estimate source depth.

4. RESULTS ON NORTH SEA DATA

North Sea data have been recorded using an airgun moving from one location to another and making one shot every 25 m. The receiver is a hydrophone on the sea bottom. As environment is range-independent, this geometry creates synthetic aperture and is equivalent to that presented in Fig. 2.

Simulations, using a finite difference algorithm or a PE algorithm, are made in an environment similar to the real environment for different source depths.

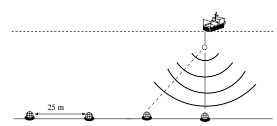

Fig. 2. North Sea data: equivalent geometry

4.1. Mode sign estimation

We present mode 3 sign estimation on PE simulations made for all source depths. For each simulation, we first use mode 1 and 2 to estimate $\hat{S}_{rel}(3, 2, z_s)$ and $\hat{S}_{rel}(3, 1, z_s)$ using all the recorded signals i on the HLA. Then, to know which mode will be used to estimate $\hat{S}_{abs}(3, z_s)$, we use figures 3-a and 3-b representing the percentage of constant $\hat{S}^i_{rel}$ on the I receivers for each simulation. Using this quantity, we choose, between mode 1 and 2, the mode used to estimate $\hat{S}_{abs}(3, z_s)$ (cf. figure 3-c) and mode 3 sign estimation is shown (cf. figure 4). We can note that for simulations with source depths close to 75 m (which is source depth where mode 2 amplitude

Source depth (m)	% of constant $\hat{S}^i_{rel}(3,2,z_s)$	% of constant $\hat{S}^i_{rel}(3,1,z_s)$	Chosen mode	Estimated sign
20	100	64	2	+
76	61	100	1	-

Table 1. Results for $z_s = 20$ m and $z_s = 76$ m

is small: $2D/3$) mode 3 sign estimation is made using mode 1 which is consistent with the method principle.

We also give in detail results obtained for simulated sources depths $z_s = 20$ m and $z_s = 76$ m in table 1. For $z_s = 20$ m, estimation of $\hat{S}^i_{rel}(3,2,z_s)$ is constant for 100% of the I signals and estimation of $\hat{S}^i_{rel}(3,1,z_s)$ is constant for 64% of the I signals. As a result mode 2 is chosen to estimate $\hat{S}_{abs}(3,z_s)$ and its sign is given by $\hat{S}_{abs}(3,z_s) = \hat{S}_{rel}(3,2,z_s)\hat{S}_{abs}(2,z_s)$.

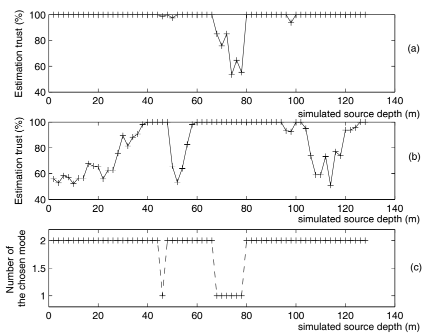

Fig. 3. a-$\%$ of constant $\hat{S}^i_{rel}(3,2,z_s)$; b-$\%$ of constant $\hat{S}^i_{rel}(3,1,z_s)$,c-mode used to estimate $\hat{S}_{abs}(3,z_s)$

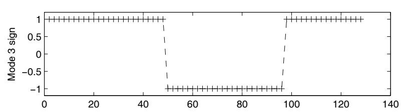

Fig. 4. mode 3 sign estimation

4.2. Source depth estimation

Pressure field of the real data is recorded on a synthetic antenna of 160 hydrophones (between 2 and 6 km in range), which will allows us to use method described in section 2.2. To evaluate the method prefomances we use the fact that the real source depth is between 10 and 20 m.Initial data are time corrected with velocity $V_1 = 1520$ m/s. We compute the $f - k$ transform, 7 modes can be identified and mode amplitude modulus is estimated [1]. We apply the same method on all the simulated data.

We then use mode amplitude signs and modulus, extracted from real and simulated data, to perform MMP. Results are shown on figures 5-top (estimation without mode amplitude sign) and 5-bottom (estimation with mode amplitude signs). For each method, we present results using simulations with a finite difference algorithm and with a PE algorithm.

In both cases, we can see that using mode amplitude signs improves source depth estimation ($\hat{z}_s = 19$ m whereas it was $\hat{z}_s = 120$ m without mode amplitude signs) as it reduces significantly ambiguous peaks. We can also note that finite difference algorithm (which has an expensive computational cost but simulates a more complex environment, closer to the real environment) can be useful if only mode amplitude modulus is extracted: with this simulation, it is possible to estimate source depth as long as it is known that source is not on the sea bottom (which is not possible with PE algorithm).

To conclude, mode sign estimation improve significantly source depth estimation using MMP and allows us to reduce computational cost using a PE algorithm for simulations.

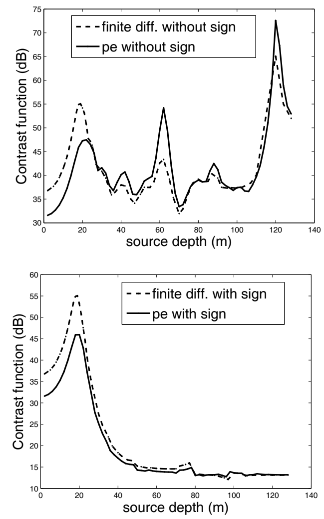

Fig. 5. Source depth estimation without (top) and with (bottom) mode sign

5. CONCLUSION

In this paper we propose an estimator of mode sign based on mode phase in the frequency domain. This estimator is an adaptation of a previous estimator developed by Le Touzé [7]. Once mode sign is estimated, this estimation is used with mode amplitude modulus estimation to perform MMP on a HLA. Results are shown on a real dataset and we see that source depth estimation is significantly improved.

6. REFERENCES

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