SUBSPACE INTERSECTION METHOD OF BEARING ESTIMATION BASED ON LEAST SQUARE APPROACH IN SHALLOW OCEAN

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ABSTRACT

In this paper, least square approach is applied in subspace intersection (SI) method for the problem of bearing estimation in shallow water. Based on that, a method called constrained least square subspace intersection method (CLS-SI) is proposed. The mathematic expressions of CLS-SI are given. In addition, the relationship between CLS-SI and MUSIC is discussed. Simulations show that the performance of the new method proposed is better than that of the original SI method.

Index Terms— DOA estimation, least square method, underwater acoustic arrays

1. INTRODUCTION

Many direction-of-arrival (DOA) estimation methods are under an assumption that signals propagate as plane-wave form in medium. However, this assumption is not always accurate in the ocean especially in shallow ocean since sound can stimulate multiple normal modes when it propagates in shallow ocean [1]. For this reason, plane-wave DOA estimation techniques yield biased bearing estimation in the ocean. In physics, it could be explained that different modes have different phase speeds [2] (also mean different wavenumbers). This phenomenon is especially remarkable when differences of phase speeds between different modes are notable (this often happens in low frequency source and some shallow water environments). To reduce the bias in bearing estimation, matched field processing methods [2] [3] are applied. But this kind of methods has two disadvantages. Firstly, they have very heavy computation. Secondly, they are sensitive to parameters of the ocean environments. Recently, a method named subspace intersection(SI) was proposed by S. Lakshmipathi and G. V. Anand [4] for bearing estimation. It reduces the computation. Besides, it requires to know the wavenumbers of various normal modes rather than exact environmental parameters. However, this SI method uses QR factorization to construct a bearing estimation function, which is not stable when the matrix is close to being singular.

In this paper, we introduce least square method into the SI method and present the constrained least square SI (CLS-SI) method which is more stable than the original SI method.

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2. SIGNAL MODEL

The signal model used is the same as that described in [4]. Horizontally stratified ocean model is used. Without loss of generality, we consider a uniform linear horizontal array located at depth z with N elements and spacing d between elements. There are J mutually uncorrelated narrowband sources of center frequency f_0. Suppose the jth source is located at depth z_j, ranges r_j with respect to the first array element and bearing θ_j with respect to the broadside direction of the horizontal array. According to the Normal Mode Theory [5], sound pressure impact on the nth (n=1,2,...,N) sensor by the jth source could be expressed as

$$p_{jn} = \sum_{m=1}^{M} b_{mj} e^{i(n-1)k_m d \cos\theta_j} \tag{1}$$

Where

$$b_{mj} = \sqrt{\frac{2\pi}{k_m r_j}} \Psi_m(z_j)\Psi_m(z) e^{-ik_m r_j - \beta_m r_j + i\pi/4} \tag{2}$$

M is the total number of the normal modes, $\Psi_m(z)$, k_m and β_m are the eigenfunction, wavenumber and attenuation coefficient of the mth normal mode, respectively. So the nth element receives the signal by the jth source could be represented by

$$s_{jn} = p_{jn}\eta(t)e^{i2\pi f_0 t} \tag{3}$$

where $\eta(t)$is the envelop of the jth source. Therefore the output of the array could be expressed as

$$\mathbf{y}(t) = \mathbf{P}(\mathbf{X})\boldsymbol{\eta}(t) + \mathbf{n}(t) \tag{4}$$

where $\mathbf{y}(t)=[y_1(t),y_2(t)...,y_N(t)]^T$, $y_n(t)$ is the output of the nth element. $\boldsymbol{\eta}(t)=[\eta_1(t),\eta_2(t),...,\eta_j(t)]^T$, $\mathbf{n}(t)=[n_1(t),n_2(t),...,n_N(t)]^T$ is the array noise vector. Suppose $\boldsymbol{\eta}(t)$ and $\mathbf{n}(t)$ are two independent Gaussian distributed processes and both are with zero mean and unit variance. $\mathbf{X}=[\mathbf{x}_1^T,\mathbf{x}_2^T,...,\mathbf{x}_j^T]$ denote unknown position parameters and $\mathbf{x}_j=[\theta_j,r_j,z_j]^T(j{=}1{,}2{,}...{,}J)$. $\mathbf{P}(\mathbf{X})=[\mathbf{p}(\mathbf{x}_1),\mathbf{p}(\mathbf{x}_2),...\mathbf{p}(\mathbf{x}_j)]$ and $\mathbf{p}(\mathbf{x}_j)=[p_{j1},p_{j2},...,p_{jN}]^T$. $\mathbf{p}(x_j)$ could also be expressed as

$$\mathbf{p}(\mathbf{x}_j)=\mathbf{A}(\theta_j)\mathbf{b}(r_j,z_j),(j=1,2,...,J) \tag{5}$$

where $\mathbf{b}(r_j,z_j)=[b_{1j},b_{2j},...b_{Mj}](j=1,2,...,J)$ are the amplitudes of normal modes excited by the same source, $\mathbf{A}(\theta)=[\mathbf{a}(k_1\cos\theta),\mathbf{a}(k_2\cos\theta),...,\mathbf{a}(k_M\cos\theta)]$. where $\mathbf{a}(k_m\cos\theta)=[1,e^{ik_m d\cos\theta},...,e^{i(N-1)k_m d\cos\theta}]^T$ is the steering vector with respect to the mth mode.

Actually, it could be concluded that each mode is in plane-wave form and signals received by array are the summation of all modes and all sources. In this paper, the wavenumbers of different normal modes are supposed known, since wavenumbers could be precisely estimated by some existing methods [6] etc.

3. REVIEW OF SI METHOD

The idea of SI method [4] is concise. Suppose $\mathcal{S}=span\{\mathbf{p}(\mathbf{x}_1),\mathbf{p}(\mathbf{x}_2),...\mathbf{p}(\mathbf{x}_J)\}$ is the signal subspace and $\mathcal{M}(\theta)=span\{\mathbf{a}(k_1\cos\theta),\mathbf{a}(k_2\cos\theta),...,\mathbf{a}(k_m\cos\theta)\}(\theta\in[0,\pi])$. Then $\mathcal{S}\bigcap\mathcal{M}(\theta)\neq 0$ if and only if $\theta\in\{\theta_1,\theta_2,...,\theta_J\}$. $\mathcal{S}$ could be obtained by decomposing the covariance matrix

$$\begin{aligned}\mathbf{R}=E[\mathbf{y}(t)\mathbf{y}^H(t)]=[\mathbf{p}(\mathbf{x}_1),\mathbf{p}(\mathbf{x}_2),...,\mathbf{p}(\mathbf{x}_J)]\mathbf{R}_s[\mathbf{p}(\mathbf{x}_1),\\ \mathbf{p}(\mathbf{x}_2),...,\mathbf{p}(\mathbf{x}_J)]^H+\sigma^2\mathbf{I}\end{aligned} \tag{6}$$

Assume $\mathbf{R}_s=E[\boldsymbol{\eta}(t)\boldsymbol{\eta}^H(t)]$ is a full rank matrix. Let $\mathbf{u}_1,\mathbf{u}_2$, ..., $\mathbf{u}_N$ indicate the unit-norm eigenvectors of $\mathbf{R}$ arranged in a descending order with respect to the value of their eigenvalues. Thus, $\mathcal{S}=span\{\mathbf{u}_1,\mathbf{u}_2,...,\mathbf{u}_J\}$. The SI method could be described as follows [4]:

(1) Estimate the covariance matrix $\hat{\mathbf{R}}$ from samples.

(2) Compute the eigenvectors of $\hat{\mathbf{R}}$ to get $\hat{\mathbf{u}}_1,\hat{\mathbf{u}}_2,...,\hat{\mathbf{u}}_J$, $\hat{\mathbf{u}}_{J+1},\hat{\mathbf{u}}_{J+2},...,\hat{\mathbf{u}}_N$, $\hat{\mathbf{U}}_s=[\hat{\mathbf{u}}_1,\hat{\mathbf{u}}_2,...,\hat{\mathbf{u}}_J]$ and $\hat{\mathbf{U}}_o=[\hat{\mathbf{u}}_{J+1},\hat{\mathbf{u}}_{J+2},...,\hat{\mathbf{u}}_N]$.

(3) Construct $\hat{\mathbf{D}}(\theta)=[\frac{1}{\sqrt{N}}\mathbf{A}(\theta),\hat{\mathbf{U}}_s]$ and do the QR factorization to get $\hat{\mathbf{D}}(\theta)=\hat{\mathbf{q}}(\theta)\hat{\mathbf{r}}(\theta)$. Let $\hat{r}_{jj}$ indicate the diagonal elements of $\hat{\mathbf{r}}(\theta)$.

(4) Compute the function $B_{SI}(\theta)=[\min\limits_{M+1\leq j\leq M+J}\hat{r}^2_{jj}]^{-1}$. The locations of the peaks correspond to the bearing of sources.

4. LEAST SQUARE SUBSPACE INTERSECTION METHOD

From Section 3, we can see that S. Lakshmipathi and G. V. Anand [4] use the QR decomposition to implement subspace intersection. Actually, the idea of subspace intersection could also be understood that there exist vector $\mathbf{x}(M\times1)$ and $\mathbf{y}(J\times 1)$, for $\theta\in\theta_1,\theta_2,...,\theta_J$

$$\mathbf{A}(\theta)\mathbf{x}=\mathbf{U}_s\mathbf{y} \tag{7}$$

Therefore, we could construct bearing estimation function via equation (7). Because there always exist error between $\mathbf{U}_s$ and $\hat{\mathbf{U}}_s$ due to finite samples and noise, equation(7) does not hold strictly. However, from the perspective of least square, equation(7) could be written as

$$||\mathbf{A}(\theta)\mathbf{x}-\mathbf{U}_s\mathbf{y}||_2^2=0 \tag{8}$$

where $||,||_2$ denotes Euclid norm.

4.1. Constrained least square SI(CLS-SI) method

Obviously, $\mathbf{x}=\mathbf{y}=0$ is the trivial solution of (8). Therefore, $\mathbf{x}$ and $\mathbf{y}$ should be constrained. It is convenient to let $\mathbf{x}$ or $\mathbf{y}$ be a unit vector. In this paper, we define $||\mathbf{y}||=1$. Finally, the question is changed as a constraint satisfaction problem that could be stated as

$$\min_{\mathbf{x},\mathbf{y}}||\mathbf{A}(\theta)\mathbf{x}-\hat{\mathbf{U}}_s\mathbf{y}||^2, subject\ to||\mathbf{y}||=1 \tag{9}$$

For convenience, θ is omitted in derivation. Use Lagrange multiplier method to construct function

$$J(x,y)=||\mathbf{A}(\theta)\mathbf{x}-\hat{\mathbf{U}}_s\mathbf{y}||_2^2+Re\{\mu(\mathbf{y}^H\mathbf{y})-1\} \tag{10}$$

then

$$\frac{\partial J}{\partial\mathbf{x}^*}=\mathbf{A}^H\mathbf{A}\mathbf{x}-\mathbf{A}^H\hat{\mathbf{U}}_s\mathbf{y}=0 \tag{11}$$

$$\frac{\partial J}{\partial\mathbf{y}^*}=(\mu+1)\mathbf{y}-\hat{\mathbf{U}}_s^H\mathbf{A}\mathbf{x}=0 \tag{12}$$

Perform a left multiply on both sides of equation(12) by $\mathbf{y}^H$ and utilize the constraint $||\mathbf{y}||=1$ to get

$$\mu=\mathbf{y}^H\hat{\mathbf{U}}_s^H\mathbf{A}\mathbf{x}-1 \tag{13}$$

Therefore, substitute μ into (12) with (13)

$$\hat{\mathbf{U}}_s^H\mathbf{A}\mathbf{x}=\mathbf{y}^H\hat{\mathbf{U}}_s^H\mathbf{A}\mathbf{x}\cdot\mathbf{y} \tag{14}$$

From (11), we could see that

$$\mathbf{x}=(\mathbf{A}^H\mathbf{A})^\dagger\mathbf{A}^H\hat{\mathbf{U}}_s\mathbf{y} \tag{15}$$

Where † denotes Moore-Penrose inverse. Use (15) to replace $\mathbf{x}$ in (14) to get

$$\hat{\mathbf{U}}_s^H\mathbf{A}(\mathbf{A}^H\mathbf{A})^\dagger\mathbf{A}^H\hat{\mathbf{U}}_s\mathbf{y}=\mathbf{y}^H\hat{\mathbf{U}}_s^H\mathbf{A}(\mathbf{A}^H\mathbf{A})^\dagger\mathbf{A}^H\hat{\mathbf{U}}_s\mathbf{y}\cdot\mathbf{y} \tag{16}$$

Let

$$\mathbf{C}=\hat{\mathbf{U}}_s^H\mathbf{A}(\mathbf{A}^H\mathbf{A})^\dagger\mathbf{A}^H\hat{\mathbf{U}}_s=\hat{\mathbf{U}}_s^H\mathbf{P}_\mathbf{A}\hat{\mathbf{U}}_s \tag{17}$$

where $\mathbf{P_A} = \mathbf{A}(\mathbf{A}^H\mathbf{A})^\dagger\mathbf{A}^H$ is a projection matrix. Plugging (17) into (16), we find that

$$\mathbf{Cy} = \mathbf{y}^H\mathbf{Cy}\cdot\mathbf{y} \tag{18}$$

Substitute (15) into $||\mathbf{Ax} - \hat{\mathbf{U}}_s\mathbf{y}||_2^2$

$$||\mathbf{Ax} - \hat{\mathbf{U}}_s\mathbf{y}||_2^2 = ||[\mathbf{P_A} - \mathbf{I}]\hat{\mathbf{U}}_s\mathbf{y}||_2^2 = 1 - \mathbf{y}^H\mathbf{Cy} \tag{19}$$

From (18) it could be deduced that $\mathbf{y}^H\mathbf{Cy}$ is the eigenvalue according to $\mathbf{C}$. Therefore

$$\min_{\mathbf{x},\mathbf{y}} ||\mathbf{Ax} - \hat{\mathbf{U}}_s\mathbf{y}||_2^2 = 1 - \lambda_{max}(\mathbf{C}) \tag{20}$$

where, $\lambda_{max}(\mathbf{C})$ denotes the largest eigenvalue of $\mathbf{C}$
Thus, the bearing estimation function could be constructed as

$$B_{CLS-SI}(\theta) = \frac{1}{1 - \lambda_{max}(\mathbf{C}(\theta))} \tag{21}$$

4.2. Relation between MUSIC and CLS-SI

In this subsection, we will prove that CLS-SI could be interpreted as an extension of MUSIC [9] to multipath environments. Suppose $\mathbf{A}(\theta)$ could be orthogonalized as

$$\mathbf{A}(\theta)\mathbf{T}(\theta) = \mathbf{V}_s(\theta) \tag{22}$$

where $\mathbf{T}(\theta)$ is a full rank square matrix.
Substituting equation(22) into equation(21), we get

$$B_{CLS-SI}(\theta) = \frac{1}{1 - \lambda_{max}(\hat{\mathbf{U}}_s^H\mathbf{V}_s(\theta)\mathbf{V}_s^H(\theta)\hat{\mathbf{U}}_s)} \tag{23}$$

Assume $\mathbf{V}_o(\theta)$ is an orthogonal basis of $\mathcal{M}^\perp$ which denotes the orthogonal subspace of $\mathcal{M}$ ($\mathcal{M} = span\{\mathbf{A}(\theta)\} = span\{\mathbf{V}_s(\theta)\}$). There exist $\mathbf{V}_o(\theta)^H\mathbf{V}_s(\theta) = 0$, $\mathbf{V}_s(\theta)\mathbf{V}_s(\theta)^H + \mathbf{V}_o\mathbf{V}_o(\theta)^H = \mathbf{I}$. According to the characteristic of matrix 2-norm

$$\lambda_{min}(\hat{\mathbf{U}}_s^H\mathbf{V}_o(\theta)\mathbf{V}_o^H(\theta)\hat{\mathbf{U}}_s) = 1 - \lambda_{max}(\hat{\mathbf{U}}_s^H\mathbf{V}_s(\theta)\mathbf{V}_s^H(\theta)\hat{\mathbf{U}}_s^H) = 1 - ||\mathbf{V}_s^H(\theta)\hat{\mathbf{U}}_s||_2^2 \tag{24}$$

$$\lambda_{min}(\mathbf{V}_s^H(\theta)\hat{\mathbf{U}}_o\hat{\mathbf{U}}_o^H\mathbf{V}_s(\theta)) = 1 - \lambda_{max}(\mathbf{V}_s^H(\theta)\hat{\mathbf{U}}_s\hat{\mathbf{U}}_s^H\mathbf{V}_s(\theta)^H) = 1 - ||\hat{\mathbf{U}}_s^H\mathbf{V}_s(\theta)||_2^2 \tag{25}$$

where $\hat{\mathbf{U}}_o$ is defined in Section 3. Therefore

$$\lambda_{min}(\hat{\mathbf{U}}_s^H\mathbf{V}_o(\theta)\mathbf{V}_o^H(\theta)\hat{\mathbf{U}}_s) = \lambda_{min}(\mathbf{V}_s^H(\theta)\hat{\mathbf{U}}_o\hat{\mathbf{U}}_o^H\mathbf{V}_s(\theta)) \tag{26}$$

Thus, equation(23) could be written as

$$\begin{aligned} B_{CLS-SI}(\theta) &= \frac{1}{\lambda_{min}(\hat{\mathbf{U}}_s^H\mathbf{V}_o(\theta)\mathbf{V}_o^H(\theta)\hat{\mathbf{U}}_s)} \\ &= \frac{1}{\sigma_{min}^2(\hat{\mathbf{U}}_s^H\mathbf{V}_o(\theta))} \end{aligned} \tag{27}$$

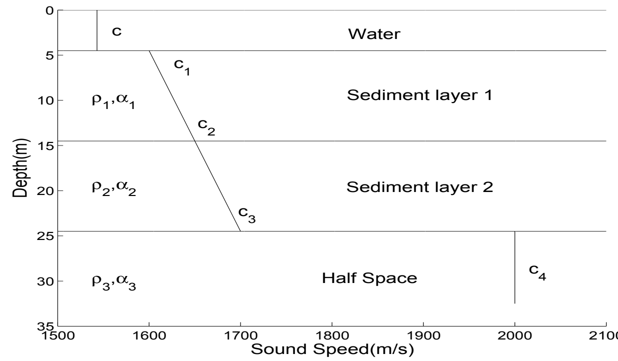

Fig. 1: Channel parameters

where $\sigma(\mathbf{A})$ denotes singular value of matrix $\mathbf{A}$. Plugging equation (26) into equation (27), we get

$$B_{CLS-SI} = \frac{1}{\sigma_{min}^2(\mathbf{V}_s^H(\theta)\hat{\mathbf{U}}_o)} \tag{28}$$

When only one normal mode wave dominates, there exists $\mathbf{V}_s(\theta) = \mathbf{A}(\theta) = \mathbf{a}(k_1\cos\theta/\sqrt{(N)})$. In this situation, equation (28) could be written as

$$B_{CLS-SI} = \frac{N}{\mathbf{a}^H(k_1\cos\theta)\hat{\mathbf{U}}_o\hat{\mathbf{U}}_o^H\mathbf{a}(k_1\cos\theta)} \tag{29}$$

Equation (29) is the same as the MUSIC [9] spectral formula except for the constant factor N.

In summary, CLS-SI is an extension for multiple normal modes of MUSIC and they are equal when only one normal mode wave dominates.

5. SIMULATION

The differences between SI methods and plane-wave methods are more remarkable when the wavenumber of each normal mode differs widely [2]. We simulate a condition on the base of actual environment in Yellow Sea [7] as shown in Fig.1. The channel parameters are as follows. Ratios of density of sediment layers and bottom are $\rho_1 = 1.5g/cm^3$,$\rho_2 = 1.8g/cm^3$,$\rho_3 = 2.0g/cm^3$; the coefficients of attenuation are $\alpha_1 = \alpha_2 = 0.2dB/\lambda$, $\alpha_3 = 0.4dB/\lambda$; sound speed in water is $c = 1543m/s$, sound speeds in sediment layers are $c_1 = 1600m/s$, $c_2 = 1650m/s$, $c_3 = 1700m/s$, $c_4 = 2000m/s$; water depth is $H = 4.5m$, depth of sediment layers are $H_1 = 10m$, $H_2 = 10m$. Uncorrelated sources of frequency $150Hz$ are located at range $5km$. The signal-to-ratio(SNR) is defined as [4]

$$SNR = 10\log_{10}\frac{\sum_{n=1}^{N}\sum_{j=1}^{J}\sigma_j^2|p_{jn}|^2}{N\sigma^2} \tag{30}$$

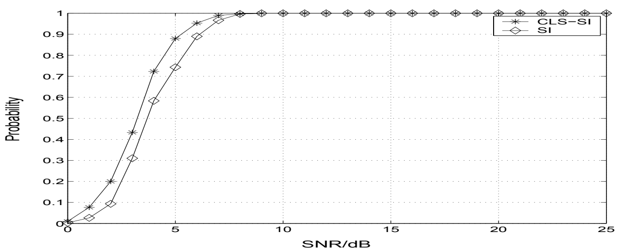

Fig. 2: Probability to distinguish two sources located at 20° and 25°

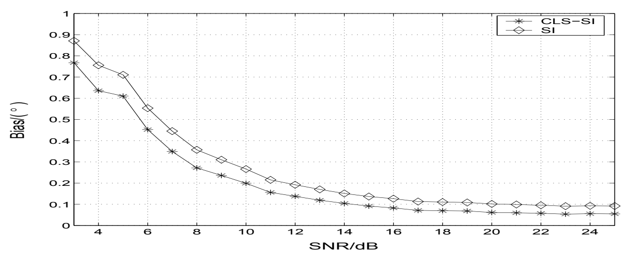

Fig. 3: Bias of estimated sources direction located at 20° and 25°

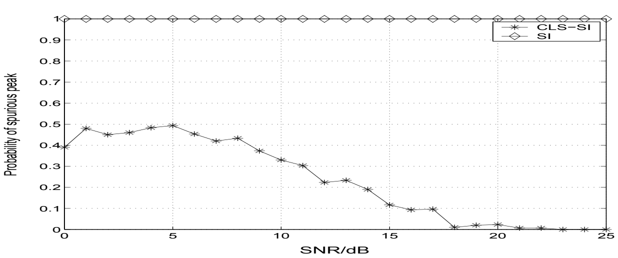

Fig. 4: Probability of appearing spurious peak for sources located at 80° and 85°

where σ^2 is the variance of the zero-mean Gaussian noise and $\sigma_j^2 = E[|\eta_j(t)|^2]$ The uniform linear array has 30 elements with interelement distance $d = 4m$. Acoustic fields are computed by Kraken program [8]. In all simulations, snapshots are 1000. Fig.2 shows the ability of SI and CLS-SI to distinguish two sources located at 20°, 25° respect to different SNR. From Fig.2, we could see that CLS-SI is about ten percentage higher than SI to distinguish the two targets when SNR is lower than $7dB$. Fig.3 shows that CLS-SI has a smaller bias (difference between estimated bearing and real bearing) in bearing estimation. Figs.2-3 demonstrate that CLS-SI could improve resolution in endfire direction.

When θ is close to 90°, the columns of $\mathbf{A}(\theta)$ would not be linear independent and $\hat{\mathbf{D}}(\theta) = [\frac{1}{\sqrt{N}}\mathbf{A}(\theta), \hat{\mathbf{U}}_s]$ would be ill-conditioned. Thus, in some bearing θ_f where no source is located would also appear a spurious peak due to $r_{jj}(\theta_f) \to 0$. In addition, $r_{jj}(\theta_f)$ is the denominator in the process of QR factorization [4], this also would reduce the stability of the SI method. Fig.4 shows that SI could not avoid the spurious peak while CLS-SI could dramatically decrease the probability of spurious peak especially in high SNR.

In summary, CLS-SI method determines the largest eigenvalue instead of QR decompositon in SI method. In addition, Moore-Penrose inverse is applied. These improvements enhance the performance of CLS-SI.

6. CONCLUSIONS

CLS-SI approach is more stable than the original SI method. Firstly, it increases the resolution near the endfire direction. Secondly, it enhances the ability in eliminating the spurious peak around the 90° direction of the array.

7. ACKNOWLEDGEMENT

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