OPTIMAL STRATEGIES FOR DISTRIBUTED DETECTION OVER MULTIACCESS CHANNELS

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ABSTRACT

In this paper, we consider the problem of distributed detection with binary sensors over a noisy multiaccess channel. Under the assumption of conditionally independent sensor observations, we investigate the optimal decision regions at the fusion centre. For a special case, it is shown that the optimal fusion rule can be reduced to a simple threshold test on the signal received by the fusion centre. We will demonstrate that the traditional amplitude-scaling approach to satisfy the power constraint is suboptimal. With a given power budget, a solution featuring in the joint optimization of the local mapping rule and the fusion rule is proposed and is shown to lead to a gain in performance.

Index Terms— Distributed detection, multisensor systems, maximum likelihood detection, multiaccess communication, energy conservation

1. INTRODUCTION

For a binary hypothesis testing problem, distributed solutions utilizing intelligent sensors with each of them making a local decision have attracted considerable attention. Traditionally, the design of such schemes is under the assumption of parallel access channels where each sensor is allocated a dedicated channel to communicate with the fusion centre. In contrast to the parallel setup, several papers (e.g. [1–3]) have considered distributed detection over a multiaccess channel. With a large number of sensors, multiaccess channels will provide higher bandwidth efficiency and better performance. Recently, [4] has applied this idea to the fusion of local sensor decisions. However, one issue that has not been addressed in those papers is how to optimize the system under the given power constraint.

In this paper, the fusion of local binary decisions, which are communicated to the fusion centre over a multiaccess channel, will be studied. Different from [1–3] which focus on systems with a large number of sensors, we would like to consider a more realistic problem where the system is composed of a small to moderate number of sensors. We will study the optimal decision regions at the fusion centre. In an energy limited system, the prevailing approach to satisfy the power constraint is to scale the amplitude of the signal to be transmitted. However, we will show that this design is suboptimal. For a fixed access scheme, we propose to jointly optimize the mapping rule, which maps the local decision into a specified waveform to be transmitted, at local sensors and the fusion rule at the fusion centre such that the best performance is achieved under the given power constraint.

The rest of the paper is organized as follows. The formulation of the problem is presented in the next section. The study of the optimal decision regions can be found in Section 3. In Section 4, a special case is discussed. And the optimal design strategy is proposed in Section 5. We conclude in Section 6.

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2. PROBLEM FORMULATION

Consider the testing of two hypotheses H_0 and H_1 using N distributed binary sensors. The prior probabilities for both hypotheses (denoted by P_0 and P_1 respectively) are assumed known. Let v_i be the observation obtained by the ith sensor. We do not assume any specific distribution for observations but do assume the observations are conditionally independent given H_0 or H_1. Based on its observation, the ith sensor makes a local decision u_i ($u_i = j \in \{0,1\}$ indicates that the sensor is in favour of H_j) and will communicate it to the fusion centre by transmitting a waveform. Such a mapping from local decision u_i to a particular waveform $M_i(u_i)$ is determined by the following mapping rule $M_i(\cdot)$

$$M_i(u_i) = \begin{cases} \sqrt{\alpha_i}(1-l_i) & \text{when } u_i = 1 \\ \sqrt{\alpha_i}(-l_i) & \text{when } u_i = 0 \end{cases},$$

where α_i is a scaling factor and l_i is the bias. Here we make the same assumption as in [1] and [2] that all the sensors are fully synchronized and are allowed to communicate with the fusion centre simultaneously over a multiaccess channel. Thus, the signal w received by the fusion centre will be a

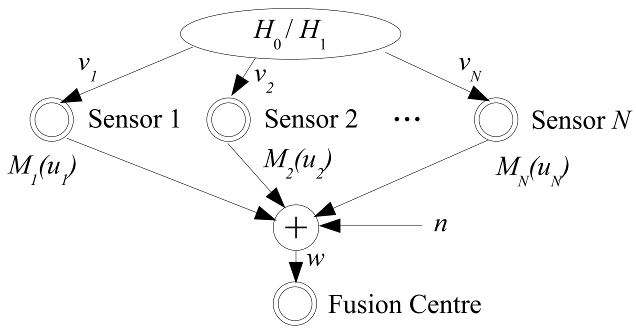

Fig. 1. System diagram

noisy version of the superposition of all $M_i(u_i)$, which can be expressed as

$$w = \sum_{i=1}^{N} M_i(u_i) + n,$$

where n is the zero-mean additive Gaussian channel noise with variance σ^2. This is illustrated in Fig. 1. Upon receiving w, the fusion centre will make a final decision u according to a fusion rule Γ. To conserve energy, each sensor is subject to a power constraint

$$E[M_i(u_i)^2] \leq C, \quad i = 1, \cdots, N, \tag{1}$$

where C is the power budget and $E[\cdot]$ denotes the expectation operation. Our objective is to minimize the system error probability $P_e = \Pr\{u = 1|H_0\} \times P_0 + \Pr\{u = 0|H_1\} \times P_1$ under the power constraint (1).

3. OPTIMAL DECISION REGIONS

In this section we study the optimal decision regions at the fusion centre. The optimal fusion rule can be written as

$$g(w) = f(w|H_1)P_1 - f(w|H_0)P_0 \underset{H_0}{\overset{H_1}{\gtrless}} 0, \tag{2}$$

which means that the fusion centre will decide H_0 if $g(w)$ is smaller than 0 and decide H_1 otherwise. We use $\mathbf{u} = (u_1, u_2, \cdots, u_N)$ to denote the local decision vector. Since each u_i can take value from $\{0, 1\}$, there are altogether 2^N possible values for $\mathbf{u}$, which implies that in general the conditional probability density function under either hypothesis will be a mixture of 2^N Gaussian distributions. Consider a simple example where $N = 2$. Under the assumption of conditionally independent sensor observations, each sensor could be uniquely characterized by its false alarm probability $P_{fi} = \Pr\{u_i = 1|H_0\}$ and its detection probability $P_{di} = \Pr\{u_i = 1|H_1\}$. Thus, $f(w|H_0)$ in (2) can be written as

$$\begin{aligned} f(w|H_0) = {} & \frac{(1-P_{f1})(1-P_{f2})}{\sqrt{2\pi\sigma^2}} e^{-\frac{[w-\sqrt{\alpha_1}(-l_1)-\sqrt{\alpha_2}(-l_2)]^2}{2\sigma^2}} \\ & + \frac{(1-P_{f1})P_{f2}}{\sqrt{2\pi\sigma^2}} e^{-\frac{[w-\sqrt{\alpha_1}(-l_1)-\sqrt{\alpha_2}(1-l_2)]^2}{2\sigma^2}} \\ & + \frac{P_{f1}(1-P_{f2})}{\sqrt{2\pi\sigma^2}} e^{-\frac{[w-\sqrt{\alpha_1}(1-l_1)-\sqrt{\alpha_2}(-l_2)]^2}{2\sigma^2}} \\ & + \frac{P_{f1}P_{f2}}{\sqrt{2\pi\sigma^2}} e^{-\frac{[w-\sqrt{\alpha_1}(1-l_1)-\sqrt{\alpha_2}(1-l_2)]^2}{2\sigma^2}}, \end{aligned} \tag{3}$$

which is a Gaussian mixture with 2^2 components. By replacing P_{fi} with P_{di}, we will obtain the expression for $f(w|H_1)$.

As we can observe, the number of components in the Gaussian mixture grows exponentially with the number of sensors. To perform such a test (2) will result in heavy computational loads and complex implementations when there are many sensors participating in the observation. In the following, we provide a numerical approach to alleviate the problem. Define Ω_0 to be the set in the domain of w such that $g(w)$ is smaller than 0 and Ω_1 to be the complement set of Ω_0. Ω_0 and Ω_1 are also known as the decision regions for H_0 and H_1 respectively. Then we have the following theorem.

Theorem 1 *The boundaries of Ω_0 and Ω_1 can be obtained by solving an equation which has the following form*

$$\beta_M x^{b_M} + \beta_{M-1} x^{b_{M-1}} + \cdots + \beta_1 x^{b_1} + \beta_0 = 0, \tag{4}$$

where $\beta_0, \beta_1, \cdots, \beta_M$ are real numbers, $b_1, b_2, \cdots, b_M$ are positive real numbers and x can take only positive real values.

The proof is omitted due to space limitations.

4. A SPECIAL CASE

In this section we consider a special case, where $P_{fi} = P_f$, $P_{di} = P_d$, $\alpha_i = \alpha$ and $l_i = l$ for all i (one possible scenario for this case will be the situation when we have homogeneous sensors which adopt the same local decision and mapping rule). In this case, the conditional probability density function of w under H_0 is given by

$$f(w|H_0) = \sum_{i=0}^{N} \binom{N}{i} \frac{(1-P_f)^{N-i} P_f^i}{\sqrt{2\pi\sigma^2}} e^{-\frac{[w-(i\sqrt{\alpha}-N\sqrt{\alpha}l)]^2}{2\sigma^2}}.$$

Here the number of components in the Gaussian mixture is reduced to $N+1$. Again, $f(w|H_1)$ can be obtained by replacing P_f with P_d. The boundaries of Ω_0 and Ω_1 can be found by solving $g(w) = 0$. After algebraic simplification, $g(w) = 0$ becomes

$$\sum_{i=0}^{N} \Bigg\{ \left[(1-P_f)^{N-i} P_f^i P_0 - (1-P_d)^{N-i} P_d^i P_1\right] \times \binom{N}{i} e^{-\frac{(i\sqrt{\alpha})^2}{2\sigma^2}} x^i \Bigg\} = 0, \tag{5}$$

where

$$x = e^{\frac{\sqrt{\alpha}(w+N\sqrt{\alpha}l)}{\sigma^2}}. \tag{6}$$

Notice that (5) has the form of (4).

Before exploring the optimal fusion rule for this special case, we provide one lemma and one theorem.

Lemma 1 *Let A_0 and A_1 be two positive real numbers, K be a positive integer, $0 < p < 1$ and $0 < q < 1$. The following sequence*

$$S_k = A_0(1-p)^{K-k}p^k - A_1(1-q)^{K-k}q^k$$

with $k = 0, 1, \cdots, K$ will change its sign at most once.

The proof is omitted due to space limitations.

Theorem 2 *If there is one sign change in the coefficients of polynomial $p(x)$, then $p(x)$ has exactly one positive root.*

Proof Since there is one sign change, the sign of $p(x)$ when $x > 0$ is sufficiently small and the sign of $p(x)$ when $x > 0$ is sufficiently large will be different. By the continuity of $p(x)$, we know that there will be at least one positive root for $p(x)$. It is also shown in [5] that $p(x)$ will have at most one positive root when there is one sign change in the coefficients of $p(x)$. And the result follows. ∎

Based on Lemma 1 and Theorem 2, we have

Theorem 3 *The optimal fusion rule will be a threshold test on w.*

Proof The coefficients of x^i on the left-hand side of (5) will change sign at most once according to Lemma 1. If the sign doesn't change, the fusion centre will either always decide H_0 or always decide H_1. So the threshold of the test will be either ∞ or $-\infty$. If the sign of the coefficients changes once, (5) will have one positive root by Theorem 2. And the left-hand side of (5) will change sign at this unique root. Since x is a strictly increasing function of w as shown by (6), $g(w)$ will also have one root and the sign of $g(w)$ will change at this root. Thus (2) can be reduced to a threshold test on w. ∎

In the following, we denote the threshold for w as T. And Theorem 3 suggests that T will be the boundary of Ω_0 and Ω_1.

5. OPTIMAL DESIGN STRATEGIES

As we have mentioned before, our objective is to minimize the error probability under the power constraint

$$\begin{aligned} &\min \quad P_e \\ &\text{s.t.} \quad E[M_i(u_i)^2] \le C, \quad i = 1, \cdots, N. \end{aligned} \tag{7}$$

For any fixed P_{fi} and P_{di} and the given multiaccess channel, P_e only depends on the local mapping rule and the fusion rule. Thus the joint optimization of the local mapping rule and the fusion rule will provide the minimum error probability.

Continuing the discussion of the special case, we provide the optimal system design in this section. Use $M(\cdot)$ to denote the common local mapping rule. Due to the symmetry of the system, the power constraints in (7) will be satisfied if and only if the power constraint for any one of the sensors is satisfied. So, (1) can be simplified as $E[M(u_1)^2] \le C$.

When (5) has a unique positive root x_r (i.e. the sign of the coefficients of x^i in (5) changes once), the threshold T can then be derived from (6) and is given by

$$T = \frac{\sigma^2 \ln x_r}{\sqrt{\alpha}} - N\sqrt{\alpha}l. \tag{8}$$

In the case where $P_d > P_f$, we have $P_f^N P_0 - P_d^N P_1 < 0$. Then for all sufficiently large x, the sign of the polynomial on the left-hand side of (5) will be negative. Therefore, $g(w) > 0$ for all sufficiently large w and the optimal fusion rule can be written as

$$w \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{9}$$

For such a test (9), the error probability can be expressed as

$$\begin{aligned} P_e &= P_F P_0 + (1-P_D)P_1 \\ &= \sum_{i=0}^{N} \Big\{ \big[(1-P_f)^{N-i}P_f^i P_0 - (1-P_d)^{N-i}P_d^i P_1\big] \\ &\quad \times \binom{N}{i} Q\left(\frac{T - (i\sqrt{\alpha} - N\sqrt{\alpha}l)}{\sigma}\right)\Big\} + P_1, \end{aligned} \tag{10}$$

where P_F and P_D denote, respectively, the false alarm probability and the detection probability of the fusion centre and $Q(\cdot)$ is the complementary cumulative distribution function of standard Gaussian. We will now establish the monotonic property of P_e.

Theorem 4 *Let $\eta = \sqrt{\alpha}/\sigma$, P_e is a monotonically decreasing function of η.*

Proof We prove this by showing that the first derivative of P_e with respect to η is negative. Since the sign of the coefficients of x^i in (5) changes once, there must be a λ such that the coefficients of x^i $(i > \lambda)$ are all negative and the coefficients of x^i $(i \le \lambda)$ are all non-negative. From (10), we have

$$\begin{aligned} \frac{dP_e}{d\eta} &= \sum_{i=0}^{N} \Big\{ \big[(1-P_f)^{N-i}P_f^i P_0 - (1-P_d)^{N-i}P_d^i P_1\big] \\ &\quad \times \binom{N}{i} \frac{-e^{-\left(\frac{\ln x_r}{\sqrt{2}\eta} - \frac{i\eta}{\sqrt{2}}\right)^2}}{\sqrt{\pi}} \frac{d\left(\frac{\ln x_r}{\sqrt{2}\eta} - \frac{i\eta}{\sqrt{2}}\right)}{d\eta}\Big\}. \end{aligned}$$

By writing

$$\frac{d\left(\frac{\ln x_r}{\sqrt{2}\eta} - \frac{i\eta}{\sqrt{2}}\right)}{d\eta} = \frac{d\left(\frac{\ln x_r}{\sqrt{2}\eta}\right)}{d\eta} - \frac{\lambda}{\sqrt{2}} + \frac{\lambda - i}{\sqrt{2}},$$

and using the fact that x_r is the root of (5), we will finally have

$$\frac{dP_e}{d\eta} = \sum_{i=0}^{N} \Bigg\{ \left[(1-P_f)^{N-i} P_f^i P_0 - (1-P_d)^{N-i} P_d^i P_1\right] \times \binom{N}{i} \frac{-1}{\sqrt{\pi}} e^{-\left(\frac{\ln x_r}{\sqrt{2}\eta} - \frac{i\eta}{\sqrt{2}}\right)^2} \frac{\lambda - i}{\sqrt{2}} \Bigg\}.$$

We have $[(1-P_f)^{N-i}P_f^iP_0 - (1-P_d)^{N-i}P_d^iP_1](\lambda - i) > 0$ when $i > \lambda$ and $[(1-P_f)^{N-i}P_f^iP_0 - (1-P_d)^{N-i}P_d^iP_1](\lambda - i) \geq 0$ when $i \leq \lambda$. As a result, $dP_e/d\eta < 0$. ∎

Following from Theorem 4, problem (7) is now equivalent to

$$\begin{aligned} \max \quad & \frac{\sqrt{\alpha}}{\sigma} \\ \text{s.t.} \quad & E[M(u_1)^2] \leq C. \end{aligned} \tag{11}$$

The constraint in (11) could be expanded as

$$\begin{aligned} E[M(u_1)^2] &= \alpha\{\hat{P}E[(1-l)^2] + (1-\hat{P})E[(-l)^2]\} \\ &= \alpha[(1-2l)\hat{P} + l^2] \\ &\leq C, \end{aligned}$$

where $\hat{P} = P_f P_0 + P_d P_1$. The maximum $\sqrt{\alpha}/\sigma$ could be achieved by setting $l = \hat{P}$ and the optimal local mapping rule $M(\cdot)$ will be given by

$$M(u_i) = \begin{cases} \sqrt{\frac{C}{\hat{P}-\hat{P}^2}}(1-\hat{P}) & \text{when } u_i = 1 \\ \sqrt{\frac{C}{\hat{P}-\hat{P}^2}}(-\hat{P}) & \text{when } u_i = 0 \end{cases}.$$

And the threshold T for the corresponding optimal fusion rule can be obtained by substituting $\alpha = C/(\hat{P} - \hat{P}^2)$ and $l = \hat{P}$ into (8). The same result can be proved for the case where $P_d < P_f$. Notice that the optimal bias l depends on P_f, P_d, P_0 and P_1 and generally will not be equal to 0 or $\frac{1}{2}$ as has been assumed in many other works.

By jointly optimizing the local mapping rule and the fusion rule, the performance will be improved. In Fig. 2, we compare the optimal scheme to two non-optimal schemes (with l set to be 0 and $\frac{1}{2}$) for the case where $N = 5$, $P_d = 0.9$, $P_f = 0.1$ and $\frac{C}{\sigma^2} = 1$. As can be observed, for local sensors with high P_d and low P_f, the performance gap between the optimal scheme and the scheme with $l = 0$ will be negligible only when P_1 is close to 0. And the performance gap between the optimal scheme and the scheme with $l = \frac{1}{2}$ will be negligible only when P_1 is close to 0.5.

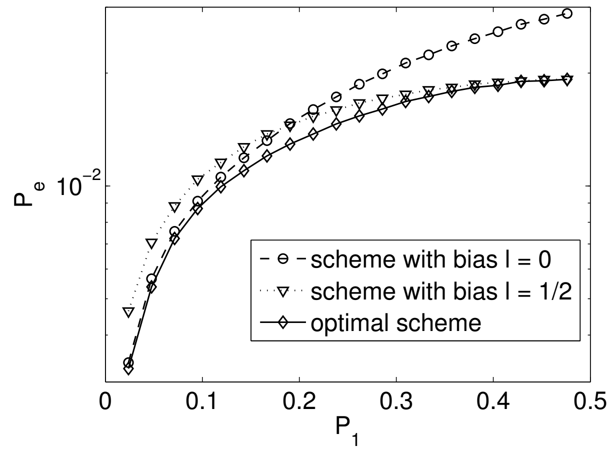

Fig. 2. Performance comparison

6. CONCLUSIONS

For distributed detection over a multiaccess channel, we investigate the design of the fusion rule at the fusion centre and the local mapping rule for binary sensors. A numerical method is proposed to find the decision regions for both hypotheses. In a special case, the optimal fusion rule is proved to be a simple threshold test on the received signal. For an energy limited detection system with an unreliable communication channel, we reveal the importance of the local mapping rule design. As demonstrated by simulations, a gain in performance is achieved by jointly optimizing the local mapping rule and the fusion rule.

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