NON-ITERATIVE MULTIUSER MIMO COORDINATED BEAMFORMING WITH LIMITED FEEDFORWARD

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ABSTRACT

This paper proposes non-iterative coordinated beamforming algorithms for a multiuser MIMO (multiple input multiple output) system with multiple antennas at the transmitter and multiple users, each with multiple receive antennas. The transmitter uses linear beamforming to convey information to each user, while each receiver uses a quantized combining vector, sent from the transmitter via a low-rate feedforward control channel. Two different algorithms for optimizing transmit beamformers and receive combining vectors are proposed: a joint optimization and a greedy search. Simulations show that the proposed methods using quantized codebooks approach the sum capacity of the MIMO broadcast channel.

Index Terms— MIMO systems, multiuser channels, space division multiplexing

1. INTRODUCTION

The multiple input multiple output (MIMO) broadcast channel achieves high capacity on the downlink by coordinating the transmissions to multiple users simultaneously [1]. The optimal transmit strategy given by information theory is dirty paper coding (DPC) [2], which is required to achieve points on the boundary of the achievable rate region. Unfortunately, DPC does not directly lead to a realizable transmission strategy [3]. Consequently, there has been substantial interest in developing linear transmission strategies that approach the performance of DPC and are easier to realize in practice [4–9].

One linear strategy is channel inversion, where the beamforming vectors are derived from the inverse of the effective multiuser channel at the transmitter. Channel inversion, though, is only known to work for one receive antenna per user and suffers from a power penalty, which is a function of the smallest singular value of the effective channel matrix [8]. A related strategy is block diagonalization (BD) [4, 6] for situations with multiple antennas and multiple data streams intended for each user. Block diagonalization enforces a zero interference property at each user but requires that the number of receive antennas is equal to the number of data streams. It is possible to improve block diagonalization through transmit antenna selection or eigenmode selection [10] when additional transmit antennas are available, and through receive antenna selection [9] or combining [11] when extra receive antennas are available but the transmitter and the receivers have not been jointly optimized. Coordinated beamforming algorithms work similarly to block diagonalization but allow fewer streams than receive antennas [5, 7, 9] by jointly optimizing the transmit beamforming vectors and receive combining vectors. Unfortunately, this requires a joint optimization and the convergence of the iteration-based algorithms in [5, 7, 9] cannot be guaranteed. Moreover, prior work on coordinated beamforming assumes that the effective channel for each user, which combines transmit beamforming and the user's channel, is known at each user [4–7]. Developing standards like 3G long term evolution (LTE), however, include only common pilots [12]. Thus in 3GPP-LTE, there is no way to estimate the effective channel gain at the mobile station (MS). Hence, the receiver cannot estimate the optimal post-processing receiver coefficients.

In this paper, we propose to quantize the receive beamforming algorithms and send the quantized beamformers to each user via a low-rate feedforward control channel. Using the fact that the receive beamformers are quantized, we propose two non-iterative algorithms for jointly designing the transmit beamformers and receive combining vectors. The first algorithm performs an exhaustive codebook search over all users, but maximizes the sum rate assuming quantized receive combining vectors. The second algorithm is a greedy quantization with lower complexity but good performance. Numerical simulations are provided to illustrate performance of the proposed system in Rayleigh fading channels with and without channel estimation error. The results show that the joint optimization and greedy optimization closely approach the performance of the iterative coordinated beamforming algorithms and the sum capacity.

2. SYSTEM MODEL

In this section we review the MIMO broadcast signal model under consideration and describe the general concept of limited feedforward receive beamforming.

2.1. MIMO Broadcast Signal Model

Consider a multi-user MIMO broadcast channel with N_t transmit antennas and N_r receive antennas of K users as shown in Fig. 1. We assume that the channel is flat fading, which can be realized practically with MIMO-OFDM. The channel between the transmitter and user k is represented by a matrix $\mathbf{H}_k$ of size $N_r \times N_t$ with complex entries [1].

We assume that the system uses time division duplex (TDD) with perfect radio frequency (RF) calibration in which the temporal variations of the channel are slow compared to the duration of

[*]This work was supported by Samsung Electronics.

[1]In this paper, we assume that upper case and lower case boldface are used to denote matrices $\mathbf{A}$ and vectors $\mathbf{a}$, respectively. If $\mathbf{A}$ denotes a complex matrix, and $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^{-1}$, and $\mathbf{A}^\dagger$ denote the transpose, Hermitian, inverse, and pseudo inverse of $\mathbf{A}$, respectively. $[\mathbf{A}]_k$ is the k-th column of matrix $\mathbf{A}$.

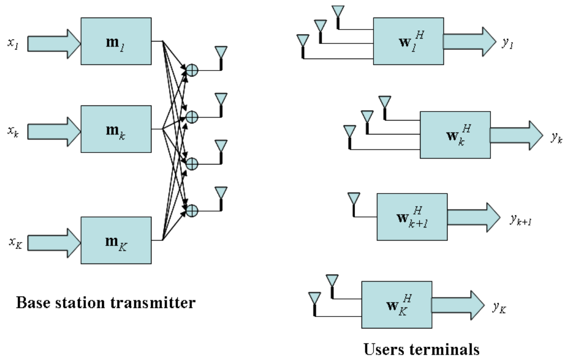

Fig. 1. Transmission model with linear processing at the transmitter and at the receivers

the downlink and uplink frames. Thus the channel can be assumed to be approximately constant over several frames, and to be the same for the downlink and uplink. With TDD, it is reasonable to assume that the transmitter and the receivers can estimate the same channel using channel sounding [13]. Thus we assume that $\{\mathbf{H}_k\}$ are known perfectly at the transmitter. This assumption is used in most other work on BD and coordinated beamforming [4–7].

Let x_k denote the k^{th} transmit symbol and $\mathbf{n}_k$ denote the additive white Gaussian noise vector with variance σ_k^2 per entry observed at the receiver. Let $\mathbf{f}_k$ denote the transmit beamformer (assumed unit norm) and $\mathbf{w}_k$ denote the combining vector for the k^{th} user. The received signal at the k^{th} user after combining is

$$y_k = \mathbf{w}_k^* \mathbf{H}_k \mathbf{f}_k x_k + \mathbf{w}_k^* \mathbf{H}_k \sum_{l=1, l \neq k}^{K} \mathbf{f}_l x_l + \mathbf{w}_k^* \mathbf{n}_k. \qquad (1)$$

2.2. Limited Feedforward Receive Beamforming

Quantized transmit beamforming, known as limited feedback beamforming, is a popular technique used to inform the transmitter of the desired transmit beamforming vector in single user MIMO systems [14]. The authors in [15] proposed to use the limited feedback concept to inform each receiver of $\mathbf{f}_k$, since the receiver can estimate $\mathbf{w}_k$ with $_k$ using maximum ratio combining (MRC). The algorithm in [15], however, performs codebook quantizatation after iterative-optimizations while the proposed algorithms which will be explained in the next section combine the quantization with non-iterative optimizations. It is important that the $\mathbf{w}_k$ is estimated based on the forwarded information $\mathbf{f}_k$ because the multiuser interference cancellation hinges on the selected $\mathbf{f}_k$.

To explain this concept in more detail, suppose that the transmitter and receiver have predesigned codebooks denoted by $C = \{\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_{2^{N_b}}\}$ having N_b-bit codewords. Further suppose that the transmitter has derived the optimum set of codewords $\{\mathbf{f}_k\}_{k=1}^{K}$. Let $\mathcal{Q}(\mathbf{f}_k)$ denote the quantized value of $\mathbf{f}_k$, then the k^{th} receiver observes after receive beamforming (i.e., matched filter)

$$\begin{aligned} y_k =& \mathcal{Q}(\mathbf{f}_k)^* \mathbf{H}_k^* \mathbf{H}_k \mathbf{f}_k x_k + \mathcal{Q}(\mathbf{f}_k)^* \mathbf{H}_k^* \mathbf{H}_k \sum_{l=1, l \neq k}^{K} \mathbf{f}_l x_l \\ &+ \mathcal{Q}(\mathbf{f}_k)^* \mathbf{H}_k^* \mathbf{n}_k. \end{aligned}$$

Note in particular that *the transmitter uses the optimal* $\mathbf{f}_k$ *while the receiver uses the quantized* $\mathcal{Q}(\mathbf{f}_k)$. The received signal-to-interference-plus-noise ratio (SINR) for the user k with the quantized beamformer can be expressed as

$$\text{SINR}_k = \frac{|\mathcal{Q}(\mathbf{f}_k)^* \mathbf{R}_k \mathbf{f}_k|^2}{\mathcal{Q}(\mathbf{f}_k)^* \left(\sum_{l=1, l \neq k}^{K} \mathbf{R}_k \mathbf{f}_l \mathbf{f}_l^* \mathbf{R}_k \right) \mathcal{Q}(\mathbf{f_k}) + \sigma_{\text{eff}}^2},$$

where $\mathbf{R}_k = \mathbf{H}_k^* \mathbf{H}_k$ and $\sigma_{\text{eff}}^2 = \mathcal{Q}(\mathbf{f_k})^H \mathbf{H}_k^* \mathbf{H}_k \mathcal{Q}(\mathbf{f_k}) \sigma^2$.

It is well known that the optimum solution for the transmit beamformer and the receive combining vectors in the interference free link are the right and left singular vectors that correspond to the maximum singular value of the channel matrix. Quantization of $\mathbf{f}_k$ thus can be solved using well-known Grassmannian codebooks in the case of an independent identically distributed complex Gaussian channel [14]. This codebook choice seems reasonable in the case of interference because in the absence of interference, the right singular vector is isotopic, and Grassmannian codebooks respect that isotropic property. Finding a provably optimal codebook is a topic of future research.

3. COORDINATED BEAMFORMING ALGORITHMS WITH QUANTIZED FEEDFORWARD

3.1. Coordinated Beamforming

In the coordinated transmission strategies [5] [7] [9], the transmitter chooses $\mathbf{f}_k$ such that the subspace spanned by its columns lies in the null space of $\mathbf{w}_l^* \mathbf{H}_l$ $(\forall l \neq k)$, that is, $\mathbf{w}_l^* \mathbf{H}_l \mathbf{f}_k = 0$ for $l = 1, \ldots, k-1, k+1, \ldots, K$. If chosen in this way, $\mathbf{f}_k$ will then cause zero interference to user l by completely removing the interference term in (1). Essentially, the algorithms in [5] [7] [9] form an equivalent channel matrix $\tilde{\mathbf{H}}_k$ for the k^{th} user

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \cdots & (\mathbf{w}_{k-1}^* \mathbf{H}_{k-1})^T & (\mathbf{w}_{k+1}^* \mathbf{H}_{k+1})^T & \cdots \end{bmatrix}^T,$$

and then they iteratively solve for a transmit beamformer $\mathbf{f}_k$ that satisfies $\tilde{\mathbf{H}}_k \mathbf{f}_k = \mathbf{0}$ to ensure that after combining, the signal received at each user is interference free.

Assuming that $K = N_t$ and that the channels are sufficiently rich, $\tilde{\mathbf{H}}_k$ will be full-rank and of dimension $(K-1) \times K$, thus the null-space has dimension one and there is only one zero singular value. Let the singular value decomposition (SVD) of $\tilde{\mathbf{H}}_k$ be

$$\tilde{\mathbf{H}}_k = \tilde{\mathbf{U}}_k \tilde{\mathbf{D}}_k \begin{bmatrix} \tilde{\mathbf{V}}_k^{(1)} & \tilde{\mathbf{v}}_k^{(0)} \end{bmatrix}^*,$$

where $\tilde{\mathbf{U}}_k$ and $\tilde{\mathbf{D}}_k$ denote the left singular matrix and the diagonal singular value matrix, respectively, and $\tilde{\mathbf{V}}_k^{(1)}$ and $\tilde{\mathbf{v}}_k^{(0)}$ are the right singular matrix and vector each corresponding to non-zero singular values and zero singular value, respectively. User k's beamforming vector has to lie in the space spanned by $\tilde{\mathbf{v}}_k^{(0)}$, consequently, we take $\mathbf{f}_k = \tilde{\mathbf{v}}_k^{(0)}$.

Assuming that maximum ratio combining (MRC) is used at the receiver, which is optimal under the zero interference constraint, $\mathbf{w}_k = \mathbf{H}_k \mathbf{f}_k$ thus the solution to $\mathbf{f}_k$ depends on $\{\mathbf{f}_n\}_{n \neq k}$. This makes it difficult for each user to compute their optimal combining vector since this requires that they know channel state information from the other users. To solve this problem, we propose joint beamforming strategies where only the quantized beamforming information is sent to each user.

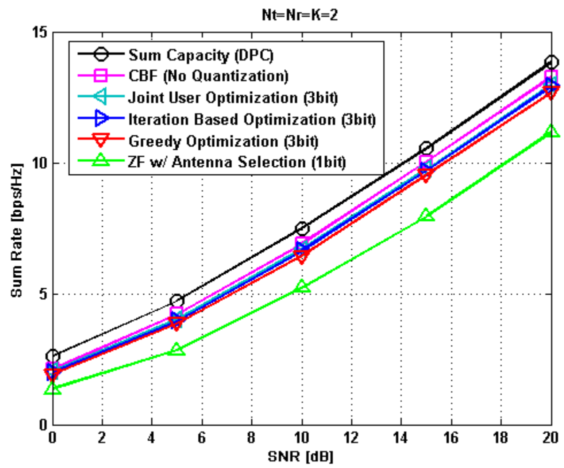

Fig. 2. Sum rates vs. SNR for the coordinated beamforming with joint user quantization, with greedy quantization, with independent user quantization, and zero-forcing beamforming with receive antenna selection

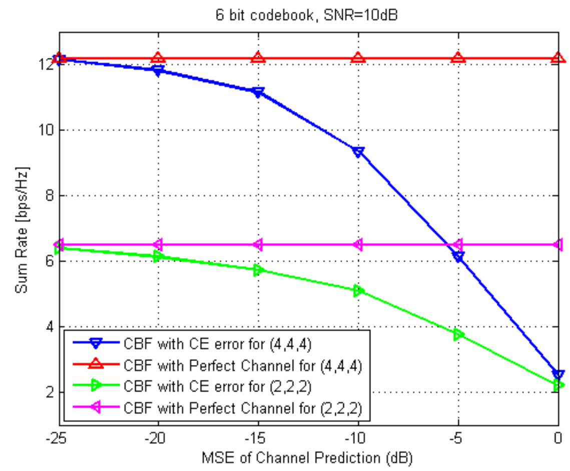

Fig. 3. Achievable rate degradation due to channel estimation and prediction error

3.2. Proposed Joint User Quantization

We assume that the transmitter and the receivers have a shared codebook set $C = \{\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_{2^{N_b}}\}$. At the initialization, the transmitter computes the matched channel matrix $\mathbf{R}_k = \mathbf{H}_k^* \mathbf{H}_k$ for each user where k is the user index. With the stored codebook set, the received SINR for each user is expressed by

$$\text{SINR}_{k,quant}(i_1, i_2, \cdots, i_K) = |\mathbf{c}_{i_k}^* \mathbf{R}_k \mathbf{c}_{i_k}|^2 / \mathbf{c}_{i_k}^H \sum_{i_l \neq i_k}^{i_K} \mathbf{R}_k \mathbf{c}_{i_l} \mathbf{c}_{i_l}^* \mathbf{R}_k \mathbf{c}_{i_k} + \mathbf{c}_{i_k}^* \mathbf{R}_k \mathbf{c}_{i_k} \sigma^2, \quad (2)$$

where i_k is the codebook index for the k^{th} user and the effective channel matrix is given by

$$\mathbf{H}_{\text{eff}}(i_1, i_2, \cdots, i_K) = [(\mathbf{c}_{i_1}^* \mathbf{R}_1)^T \cdots (\mathbf{c}_{i_K}^* \mathbf{R}_K)^T]^T. \quad (3)$$

Based on the effective channel matrix combined with the quantized receive combining vectors, the transmitter computes the transmit beamformers $\hat{\mathbf{f}}_k$ with $\mathbf{R}_k$ and $\mathbf{c}_{i_k}$ where $k = 1, \cdots, K$ as follows:

$$\hat{\mathbf{f}}_k(i_1, \cdots, i_K) = \frac{[\mathbf{H}_{\text{eff}}^\dagger(i_1, i_2, \cdots, i_K)]_k}{\|[\mathbf{H}_{\text{eff}}^\dagger(i_1, i_2, \cdots, i_K)]_k\|}.$$

The transmitter computes the achievable rate for each user with the computed transmit beamformer $\hat{\mathbf{f}}_k$ and the quantized codebook $\mathbf{c}_{i_k}$ as

$$R_k(i_1, i_2, \cdots, i_K) = \log_2 \left(1 + \frac{|\mathbf{c}_{i_k}^* \mathbf{R}_k \hat{\mathbf{f}}_{i_k}|^2}{\mathbf{c}_{i_k}^* \sum_{l \neq i_k}^{i_K} \mathbf{R}_k \mathbf{c}_{i_l} \mathbf{c}_{i_l}^* \mathbf{R}_k \mathbf{c}_{i_k} + \mathbf{c}_{i_k}^* \mathbf{R}_k \mathbf{c}_{i_k} \sigma^2} \right),$$

and finds the codebook indices for all users maximizing the sum rate as follows:

$$(\hat{i}_1, \hat{i}_2, \cdots, \hat{i}_K) = \arg \max_{i_1, i_2, \cdots, i_K} \sum_{k=1}^{K} R_k(i_1, i_2, \cdots, i_K).$$

With the codebook indices $(\hat{i}_1, \hat{i}_2, \cdots, \hat{i}_K)$, the transmitter recomputes the effective channel matrix $\mathbf{H}_{\text{eff}}(\hat{i}_1, \cdots, \hat{i}_K) = [(\mathbf{c}_{\hat{i}_1}^* \mathbf{R}_1)^T \cdots (\mathbf{c}_{\hat{i}_K}^* \mathbf{R}_K)^T]^T$ and the final transmit beamformers $\bar{\mathbf{f}}_k = [\mathbf{H}_{\text{eff}}^\dagger(\hat{i}_1, \hat{i}_2, \cdots, \hat{i}_K)]_k / \|[\mathbf{H}_{\text{eff}}^\dagger(\hat{i}_1, \hat{i}_2, \cdots, \hat{i}_K)]_k\|$. This is the best solution for the limited feedforward problem for the multi-user MIMO system since the transmitter searches the combining vector over all codeword combinations and computes the transmit beamformers based on the combining vector achieving the max sum rate. The search complexity is dominated by the number of transmit antennas, codebook size, number of users, and sum rate computation but it does not require any iteration used in [15].

3.3. Proposed Greedy-based Quantization

The exhaustive search solution introduced in the previous section exponentially increases in complexity as the number of transmit antennas N_t and the codebook size N_b increase. In this section, we propose an alternative solution where the exhaustive search is simplified to greedy selection.

The beamforming vector that maximizes the numerator term in (2) is given by the eigen vector corresponding to the largest eigenvalue of the channel matrix $\mathbf{H}_k$. That is,

$$\bar{\mathbf{f}}_k = [\bar{\mathbf{U}}_k]_1.$$

The interference term in the denominator is minimized by selecting $\bar{\mathbf{f}}_l$ so that $\bar{\mathbf{f}}_l$ lies in the space orthogonal to $\bar{\mathbf{f}}_k$. That is, $[\mathbf{U}_k]_p$ with $p \neq 1$ can be chosen to minimize the denominator. This selection criterion guarantees that the desired signal is transmitted along the channel mode with the highest gain while nulling the interference with respect to user k.

Based on the above selection, we propose the following algorithm that optimizes the beamforming vector for user k and successively assigns null space to the other users. At the initialization, the transmitter computes the matched channel matrix identical to the joint user quantization case and performs eigen value decomposition to get the eigenvectors corresponding to the largest eigenvalue. The transmitter then orders all users from the highest channel gain to the lowest lowest channel gain. The transmit beamformer for the first user who has the best channel gain is given by $\bar{\mathbf{f}}_1 = [\bar{\mathbf{U}}_1]_1$. For the second user, we compute the correlation between the second user's principal eigenvector and the first user's eigenvectors to find the transmit beamformer and repeat this procedure until all users are allocated to each eigenvector of $\bar{\mathbf{U}}_k$. The transmit beamformer for the k^{th} user can be found by

$$\bar{\mathbf{f}}_k = \arg \min_{p \in \{1,2,\cdots,K\}-\{\text{selected index}\}} \left| [\bar{\mathbf{U}}_k]_1^* [\bar{\mathbf{U}}_1]_p \right| \qquad (4)$$

where, $[\bar{\mathbf{U}}_1]_p$ is the p^{th} column of the eigenvector matrix $\bar{\mathbf{U}}_1$ for the first user. We next compute SINR with the transmit beamformers in (4) as

$$R_k(i_k) = \log_2 \left(1 + \frac{|\mathbf{c}_{i_k}^* \mathbf{R}_k \bar{\mathbf{f}}_k|^2}{\mathbf{c}_{i_k}^* \sum_{l \neq i_k}^{i_K} \mathbf{R}_k \bar{\mathbf{f}}_l \bar{\mathbf{f}}_l^* \mathbf{R}_k \mathbf{c}_{i_k} + \mathbf{c}_{i_k}^H \mathbf{R}_k \mathbf{c}_{i_k} \sigma^2} \right),$$

and the codebook index for maximizing the user k's sum rate is given by

$$\hat{i}_k = \arg \max_{i_k} R_k(i_k).$$

After finding the quantized combining vectors, the effective channel matrix in (3) is recalculated and the final transmit beamformer can be found in the same manner as in the joint user quantization problem.

4. SUM RATE COMPARISON

For simulations we model the elements of each user's channel matrix as independent complex Gaussian random variables with zero mean and unit variance $\mathcal{CN}(0, 1)$. Fig. 2 compares the sum rate of the proposed coordinated beamforming algorithms (*i. joint user quantization*, *ii. greedy-based quantization*) with DPC [2], coordinated beamforming [9, 15], and zero-forcing beamforming with receive antenna selection [9]. Note that the transmitter needs to feedforward only a one bit index for the zero-forcing beamforming with receive antenna selection where 2 transmit antennas at the transmitter, 2 receive antennas at the receiver and 2 users in the network. The proposed joint user quantization approaches to the sum capacity even with 3 bit quantization and the gap is decreased as the quantization level increases but the computational complexity also would be increased. For this scenario, there is a small gap between the proposed greedy-based quantization and the iteration-based optimization in [15].

In real systems, the transmitter and the receivers may not be able to utilize perfect channel information due to calibration error and practical channel estimators such as minimum mean square error (MMSE) and it causes performance degradation. To study this effect, we use the first order autoregressive model given by

$$\tilde{h}_{i,j} = \gamma h_{i,j} + \sqrt{1-\gamma^2} u_{i,j}$$

where $\tilde{h}_{i,j}$ is the estimated channel, $h_{i,j}$ is the (i, j)-th element of $\mathbf{H}$, $u_{i,j}$ is i.i.d. complex Gaussian noise with zero mean and unit variance, and $0 < \gamma < 1$. The mean square error (MSE) between $\tilde{h}_{i,j}$ and $h_{i,j}$ is then given by

$$E[|\tilde{h}_{i,j} - h_{i,j}|^2] = (1-\gamma)^2 + 1 - \gamma^2 = 2 - 2\gamma$$

where $E[\cdot]$ denotes expectation. This model accounts for channel estimation error with suitable choice of parameter γ. Fig. 3 illustrates the achievable sum rate degradation caused by channel estimation and prediction error at SNR$= 10\ dB$. We assume that the transmitter and the receiver have the same MSE for simplicity.

5. CONCLUSION

In this paper, we investigated jointly optimized linear transmit beamforming and receive combining for the downlink of a multiuser MIMO systems. The full search and greedy search algorithms for finding beamforming and combing vectors were derived assuming that the combining vector is quantized so that it can be sent on a feedforward control channel to each user. Our approach is a practical solution to the multiuser joint beamforming and combining problem for time division duplexing systems where reciprocity provides channel state information at the transmitter. In the next step, we will consider the combination of limited feedback and limited feedforward quantization to allow our approach to operate with only limited channel state information at the transmitter.

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