EFFICIENT CHANNEL QUANTIZATION SCHEME FOR MULTI-USER MIMO BROADCAST CHANNELS WITH RBD PRECODING

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Abstract — Regularized block diagonalization (RBD) is a new linear precoding technique for the multi-antenna broadcast channel and has a significantly improved sum rate and diversity order compared to all previously proposed linear precoding techniques. We consider a limited feedback system with RBD precoding, in which each receiver has perfect channel state information (CSI) and quantizes its channel. The transmitter receives the quantized CSI with a finite number of feedback bits from each receiver. In contrast to zero-forcing (ZF) or block diagonalization (BD) precoding, where the transmitter only requires the channel direction information which refers to the knowledge of subspaces spanned by the users' channel matrices, for RBD precoding the transmitter additionally requires the channel magnitude information which defines the strength of the eigenmodes of the users' channel matrices. The key contribution of our work is that we propose a new scheme for the channel quantization to supply the transmitter with both channel direction and magnitude information. Based on this new scheme, firstly, we investigate a random vector quantization (RVQ). We derive a bound for the throughput loss due to imperfect CSI and find a way to achieve the bound by linearly increasing the number of feedback bits with the system SNR. Secondly, we modify the LBG vector quantization algorithm to obtain a dominant eigenvector based LBG (DE-LBG) vector quantization which can significantly reduce the number of feedback bits compared to RVQ. Finally, we demonstrate that the DE-LBG vector quantization can be applied to an OFDM-based multi-user MIMO system.

Index Terms— MU-MIMO, Precoding, RVQ, DE-LBG

1. INTRODUCTION

In the multi-antenna broadcast channel, space division multiple access (SDMA) allows to simultaneously transmit data streams to a group of users and leads to a significant throughput improvement relative to the single user case. Compared to the optimal SDMA strategy that uses dirty paper coding (DPC), SDMA with linear precoding has a suboptimal performance, but it is more robust to erroneous channel state information (CSI) and the transmitter complexity is lower.

The RBD precoding technique has gained more attention recently [1, 2], because it promises a significantly improved sum rate and diversity order compared to all previously proposed linear precoding techniques. When the transmitter has perfect CSI, at high signal-to-noise ratios (SNRs) and under the condition that the total number of antennas at the user terminals is less or equal than the number of antennas at the base station, the effective combined channel matrix is block diagonal and the multi-user interference (MUI) can be entirely eliminated. But at low SNRs the receivers still experience a small amount of MUI.

In order to correctly perform RBD, the transmitter requires not only the channel direction information, but also the channel magnitude information which is used to avoid the noise enhancement and improve the diversity. In this paper, we consider a limited feedback model, in which each user terminal quantizes its channel and feeds back a finite number of bits to the transmitter. Instead of the channel quantization scheme proposed previously [3, 4], which only provides channel direction information, we present a new efficient channel quantization scheme to provide both channel direction and magnitude information. The amount of CSI available at the transmitter critically affects the multiplexing gain in the MIMO downlink channel.

Based on the new channel quantization scheme, two vector quantization methods are investigated. First, for the random vector quantization (RVQ), the throughput loss due to imperfect CSI is bounded and a simple expression for the number of feedback bits is given to maintain the bound. Second, for the dominant eigenvector based Linde, Buzo and Gray (DE-LBG) vector quantization, we improve the algorithm from [5, 6] by constructing an efficient codebook in order to reduce the number of feedback bits.

2. SYSTEM MODEL

We consider a multi-user MIMO system with a single base station (BS) and K users, where the BS has M_{T} transmit antennas and the ith user has M_{R_i} receive antennas. The BS separates the data streams of multiple users by using RBD precoding. The received signal of the ith user is expressed as

$$\boldsymbol{y}_i = \boldsymbol{D}_i(\boldsymbol{H}_i \sum_{k=1}^{K} \boldsymbol{F}_k \boldsymbol{s}_k + \boldsymbol{n}_i) \tag{1}$$

where the vector $\boldsymbol{s}_k \in \mathbb{C}^{r_k \times 1}$ contains the data symbols and r_k represents the number of data streams for user k, $k = 1, 2, \ldots, K$. The matrix $\boldsymbol{F}_k \in \mathbb{C}^{M_{\mathrm{T}} \times r_k}$ denotes the RBD precoding matrix. Here the matrix $\boldsymbol{H}_i \in \mathbb{C}^{M_{\mathrm{R}_i} \times M_{\mathrm{T}}}$ is the channel matrix from the BS to the ith user. The vector $\boldsymbol{n}_i \in \mathbb{C}^{M_{\mathrm{R}_i} \times 1}$ represents the complex Gaussian

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noise vector with unit variance, which is independent of $\boldsymbol{s}_k$. The matrix $\boldsymbol{D}_i \in \mathbb{C}^{r_i \times M_{\mathrm{R}_i}}$ denotes the decoding matrix and $\boldsymbol{y}_i \in \mathbb{C}^{r_i \times 1}$ is the receive signal vector of user i.

We assume that the total transmit power of all users is constrained by P_{T}, i.e., $\mathrm{E}\left\{ \left| \sum_{k=1}^{K} \boldsymbol{F}_k \boldsymbol{s}_k \right|^2 \right\} \leq P_{\mathrm{T}}$ ($P_{\mathrm{T}} \geq 0$). Each of the receivers is assumed to have perfect and instantaneous knowledge of its own channel matrix.

2.1. Regularized Block Diagonalization (RBD)

RBD precoding [1, 2] can extract the maximum diversity order of the channel at high data rates, remove any constraint regarding the number of antennas at the receivers and can be also used for precoding with long-term CSI. Compared to non-linear techniques, RBD is less sensitive to channel estimation errors.

The RBD precoding matrix is described as

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F}_1 & \boldsymbol{F}_2 & \cdots & \boldsymbol{F}_K \end{bmatrix} = \gamma \boldsymbol{F}_a \boldsymbol{F}_b \, , \tag{2}$$

where

$$\boldsymbol{F}_a = \begin{bmatrix} \boldsymbol{F}_{a_1} & \boldsymbol{F}_{a_2} & \cdots & \boldsymbol{F}_{a_K} \end{bmatrix} \in \mathbb{C}^{M_{\mathrm{T}} \times M_{\mathrm{R}}}$$

and

$$\boldsymbol{F}_b = \begin{bmatrix} \boldsymbol{F}_{b_1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F}_{b_2} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{F}_{b_K} \end{bmatrix} \in \mathbb{C}^{M_{\mathrm{R}} \times r}.$$

Here, $r \leq \min(M_{\mathrm{R}}, M_{\mathrm{T}})$ is the total number of transmitted data stream sequences and M_{R} is the total number of receive antennas. The parameter γ is chosen to set the total transmit power to P_{T}.

RBD precoding is performed in two steps. In the first step we suppress MUI while balancing it with noise enhancement by using the matrix $\boldsymbol{F}_a$. We define $\boldsymbol{F}_{a_i}$ as the ith user's precoding matrix

$$\boldsymbol{F}_{a_i} = \widetilde{\boldsymbol{V}}_i (\widetilde{\boldsymbol{\Sigma}}_i^2 + \alpha \boldsymbol{I}_{M_{\mathrm{T}}})^{-1/2} \tag{3}$$

where $\widetilde{\boldsymbol{V}}_i$ and $\widetilde{\boldsymbol{\Sigma}}_i$ are the matrix of the right singular vectors and the diagonal matrix of the eigenvalues of the combined channel matrix of all other users, respectively, and $\alpha = \left(\frac{P_{\mathrm{T}}}{M_{\mathrm{R}} \sigma_n^2} \right)^{-1}$.

From equation (3) we see that each user transmits on the eigenmodes of the combined channel matrix of all other users with the power α. At high SNRs each user transmits only in the null subspace of all other users. In the step 2, the system performance is further optimized by the specific optimization criterion assuming a set of parallel single user MIMO channels.

3. CHANNEL QUANTIZATION SCHEME

From the above introduction to RBD, we see that in order to correctly perform RBD, both channel direction and magnitude information are needed at the transmitter. In this section we propose a new efficient scheme for the channel quantization to satisfy this request.

Our channel quantization scheme is based on the equation

$$\mathrm{vec}\{\hat{\boldsymbol{H}}_i\} = \arg\min_{\boldsymbol{w}_j \in \mathcal{C}} d^2(\mathrm{vec}\{\boldsymbol{H}_i\}, \boldsymbol{w}_j) \tag{4}$$

where the vector $\boldsymbol{w}_j \in \mathbb{C}^{M_{\mathrm{R}_i} \cdot M_{\mathrm{T}} \times 1}$ is one codeword of the quantization codebook $\mathcal{C}$ used at the ith user, which is fixed beforehand and known to the transmitter and user i. Here, $\mathrm{vec}\{\boldsymbol{H}_i\}$ denotes the stacked vector of the channel matrix $\boldsymbol{H}_i$ and $d(\mathrm{vec}\{\boldsymbol{H}_i\}, \boldsymbol{w}_j)$ is the distance metric. We consider the chordal distance [7] which leads to an efficient vector quantization algorithm for designing the codebook.

$$d(\mathrm{vec}\{\boldsymbol{H}_i\}, \boldsymbol{w}_j) = \sqrt{\sin^2 \Theta} \tag{5}$$

Here, Θ is the principal angle between the two vectors. The matrix $\hat{\boldsymbol{H}}_i \in \mathbb{C}^{M_{\mathrm{R}_i} \times M_{\mathrm{T}}}$ is the quantized version of the channel matrix $\boldsymbol{H}_i$. Instead of directly quantizing the channel matrix, we first quantize the stacked vector of the channel matrix $\boldsymbol{H}_i$ according to equation (4), i.e., the codeword which is closest to $\mathrm{vec}\{\boldsymbol{H}_i\}$ is chosen as $\mathrm{vec}\{\hat{\boldsymbol{H}}_i\}$. Then we reshape $\mathrm{vec}\{\hat{\boldsymbol{H}}_i\}$ to get $\hat{\boldsymbol{H}}_i$. The advantages of our proposed channel quantization scheme are:

- To perform quantization, instead of calculating the minimum value of the sum of the principal angles spanned by columns of the channel matrices, we calculate the minimum value of the angle spanned by the two vectors.
- By quantizing the stacked vector of the channel matrix, we keep the relative magnitude information for the columns of the channel matrix and avoid the loss of the channel magnitude information which is caused by quantizing the channel matrix as a unitary codeword matrix directly.

The quantization codebook $\mathcal{C}$ consists of 2^B unit norm vectors ($\mathcal{C} = \{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{2^B}\}$). Each of the codewords has the dimension $M_{\mathrm{R}_i} \cdot M_{\mathrm{T}} \times 1$. Here B is the number of feedback bits per user. Clearly, the choice of the codebook significantly affects the quality of the CSI provided to the transmitter. In this paper, we study two quantization codebook designs and their performance.

4. RANDOM VECTOR QUANTIZATION (RVQ)

In the RVQ codebook design, we choose 2^B codewords independently and uniformly from $\mathbb{G}_{n,p}(\mathbb{C})$. Here $\mathbb{G}_{n,p}(\mathbb{C})$ is the set of all p-dimensional planes in the n-dimensional Euclidean space. In this work $p = 1$ and $n = M_{\mathrm{R}_i} \cdot M_{\mathrm{T}}$. One random codebook is generated for each user. We analyze the performance averaged over all possible random codebooks.

4.1. Quantization Distortion

The distortion associated with a given random codebook $\mathcal{C}$ for the quantization of $\boldsymbol{H}_i$ is defined as

$$D(\mathcal{C}) = \mathrm{E}\left[\min_{\boldsymbol{w} \in \mathcal{C}} d^2(\mathrm{vec}\{\boldsymbol{H}_i\}, \boldsymbol{w}) \right] \tag{6}$$

From [8] an upper bound to the quantization distortion is given by

$$\begin{aligned} D(\mathcal{C}) &\leq \bar{D}(\mathcal{C}) \\ &= \frac{\Gamma(\frac{1}{n-1})}{n-1} \cdot (C_{n,p,\beta})^{-\frac{1}{n-1}} \cdot 2^{-\frac{B}{n-1}} \end{aligned} \tag{7}$$

where $C_{n,p,\beta} = \frac{1}{(n-1)!} \prod_{i=1}^{p} \frac{\Gamma(\frac{\beta}{2}(n-i+1))}{\Gamma(\frac{\beta}{2}(p-i+1))} = 1$, if $\beta = 2$, $p = 1$. Here, $\Gamma(\cdot)$ represents the Gamma function. Therefore the upper bound can be simpled to

$$\bar{D}(\mathcal{C}) = \frac{\Gamma(\frac{1}{n-1})}{n-1} \cdot 2^{-\frac{B}{n-1}} \tag{8}$$

4.2. Throughput Loss Analysis

A limited feedback of B bits per user ultimately leads to a throughput loss. First we define the throughout loss gap to be the difference between the per user throughput achieved by perfect CSI for RBD and quantized CSI for RBD. Then we derive an upper bound of it.

4.2.1. Throughput Loss Gap

Theorem 1. The throughput loss per user incurred due to limited feedback relative to RBD with perfect CSI can be upper bounded by

$$\begin{aligned} \Delta R_i(P_{\mathrm{T}}) &= \left[R_i(P_{\mathrm{T}}) - R_{i,\mathrm{LF}}(P_{\mathrm{T}}) \right] \\ &\leq \log_2(1 + \Delta I_i + P_{\mathrm{T}} \cdot (\Delta I_i + D(\mathcal{C})) \end{aligned} \tag{9}$$

where $R_i(P_\mathrm{T})$ is the throughput achieved by perfect CSI based RBD and $R_{i,\mathrm{LF}}(P_\mathrm{T})$ refers to the throughput achieved by the limited feedback CSI based RBD. Here, ΔI_i is an average value of MUI over the system SNR for the desired user i by using RBD precoding with perfect CSI. We study the long-term average throughput, and thus the rate $R_i = \mathrm{E}\left\{\log_2(1+\mathrm{SINR}_i)\right\}$ can be transmitted to user i if Gaussian inputs are used.

Proof: $\Delta R_i(P_\mathrm{T}) = [R_i(P_\mathrm{T}) - R_{i,\mathrm{LF}}(P_\mathrm{T})]$

$$
\begin{aligned}
&=\mathrm{E}\Big\{\log_2(1+\frac{\frac{P_\mathrm{T}}{M_\mathrm{T}}\left\|\boldsymbol{H}_i\cdot\boldsymbol{F}_{\mathrm{RBD},i}\right\|_\mathrm{F}^2}{1+\Delta I_i})\Big\}-\\
&\quad\mathrm{E}\Big\{\log_2(1+\frac{\frac{P_\mathrm{T}}{M_\mathrm{T}}\|\boldsymbol{H}_i\cdot\boldsymbol{F}_i\|_\mathrm{F}^2}{1+\Delta I_i+\sum_{j\neq i}\frac{P_\mathrm{T}}{M_\mathrm{T}}\|\boldsymbol{H}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2})\Big\}\\
&\overset{(1)}{=}\mathrm{E}\Big\{\log_2(1+\Delta I_i+\frac{P_\mathrm{T}}{M_\mathrm{T}}\left\|\boldsymbol{H}_i\cdot\boldsymbol{F}_{\mathrm{RBD},i}\right\|_\mathrm{F}^2)\Big\}-\\
&\quad\underbrace{\mathrm{E}\left\{\log_2(1+\Delta I_i)\right\}}_{(a)}-\\
&\quad\mathrm{E}\Big\{\log_2(1+\Delta I_i+\frac{P_\mathrm{T}}{M_\mathrm{T}}\|\boldsymbol{H}_i\cdot\boldsymbol{F}_i\|_\mathrm{F}^2+\overbrace{\sum_{j\neq i}\frac{P_\mathrm{T}}{M_\mathrm{T}}\|\boldsymbol{H}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2)}^{(b)}\Big\}+\\
&\quad\mathrm{E}\Big\{\log_2(1+\Delta I_i+\sum_{j\neq i}\frac{P_\mathrm{T}}{M_\mathrm{T}}\|\boldsymbol{H}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2)\Big\}\\
&\overset{(2)}{\leq}\mathrm{E}\Big\{\log_2(1+\Delta I_i+\sum_{j\neq i}\frac{P_\mathrm{T}}{M_\mathrm{T}}\|\boldsymbol{H}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2)\Big\}\\
&\overset{(3)}{\leq}\log_2(1+\Delta I_i+\frac{P_\mathrm{T}}{M_\mathrm{T}}\sum_{j\neq i}\mathrm{E}\left\{\|\boldsymbol{H}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2\right\})\\
&\overset{(4)}{\approx}\log_2(1+\Delta I_i+\frac{P_\mathrm{T}(M_\mathrm{T}-1)}{M_\mathrm{T}}(\Delta I_i+D(\mathcal{C})))\\
&\leq\log_2(1+\Delta I_i+P_\mathrm{T}\cdot(\Delta I_i+D(\mathcal{C})))
\end{aligned}
$$

Here, the matrix $\boldsymbol{F}_{\mathrm{RBD},i}$ is the RBD precoding matrix for user i which is calculated from perfect CSI and $\boldsymbol{F}_i$ denotes the RBD precoding matrix of user i calculated from quantized CSI. After (1) we neglect the positive terms (a) and (b), and consider that both $\boldsymbol{F}_{\mathrm{RBD},i}$ and $\boldsymbol{F}_i$ are unitary matrices and independent of $\boldsymbol{H}_i$, which leads to bound (2). Then we use Jensen's inequality to get bound (3). Note that $\mathrm{E}\left\{\|\boldsymbol{H}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2\right\}$ is determined by the average MUI $\Delta I_{i,j}$ caused from the $j\mathrm{th}$ user to the $i\mathrm{th}$ user and quantization distortion $D(\mathcal{C})_i$ of user i, which can be upper bounded as follows

$$
\begin{aligned}
\mathrm{E}\left\{\|\boldsymbol{H}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2\right\} &= \mathrm{E}\left\{\left\|(\hat{\boldsymbol{H}}_i+\boldsymbol{\epsilon}_i)\cdot\boldsymbol{F}_j\right\|_\mathrm{F}^2\right\}\\
&= \mathrm{E}\left\{\left\|(\hat{\boldsymbol{H}}_i\cdot\boldsymbol{F}_j+\boldsymbol{\epsilon}_i\cdot\boldsymbol{F}_j)\right\|_\mathrm{F}^2\right\}\\
&\leq \mathrm{E}\left\{\left\|\hat{\boldsymbol{H}}_i\cdot\boldsymbol{F}_j\right\|_\mathrm{F}^2+\|\boldsymbol{\epsilon}_i\cdot\boldsymbol{F}_j\|_\mathrm{F}^2\right\}\\
&\approx(\Delta I_{i,j}+D(\mathcal{C})_i)
\end{aligned}
$$

where $\boldsymbol{\epsilon}_i$ is the error introduced by the quantization of $\boldsymbol{H}_i$.

4.2.2. *Feedback Rate*

In order to maintain a throughput loss $\Delta R_i(P_\mathrm{T})$ that is not larger than a given bound, we need to scale the number of feedback bits per user with the system SNR. We assume $\log_2 b$ as the upper bound of the throughput loss and replace $\Delta R_i(P_\mathrm{T})$ in equation (9) with it. Instead of $(\Delta I_i + D(\mathcal{C}))$, we use the expression of the quantization distortion upper bound $\bar{D}(\mathcal{C})$ in equation (9). Then we solve the equation for B as a function of P_T and b. We get

$$
\begin{aligned}
B &= (n-1)\cdot\log_2 P_\mathrm{T} - (n-1)\cdot\log_2(b-1-\Delta I_i) +\\
&\quad (n-1)\cdot\log_2\left(\frac{\Gamma(\frac{1}{n-1})}{n-1}\right)\\
&\approx \frac{n-1}{3}\mathrm{SNR} - (n-1)\cdot\log_2(b-1-\Delta I_i) \qquad (10)
\end{aligned}
$$

For RBD precoding with perfect CSI, ΔI_i is approximately equal to zero at high SNRs and equal to a small number at low SNRs. In our simulations, a two user MIMO system with $M_\mathrm{T}=4$, $M_{\mathrm{R}_i}=2$ is considered. We average the MUI over the system SNR and obtain an experimental value of ΔI_i as 0.25. The bound of the throughput loss gap can be adjusted by the value of b. Here we set $b=2$ which means 1 bps/Hz rate offset and refers to a bound of 3 dB loss gap. In Figure 1(a), we compare the performance of the limited feedback system based on the channel quantization scheme we proposed here with the channel quantization scheme proposed in [4], which can only provide channel direction information. To ensure a fair comparison, we use the same number of feedback bits and a random quantization codebook for both channel quantization schemes.

The simulation result in Figure 1(a) shows that the 3 dB throughput loss gap can be maintained by the channel quantization scheme we propose here. In the absence of channel magnitude information, the throughput loss bound cannot be satisfied.

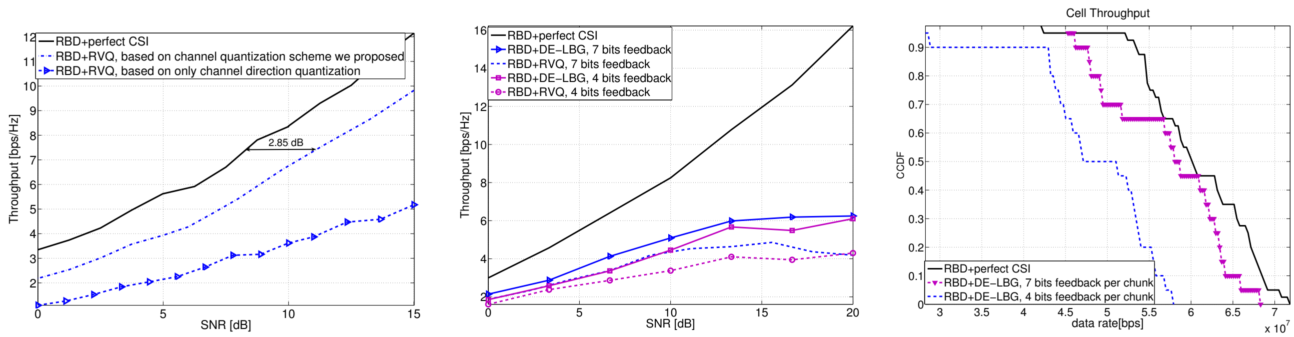
5. DE-LBG VECTOR QUANTIZATION

In this section we propose an efficient quantization codebook design based on the LBG vector quantization (VQ) algorithm in order to reduce the number of feedback bits. We modify the optimality criteria of the LBG VQ algorithm [5] as follows.

- Nearest Neighbor Condition:
  This condition states that the encoding region $\mathbb{S}_n$ should consist of all vectors that are closer to $\boldsymbol{c}_n$ than any of the other code vectors in the chordal distance sense.
- Centroid Condition:
  In contrast to the original LBG algorithm, where the centroid of one encoding region is determined by the arithmetic average of all the training vectors in this region, we modify this condition by requiring the code vector $\boldsymbol{c}_n$ of the encoding region $\mathbb{S}_n$ to be equal to the dominant eigenvector of the covariance matrix $\boldsymbol{R}_n$ of all training vectors in this encoding region. This scheme efficiently captures the statistics of the training vectors in the encoding region.

The DE-LBG VQ algorithm is an iterative algorithm which satisfies the above two optimality criteria. The algorithm requires an initial codebook $\mathcal{C}^{(0)}$ which is obtained by the splitting method. In this method an initial code vector is split into two code vectors. The iterative algorithm is run with these two vectors as the initial codebook. At the end of this step the two code vectors are split into four and the iterative algorithm is run again. The process is repeated until the desired number of code vectors is obtained. The algorithm is summarized below.

1. Generate a training sequence $\boldsymbol{\mathcal{T}}$ which captures the statistical properties of the stacked vectors $\boldsymbol{h}_i \in \mathbb{C}^{M_{\mathrm{R}_i}\cdot M_\mathrm{T}\times 1}$ of channel matrix samples. Channel matrix samples are generated by Monte-Carlo simulations in this work.
$$\boldsymbol{\mathcal{T}} = \{\boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_M\} \qquad (11)$$
   Here M is the number of channel samples.
2. Generate the initial code vector $\boldsymbol{c}_1^*$ by choosing it as the dominant eigenvector of the covariance matrix $\boldsymbol{R}$ of the entire training sequence. Set $N=1$.
$$\boldsymbol{R} = \frac{1}{M}\sum_{m=1}^{M}\boldsymbol{h}_m\boldsymbol{h}_m^H \qquad (12)$$
3. Splitting: for $i=1,2,\ldots,N$, set $\boldsymbol{c}_i^{(0)} = (1+\epsilon)\boldsymbol{c}_i^*$, $\boldsymbol{c}_{N+i}^{(0)} = (1-\epsilon)\boldsymbol{c}_i^*$, here $\epsilon > 0$ is very small number and we choose $\epsilon = 0.002$. Then set $N = 2N$.



(a) (b) (c)

Fig. 1. Performance comparison for limited feedback system with channel quantization: (a) RVQ for 2 users, $4 \times \{2, 2\}$ Rayleigh fading channel; (b) DE-LBG for 2 users, $4 \times \{2, 2\}$ Rayleigh fading channel; (c) DE-LBG for 4 users, $8 \times \{2, 2, 2, 2\}$ OFDM channel.

4. Iteration: set the iteration index $k = 0$ and calculate the minimum chordal distance of the initial codebook
$$d^{(0)}_{c,\min}(\mathcal{C}) = \min d_c(\boldsymbol{c}_i, \boldsymbol{c}_j) \quad (13)$$
   (a) Assign the source vector $\boldsymbol{h}_m$ to the N encoding regions by finding $n^* = \arg\min_{n\in 1,2,\ldots,N} d(\boldsymbol{h}_m, \boldsymbol{c}^{(k)}_n)$ for $m = 1, 2, \ldots, M$.

   (b) Update the code vector in each region by using the centroid condition.

   (c) Set $k = k + 1$

   (d) Calculate $d^{(k)}_{c,\min}(\mathcal{C})$, if $d^{(k)}_{c,\min}(\mathcal{C}) > d^{(k-1)}_{c,\min}(\mathcal{C})$ go back to step (a). Otherwise, go to step (e).

   (e) Set $\boldsymbol{c}^*_n = \boldsymbol{c}^{(k-1)}_n$ as the final code vectors.

5. Repeat steps 3 and 4 until the desired number of code vectors is obtained.

In Figure 1(b) we can see the system performance improvement of RBD with DE-LBG comparing it with RVQ for a fixed number of feedback bits.

The DE-LBG vector quantization can be applied to an OFDM system. We assume a microcellular scenario based on the Manhattan grid where users with fixed velocities ($|v| \leq 10$ km/h) are uniformly distributed in the streets. An OFDM channel with 128 subcarriers and 30 symbols is considered. The total bandwidth is 5.86 MHz and the carrier frequency $f_c = 3.95$ GHz. RBD precoding is performed per chunk which contains 8 subcarriers and 15 symbols. The channel model is the WINNER B1 channel [9].

The result is shown in Figure 1(c). The complementary CDF of the cell throughput shows us that with the DE-LBG codebook design, which can be adapted to the statistics of the channel matrices, the system performance is still not significantly degraded for only 7 feedback bits per chunk for one user compared to the case that the transmitter has perfect CSI.

6. CONCLUSIONS

We propose a new efficient channel quantization scheme for the multi-user MIMO broadcast channel with RBD precoding. By using this scheme, both channel direction and magnitude information can be obtained at the transmitter through a finite rate feedback from the receivers.

Based on the new scheme for channel quantization, first we investigate the random vector quantization. The objective of the RVQ is to effectively bound the throughput loss due to the imperfect CSI. A simple expression in terms of system SNR is provided to scale the number of feedback bits per user in order to fulfill this objective. Also, we investigate the dominant eigenvector based LBG (DE-LBG) vector quantization and propose an efficient codebook design.

Furthermore, we apply the DE-LBG vector quantization to an OFDM system. Under the assumption that each receiver has perfect channel knowledge and the statistical properties of the channel matrices can be efficiently captured by training sequences, we only need a very small number of feedback bits.

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