HIGH-RATE SPACE-TIME BLOCK CODES WITH FAST MAXIMUM-LIKELIHOOD DECODING

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ABSTRACT

Orthogonal space-time block codes (OSTBCs) represent an attractive choice of space-time coding scheme because of their simple maximum-likelihood (ML) decoding and full diversity property. However, the code orthogonality property limits their achievable transmission rate. In this paper, new high-rate block codes are proposed that are referred to as *orthogonal structure based* STBCs. To obtain these codes, the proposed design adds extra-symbols to the OSTBC matrix using different reasonable strategies. Because of the internal OSTBC structure of the proposed designs, the ML decoder can be implemented in a fast way. Simulations validate an improved performance-to-complexity tradeoff of the proposed codes as compared to several other popular choices of space-time codes.

Index Terms— Space-time coding, fast maximum-likelihood decoding

1. INTRODUCTION

OSTBCs [1] are a popular choice of space-time codes, because they achieve full diversity at a low maximum likelihood (ML) decoding complexity. However, their transmission rate is limited by the code orthogonality.

To overcome this drawback while retaining low decoding complexity, quasi-orthogonal space-time block codes (QOSTBCs) have been proposed in [2]-[4]. The latter codes achieve higher rates than OSTBCs but at the price of losing the full diversity property. However, the full diversity property of QOSTBCs can be recovered by using symbol rotation [5]-[7].

Several other approaches to the design of high-rate space-time codes achieving full diversity for any number of transmit antennas have been proposed, but the decoding complexity of most of these codes is quite high. Recently, several space-time codes with a lower ML decoding complexity than that of the standard QOSTBCs have been proposed [8], [9].

Below, we develop a new class of STBCs that offer an attractive tradeoff between the performance and decoding complexity for several practically important cases. The general idea of our designs is related to that of [10]-[11] in the sense that we also use the OSTBC structure to construct higher-rate codes. Our specific approach to design the code generator matrix is, however, different from that used in [10] and [11], and mainly follows the idea of [12].

To demonstrate the advantages of the proposed designs, we develop several useful code design strategies in the case of four transmit antennas, and additionally show that the latter designs can be used in the case of three transmit antennas as well. It is shown that the resulting rate-one codes achieve full diversity and their performance is comparable to the best rate-one STBCs known so far. At the same time, the proposed codes have a substantially reduced ML decoding complexity as compared to the current state-of-the-art rate-one STBCs.

2. SYSTEM MODEL

Let us consider a wireless MIMO communication system with N_t transmit and N_r receive antennas. We assume a flat block-fading channel with the block length T. The input-output relation for such a MIMO system can be expressed as [1]

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V} \tag{1}$$

where $\mathbf{X}$ is the $N_t \times T$ complex matrix of the transmitted signals, $\mathbf{H}$ is the $N_r \times N_t$ complex channel matrix, $\mathbf{V}$ is the $N_r \times T$ complex noise matrix, and $\mathbf{Y}$ is the $N_r \times T$ complex matrix of the received signals. The entries of $\mathbf{H}$ and $\mathbf{V}$ are assumed to be i.i.d. random variables with the probability density functions (pdf's) $\mathcal{CN}(0,1)$ and $\mathcal{CN}\left(0,\sigma^2\right)$, respectively. Here, $\mathcal{CN}(\cdot,\cdot)$ denotes the complex Gaussian pdf and σ^2 is the noise variance.

It is assumed that the channel is perfectly known at the receiver, and that the K symbols s_k, $k = 1 \dots K$ transmitted per block are drawn from an M-point constellation $\bar{\mathcal{S}}$ and encoded to form the matrix $\mathbf{X}$ as [14]

$$\mathbf{X} = \sum_{k=1}^{K} (s_{\mathrm{r}k}\mathbf{C}_{2k-1} + s_{\mathrm{i}k}\mathbf{C}_{2k}) \tag{2}$$

where $s_{\mathrm{r}k}$ and $s_{\mathrm{i}k}$ are the real and imaginary parts of s_k, respectively, $\{\mathbf{C}_k\}_{k=1}^{2K}$ is a set of complex $N_t \times T$ matrices that are subject to the following constraint

$$\sum_{k=1}^{2K} \mathrm{tr}(\mathbf{C}_k^H \mathbf{C}_k) = 2TN_t\,. \tag{3}$$

Here $(\cdot)^H$ denotes the Hermitian transpose and $\mathrm{tr}(\cdot)$ stands for the trace of a matrix.

Now, let us obtain an equivalent real-valued system model of (1) by defining the "underline" operator which transforms any $I \times J$ matrix $\mathbf{Z}$ into a $2IJ \times 1$ real column vector as follows [12], [13]

$$\underline{\mathbf{Z}} \triangleq [\mathrm{Re}\{Z_{11}\}, \mathrm{Im}\{Z_{11}\}, \mathrm{Re}\{Z_{21}\}, \mathrm{Im}\{Z_{21}\}, \dots, \mathrm{Re}\{Z_{IJ}\}, \mathrm{Im}\{Z_{IJ}\}]^T \tag{4}$$

where $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real and imaginary parts, respectively. Applying (4) to (1), we have [12]

$$\underline{\mathbf{Y}} = \mathbb{H}\underline{\mathbf{X}} + \underline{\mathbf{V}} \tag{5}$$

where $\mathbb{H} = \frac{1}{2}\mathbf{I}_T \otimes (\mathbf{H} \otimes \mathbf{E} + \mathbf{H}^* \otimes \mathbf{E}^*)$, $\mathbf{E} = \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}$, $j = \sqrt{-1}$, $\mathbf{I}_T$ is the $T \times T$ identity matrix, $\otimes$ denotes the Kronecker matrix product, and $(\cdot)^*$ stands for the complex conjugate. Inserting (2) in (5), we obtain $\underline{\mathbf{X}} = \mathbb{G}\underline{\mathbf{s}}$ where the $2N_tT \times 2K$ real matrix

$$\mathbb{G} \triangleq [\underline{\mathbf{C}}_1 \ \underline{\mathbf{C}}_2 \ \cdots \ \underline{\mathbf{C}}_{2K}]$$

is the code generator matrix, and $\underline{\mathbf{s}}$ is the underline version of the symbol vector $\mathbf{s} = [s_1 \dots s_K]^T$.

3. THE ML DECODER

Let us introduce the constellation $\underline{\mathcal{S}} \triangleq \{\underline{\mathbf{s}}_1, \dots, \underline{\mathbf{s}}_L\}$ for the vector $\underline{\mathbf{s}}$, where where the cardinality of $\underline{\mathcal{S}}$ is $L = M^K$ [12]. For any received signal matrix $\underline{\mathbf{Y}}$ in (5), the coherent ML decoder finds

$$\hat{\underline{\mathbf{s}}} = \arg\min_{\underline{\mathbf{s}} \in \underline{\mathcal{S}}} \|\underline{\mathbf{Y}} - \mathbb{H}\mathbb{G}\underline{\mathbf{s}}\| \quad (6)$$

where $\|\cdot\|$ is the Euclidean norm. If we apply the QR-decomposition to the matrix $\mathbb{H}\mathbb{G}$, (6) can be expressed as

$$\hat{\underline{\mathbf{s}}} = \arg\min_{\underline{\mathbf{s}} \in \underline{\mathcal{S}}} \left\| \underline{\mathbf{Y}} - \mathbf{Q} \begin{bmatrix} \mathbf{R} \\ \mathbf{O} \end{bmatrix} \underline{\mathbf{s}} \right\| \quad (7)$$

where $\mathbf{Q}$ is a $2N_rT \times 2N_rT$ orthogonal matrix, $\mathbf{R}$ is a $2K \times 2K$ upper-triangular matrix, and $\mathbf{O}$ is a $(2N_rT - 2K) \times 2K$ matrix of zeros. Using the orthogonality property of $\mathbf{Q}$, (7) can be reduced to

$$\hat{\underline{\mathbf{s}}} = \arg\min_{\underline{\mathbf{s}} \in \underline{\mathcal{S}}} \|\breve{\underline{\mathbf{Y}}} - \mathbf{R}\underline{\mathbf{s}}\| \quad (8)$$

where $\breve{\underline{\mathbf{Y}}}$ is composed by the first $2K$ entries of $\mathbf{Q}^T\underline{\mathbf{Y}}$.

To guarantee the uniqueness of the optimal $\hat{\underline{\mathbf{s}}}$, the matrix $\mathbf{R}$ should be full rank. Hence, $\mathbb{H}$ should be full-rank and $N_rT \geq K$.

4. ORTHOGONAL STRUCTURE BASED STBCS

Using the fact that for any OSTBC $\mathbf{X}$, the matrices $\mathbf{C}_k$ should fulfill [15]

$$\mathbf{C}_k\mathbf{C}_k^H = \mathbf{I}_{N_t}, \quad \mathbf{C}_k\mathbf{C}_p^H = -\mathbf{C}_p\mathbf{C}_k^H, \quad k \neq p \quad (9)$$

for $k = 1, \dots, 2K$ and $p = 1, \dots, 2K$. It can be proved that

$$\mathbb{G}_{\mathrm{ostbc}}^T\mathbb{H}^T\mathbb{H}\mathbb{G}_{\mathrm{ostbc}} = \|\mathbf{H}\|_F^2\mathbf{I}_{2K} \quad (10)$$

where $\|\cdot\|_F$ is the Frobenius norm and $\mathbb{G}_{\mathrm{ostbc}}$ is the OSTBC generator matrix. According to the constellation space invariance property of OSTBCs [13], the orthogonality of $\mathbb{G}_{\mathrm{ostbc}}$ remains invariant to the skewing effects of the channel matrix, guaranteeing that the coherent ML decoder can be implemented as a simple real symbol-by-symbol decoder.

In order to take advantage of the constellation space invariance property of OSTBCs to simplify the decoder, we choose as many first columns of $\mathbb{G}$ as possible from a proper OSTBC (or equivalently, as many matrices $\mathbf{C}_k$ as possible from this OSTBC) so that K_o complex symbols are encoded. Now, we add the rest of $2(K - K_o)$ linearly independent columns that are necessary to complete the matrix $\mathbb{G}$. Hence, $\mathbb{G}$ takes the form

$$\mathbb{G} = [\mathbb{G}_{\mathrm{ostbc}}, \mathbb{G}_{\mathrm{add}}]$$

where $\mathbb{G}_{\mathrm{add}}$ is the $2N_tT \times 2(K - K_o)$ matrix containing extra-columns added to the code generator matrix to design orthogonal structure based STBCs (OSB-STBCs).

According to (10), for the resulting OSB-STBC, the matrix $\mathbf{R}$ in (8) has the following form

$$\mathbf{R} = \begin{bmatrix} \gamma\mathbf{I}_{2K_o} & \mathbf{A} \\ \mathbf{O} & \mathbf{B} \end{bmatrix} \quad (11)$$

where $\mathbf{A}$ is a $2K_o \times 2(K - K_o)$ general-type matrix, $\mathbf{O}$ is a $2(K - K_o) \times 2K_o$ matrix of zeros, $\mathbf{B}$ is a $2(K - K_o) \times 2(K - K_o)$ upper-triangular matrix, and γ is some constant. Let $\tilde{\underline{\mathbf{s}}} \triangleq [\underline{s}_1, \dots, \underline{s}_{2K_o}]^T$ and $\breve{\underline{\mathbf{s}}} \triangleq [\underline{s}_{2K_o+1}, \dots, \underline{s}_{2K}]^T$ so that $\underline{\mathbf{s}} = [\tilde{\underline{\mathbf{s}}}^T, \breve{\underline{\mathbf{s}}}^T]^T$. Using (11), we can rewrite (8) as

$$\hat{\underline{\mathbf{s}}} = \arg\min_{\underline{\mathbf{s}} \in \underline{\mathcal{S}}} \left\| \breve{\underline{\mathbf{Y}}} - \begin{bmatrix} \gamma\tilde{\underline{\mathbf{s}}} + \mathbf{A}\breve{\underline{\mathbf{s}}} \\ \mathbf{B}\breve{\underline{\mathbf{s}}} \end{bmatrix} \right\|. \quad (12)$$

We observe that for any given $\breve{\underline{\mathbf{s}}}$, the value of $\tilde{\underline{\mathbf{s}}}$ that minimizes the metric in (12) can be found by a real symbol-by-symbol decoding procedure. Therefore, in (12) it is only necessary to inspect the metric for all possible combinations of $\breve{\underline{\mathbf{s}}}$ in order to find $\hat{\underline{\mathbf{s}}}$. This reduces the decoding complexity.

4.1. Rate-One OSB-STBC

A reduced complexity of the ML decoder makes OSB-STBCs attractive in the cases when OSTBCs can achieve a reasonably high rate. For example, using the idea of OSB codes, a full-rate full-diversity 2×2 OSB-STBC with non-vanishing determinants has been developed in [12]. However, in the case of three or more transmit antennas and full rate, the ratio K_o/K decreases and, therefore, the complexity advantage of the decoder becomes insignificant.

In the practically important case of three and four antennas, we can keep the value of K_o/K high enough by restricting the code rate to be one. For this case, it is only necessary to aggregate one complex symbol to the OSTBC matrix (or, equivalently, two columns to the code generator matrix) to construct our OSB-STBC.

In what follows, we will present different strategies for designing the additional matrices $\mathbf{C}_k$, $k = K_o + 1, \dots, K$ for the case of $N_t = 4$. For $N_t = 3$, the same code can be used by removing, for instance, the last row of the matrix $\mathbf{X}$. Our design is based on the following 4×4 OSTBC [15]:

$$\mathbf{X} = \frac{4}{\sqrt{3}} \begin{bmatrix} s_1 & -s_2^* & s_3^* & 0 \\ s_2 & s_1^* & 0 & s_3^* \\ s_3 & 0 & -s_1^* & -s_2^* \\ 0 & s_3 & s_2 & -s_1 \end{bmatrix}. \quad (13)$$

Our first strategy is to constrain the columns added to $\mathbb{G}$ to be orthogonal to the original "OSTBC" columns of $\mathbb{G}$ and to each other. In the full-rate case ($N_tT = K$), it has been shown that an orthogonal $\mathbb{G}$ is a sufficient and necessary condition for the code to be information lossless [16], [17]. Let $\mathbf{N}$ be a $2N_tT \times 2(N_tT - K_o)$ matrix whose columns form an orthogonal basis for the null-space of $\mathbb{G}_{\mathrm{ostbc}}^T$. Orthogonal columns (that are the candidate columns for $\mathbb{G}_{\mathrm{add}}$) can be obtained as those of the $2N_tT \times 2(N_tT - K_o)$ matrix $\mathbf{NU}$ where $\mathbf{U}$ is any $2(N_tT - K_o) \times 2(N_tT - K_o)$ orthogonal matrix that can be parameterized using, for instance, Givens rotations [18]. Since only two columns are added in the rate-one STBC case with $N_t = 4$, only the first two columns of $\mathbf{U}$ are needed. Using Givens rotations parametrization for the first two columns of $\mathbf{U}$, we can take into account only the rotations that are involved in these two columns, thereby reducing the number of parameters to be optimized. The Givens rotations parameters are angles between $-\pi$ and π, and any global search algorithm (such as the genetic algorithm) can be employed to optimize the OSB-STBC according to a certain criterion. A suitable criterion for such a design is the *diversity product* [7], [19] that for $T = N_t$ is defined as

$$\zeta = \frac{1}{2\sqrt{N_t}} \min_{\substack{\mathbf{X},\mathbf{X}' \in \mathcal{X} \\ \mathbf{X} \neq \mathbf{X}'}} \left|\det\left(\mathbf{X} - \mathbf{X}'\right)\right|^{\frac{1}{N_t}} \tag{14}$$

where $\mathcal{X} \triangleq \{\mathbf{X}_1, \ldots, \mathbf{X}_L\}$ is the codebook of the codeword matrices $\mathbf{X}$ and $\mathbf{X}'$. Maximizing ζ, we ensure that the designed OSB-STBC will provide full diversity with a high coding gain. In the maximization of ζ, the codeword matrix $\mathbf{X}$ is a function of $\mathbb{G}$, whose last two columns depend on the optimization variables (the Givens rotations parameters). We have maximized ζ for the 4-QAM constellation over these parameters using the genetic algorithm and then used the output of such optimization as a starting point for a local search. The resulting code is denoted as OSB-STBC-1.

Our second strategy of the OSB-STBC design is to add s_4 to the anti-diagonal of (13) to obtain the code matrix

$$\mathbf{X} = \begin{bmatrix} s_1 & -s_2^* & s_3^* & s_4 \\ s_2 & s_1^* & s_4 & s_3^* \\ s_3 & s_4 & -s_1^* & -s_2^* \\ s_4 & s_3 & s_2 & -s_1 \end{bmatrix}. \tag{15}$$

Clearly, this operation corresponds to adding two orthogonal columns to $\mathbb{G}_{\mathrm{ostbc}}$ and, therefore, the resulting code belongs to the class of OSB-STBC codes. However, as the minimal determinant of the codeword difference matrix is zero in this case, the resulting code will not achieve full diversity. Using (14), it can be readily shown that to achieve full diversity, we need to satisfy $\tilde{s}_{\mathrm{i}4}^2 \neq \tilde{s}_{\mathrm{r}1}^2 + \tilde{s}_{\mathrm{i}2}^2 + \tilde{s}_{\mathrm{i}3}^2$ and $\tilde{s}_{\mathrm{r}4}^2 \neq \tilde{s}_{\mathrm{i}1}^2 + \tilde{s}_{\mathrm{r}2}^2 + \tilde{s}_{\mathrm{r}3}^2$, where $\tilde{s}_k \triangleq s_k - s_k'$, $k = 1, \ldots, 4$ and $\mathbf{s} \neq \mathbf{s}'$. These inequalities can be easily satisfied by rotating the constellation for s_4.

Note that it is desirable not only to satisfy the full diversity property, but also to obtain a high coding gain. In the particular 4-QAM case, both objectives can be achieved by maximizing ζ as follows

$$\beta_{\mathrm{opt}} = \arg\max_{\beta} \zeta\,. \tag{16}$$

Using this strategy, we have obtained that the optimal rotation angles are $\beta_{\mathrm{opt}} = \{\pi/6,\ \pi/3\}$. We denote the OSB-STBC with one of this rotation angles as OSB-STBC-2. A similar strategy was presented in [10] where the two upper anti-diagonal entries were s_4^* rather than s_4 and the coding gain was not optimized.

The previous design has only one degree of freedom that can be utilized by optimizing ζ. Although the optimal ζ can be easily obtained, the performance can be further improved by designing a code with a reduced *kissing number*, that is, a reduced number of codeword difference matrices that result in the same worst-case value of ζ.

If we let each entry in the anti-diagonal of (15) to have an independent rotation, denote the vector of rotation angles as $\boldsymbol{\beta} = [\beta_{1,4}\ \ \beta_{2,3}\ \ \beta_{3,2}\ \ \beta_{4,1}]^T$, where $\beta_{m,n}$ is the rotation angle for the (m,n)th entry of $\mathbf{X}$, and optimize the code over $\boldsymbol{\beta}$, the kissing number can be reduced. Doing so, we obtain the following optimal value of $\boldsymbol{\beta}$:

$$\boldsymbol{\beta}_{\mathrm{opt}} = [1.9267\ \ -0.9526\ \ -1.0622\ \ -0.9493]^T. \tag{17}$$

This code corresponds to our third strategy and is denoted as OSB-STBC-3.

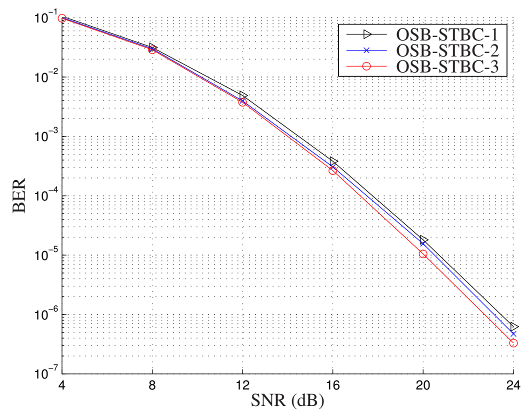

Fig. 1. BERs of the proposed OSB-STBCs versus SNR.

5. SIMULATIONS

We assume a multiple-input single-output (MISO) system with $N_t = T = 4$ and 4-QAM symbols. In the first example, we compare only the three proposed OSB-STBCs to identify the best code among them. Fig. 1 shows the bit error rates (BERs) for these codes versus the signal-to-noise ratio (SNR). As it can be seen from this figure, OSB-STBC-3 achieves the best performance among the three proposed codes. This fact can be explained by a lowered kissing number of this code.

Next, we compare the performance of OSB-STBC-3 with the following popular rate-one STBCs:

- QOSTBC with rotated constellation [7];
- coordinate interleaved orthogonal design (CIOD) [9];
- STBC based on linear constellation precoding (LCP) [20];
- diagonal algebraic space-time (DAST) code with real-valued rotation [21].

The Schnorr-Euchner variant of the sphere decoder (SD) has been employed for decoding [22]. In CIOD of [9], four parallel SDs were used to decode each symbol. In QOSTBC, two parallel SDs were employed to decode each pair s_1, s_4 and s_2, s_3. In the DAST code [21], two parallel SDs were used to decode the imaginary and real parts of each symbol separately. In the LCP code of [20] only one SD was employed. Note that the proposed OSB-OSTBCs also require one SD which is used to decode s_4, and the other symbols are decoded in the symbol-by-symbol way [12]. From Fig. 2, it can be observed that the performances of OSB-STBC-3, QOSTBC, and CIOD are very close to each other and are the best among the codes tested.

Fig. 3 displays the average number of points (i.e., the number of vectors $\underline{\mathbf{s}}$) visited by the decoder for all the codes tested versus SNR. From this figure, it can be observed that OSB-STBC-3 has the lowest decoding complexity among these codes.

6. CONCLUSIONS

New high-rate orthogonal structure based STBCs are developed that enjoy fast ML decoding. To obtain these codes, the proposed design

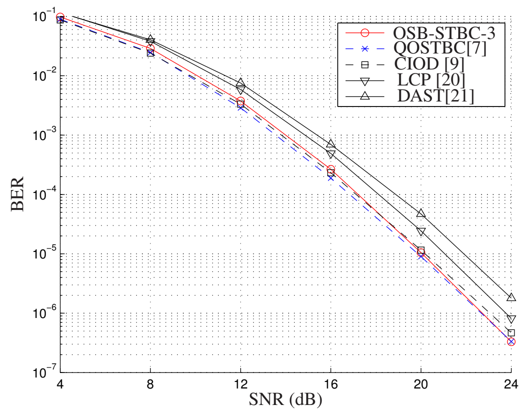

Fig. 2. BER of different rate-one STBCs versus SNR.

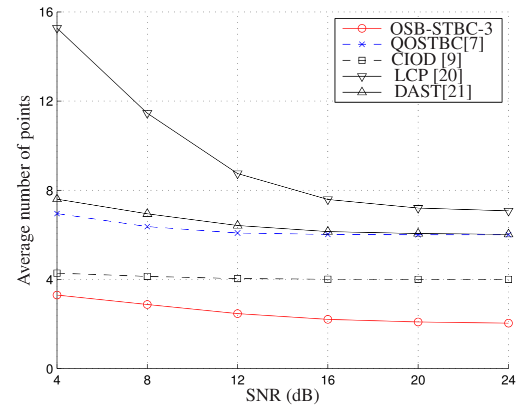

Fig. 3. Average number of points visited by the decoder versus SNR.

adds extra-symbols to the OSTBC matrix using different theoretically motivated strategies. The developed codes have been demonstrated to achieve a substantially improved performance-to-complexity tradeoff as compared to several current state-of-the-art rate-one STBCs.

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