COMBINED RECEIVER FOR MULTI-BLOCK TRANSMISSION

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ABSTRACT

We proposed a new signal transmission scheme for a multi-input multi-output (MIMO) communication system, in which the signals are encoded to span multi-block channel realizations and received by a linear equalizer followed by a maximum likelihood (ML) detector. The proposed system achieves a diversity gain proportional to the number of blocks of channels over which the signals span. The detection complexity is comparable to that of a linear receiver, with additional complexity independent of the number of antennas. A general criterion for designing the optimal precoder is presented. A design example is provided. Simulation results verify the effectiveness of such a combination.

***Index Terms*—** MIMO, multi-block transmission, combined linear and ML receivers, diversity gain

I. INTRODUCTION

Consider a MIMO communication system with M transmitter and N receiver antennas. The channel is assumed to be of flat fading with the fading coefficient between the mth transmitter, and nth receiver antennas being h_{nm}. These h_{nm} constitute an $N \times M$ channel matrix, denoted by $\boldsymbol{H}$, each element of which is i.i.d. Gaussian distributed, i.e., $\mathcal{CN}(0, 1)$. Temporally, $\boldsymbol{H}$ is assumed to remain constant for T time slots and may change independently to other states after these time slots elapse. In other words, if we let $\boldsymbol{H}_\ell$ denote the channel states for the ℓth block of T time slots, then $\boldsymbol{H}_\ell$ is assumed to be statistically independent from $\boldsymbol{H}_\jmath$ for $\ell \neq \jmath$. We also assume that the transmitter has no channel state information (CSI) but only the statistical properties of the channel, while the receiver has perfect knowledge of CSI.

In traditional MIMO communications, signals are transmitted and received block by block as depicted in Fig. 1-(a). Each block corresponds to one channel state, $\boldsymbol{H}_\ell$, and there is no inter-relationship between different signal blocks. For such a system, most available space-time block code (STBC) designs ([1]–[7]) focus on the communication systems in which signals are detected by an ML detector. The use of an ML detector may enable a MIMO system to achieve full diversity MN, yielding superior performances to other receivers. However, it also requires high computational cost rendering its application impractical. On the other hand, a linear receiver is simple in implementation and for a MIMO system with a linear receiver, the optimal minimum bit error rate (BER) STBC has been developed in [8]. However, even with the best STBC, a MIMO system with linear receiver can only attain a diversity gain of $(N-M+1)$ [9] and is still inferior to one with an ML detector. Due to its potential to provide higher diversity gain, multi-block transmission scheme was proposed to improve the system performance. As shown in Fig. 1-(b), the signals are coded and jointly transmitted through L independent channel blocks, $\boldsymbol{H}_1, \cdots, \boldsymbol{H}_L$, so that each symbol may experience L channel realizations in its transmission. At the receiver, the received signals from these L independent channel blocks are jointly detected. For such a multi-block system, with properly designed STBC and an ML detector, the full diversity which is equal to MNL can be achieved [10]. However, due to the extremely high detection complexity ($\mathcal{K}^{MNL}$ where $\mathcal{K}$ is the cardinality of the signal constellation), no design of STBC for such a system has been presented. A linear receiver is advantageous in this system due to its low computational cost. The optimal STBC for a linear receiver was proposed in [9] that minimizes the detection BER. It has also been shown in [9] that the diversity gain monotonically grows with L. However, the diversity gain increases only very slowly with L. This implies that a simple linear receiver is not able to utilize efficiently the total degrees of freedoms offered by a multi-block MIMO system.

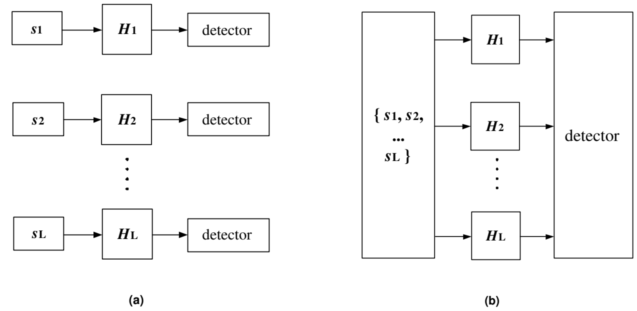

Fig. 1. (a) Single block transmission, (b) Multiple block transmission

In this paper, we propose a new scheme in which a linear and an ML receiver are combined to facilitate the detection of the coded data in a multi-block MIMO system. The linear equalizer is employed here to grossly reduce the search space of the ML detector. As a result, with a moderate increase in the computational cost, the new system has much improved performance than one with a linear receiver only. For the proposed new system, the achievable diversity gain can be shown to be $(NL-M+1)$, i.e., it increases linearly with L. In designing the STBC to improve the system performance, we simplify the problem by separating the coding of the signals into two stages, designated here as Precoders I and II, the functions of which correspond to those of the ML and linear receivers respectively. We show that the optimal Precoder II is the unitary trace-orthogonal code [8]. As well, a general design criterion on the optimal Precoder I is presented. Specifically, the optimal Precoder I is derived for the case of $L = 2$ and the signals are from a 4ϕ-PSK/4-QAM constellation.

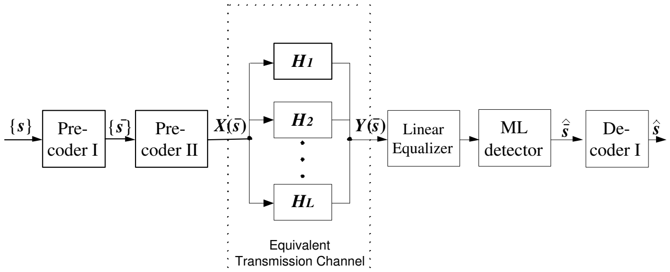

Fig. 2. Combined precoders/detectors for a multi-block MIMO system.

II. NEW TRANSMISSION SCHEME AND SYSTEM PERFORMANCE ANALYSIS

To achieve higher diversity gain with moderate detection complexity, we propose the full-rate multi-block transmission scheme with combined precoders and detectors as shown in Fig. 2. The symbols $\{s(i), i = 1, 2\cdots, MTL\}$ to be transmitted are selected from a constellation $\mathcal{S}$ of cardinality $\mathcal{K}$ and are first processed by a linear transformation Precoder I such that $\bar{s}(k) = \sum_{\ell=1}^{L} a_\ell s_k(\ell) e^{j\theta_\ell}$, where $s_k(\ell)$ are chosen L at a time without repeat, from the input symbols $\{s(i)\}$ to form $\bar{s}(k)$, and a_ℓ and θ_ℓ are the amplitude and phase to be determined. For the averaged signal power to remain unchanged, $\sum_{\ell=1}^{L} a_\ell^2 = L$. Thus, Precoder I maps the information symbol set $\{s(i)\}$ into another symbol set $\{\bar{s}(k), k = 1, 2\cdots, MT\}$ which is then processed by Precoder II to generate a linear STBC $\boldsymbol{X}(\bar{\boldsymbol{s}}) = \sum_{k=1}^{MT} \bar{s}(k)\boldsymbol{C}_k$, where $\boldsymbol{C}_k$ is an $M \times T$ matrix to be designed. The same coded signals $\boldsymbol{X}(\bar{\boldsymbol{s}})$ are then repeatedly transmitted at different blocks of time slots through the channels $\boldsymbol{H}_\ell, \ell = 1, \cdots, L$, each having different independent channel states. At the receiver, all the repeatedly transmitted coded signals are collected and jointly processed by a linear equalizer followed by an ML detector to obtain $\{\hat{s}(i)\}$, the estimate of $\{s(i)\}$.

II-A. System Model

Consider the space-time coded signals $\boldsymbol{X}(\bar{\boldsymbol{s}})$ transmitted through the ℓth state of channel, $\boldsymbol{H}_\ell$. Let the $N \times T$ matrix $\boldsymbol{Y}_\ell(\bar{\boldsymbol{s}})$ denote the corresponding block of received signals, i.e.,

$$\boldsymbol{Y}_\ell(\bar{\boldsymbol{s}}) = \sqrt{\frac{\rho}{M}}\boldsymbol{H}_\ell \boldsymbol{X}(\bar{\boldsymbol{s}}) + \boldsymbol{W}_\ell, \qquad \ell = 1, \cdots, L$$

where ρ is the signal to noise ratio (SNR) at each receiver antenna and $\boldsymbol{W}_\ell$ is the noise matrix each element of which is assumed to be i.i.d. $\mathcal{CN}(0, 1)$ distributed. At the receiver, we wait until the transmission of all the L blocks of signals is complete and stack all them into a tall $NL \times T$ matrix such that $\boldsymbol{Y}(\bar{\boldsymbol{s}}) = [\boldsymbol{Y}_1^T\ \boldsymbol{Y}_2^T\ \cdots\ \boldsymbol{Y}_L^T]^T$, where $[\cdot]^T$ stands for transpose. Correspondingly, we define an $NL \times M$ channel matrix $\boldsymbol{H} = [\boldsymbol{H}_1^T\ \boldsymbol{H}_2^T\ \cdots\ \boldsymbol{H}_L^T]^T$, and an $NL \times T$ noise matrix $\boldsymbol{W} = [\boldsymbol{W}_1^T\ \boldsymbol{W}_2^T\ \cdots\ \boldsymbol{W}_L^T]^T$, and obtain

$$\boldsymbol{Y}(\bar{\boldsymbol{s}}) = \sqrt{\frac{\rho}{M}}\boldsymbol{H}\boldsymbol{X}(\bar{\boldsymbol{s}}) + \boldsymbol{W} \tag{1}$$

The stacking of the received signal block matrices $\boldsymbol{Y}_\ell^T$ to form $\boldsymbol{Y}(\bar{\boldsymbol{s}})$ is equivalent to transmitting $\boldsymbol{X}(\bar{\boldsymbol{s}})$ in parallel as indicated in Fig. 2. We now vectorize the stacked matrix $\boldsymbol{Y}(\bar{\boldsymbol{s}})$ in Eq. (1) and obtain

$$\boldsymbol{y} = \mathrm{vec}(\boldsymbol{Y}) = \sqrt{\frac{\rho}{\mathrm{M}}}(\boldsymbol{I} \otimes \boldsymbol{H})\boldsymbol{F}\bar{\boldsymbol{s}} + \boldsymbol{w} \tag{2}$$

where $\boldsymbol{F} = [\mathrm{vec}(\boldsymbol{C}_1), \mathrm{vec}(\boldsymbol{C}_2), \cdots, \mathrm{vec}(\boldsymbol{C}_{MT})]$, "$\otimes$" stands for Kronecker product,and $\boldsymbol{w}$ is the vectorized noise. We can now perform linear equalization followed by an ML detector on the received signal vector $\boldsymbol{y}$.

II-B. Design Criteria and Performance Analysis

Let us examine the function of Precoder I. Suppose the original signals are selected, L at a time without repeat, from a constellation $\mathcal{S}$ of cardinality $\mathcal{K}$, then for the signal set $\bar{\mathcal{S}}$ generated by linear transformation, there are $\mathcal{K}^L$ elements. One element $\bar{s}_k$ corresponds uniquely to one group of ordered original signals, $\{s_{k1}, \cdots, s_{kL}\}$. Therefore, at the receiver end of the system, once $\bar{s}_k$ is correctly detected, the corresponding L original symbols are also correctly known. Hence, to examine the system performance, it is sufficient to consider the error probability in detecting $\bar{s}_k$.

From Fig. 2, an ML detector is used to make a decision for $\hat{\bar{s}}$. The pair-wise error probability (PEP) for this is given by,

$$P\left(\bar{s}_k \to \bar{s}_\ell\right) = Q\left(\frac{d_{k\ell}}{2}\sqrt{\gamma}\right) \tag{3}$$

where $Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty e^{-x^2/2}dx$, $d_{k\ell}^2 = \|\bar{s}_k - \bar{s}_\ell\|^2$, and γ is the SNR after linear equalization. Eq. (3) indicates that PEP depends on the two parameters $d_{k\ell}$ and γ. The distance $d_{k\ell}$ is decided by how $\{\bar{s}_k\}$ are generated, i.e., it depends only on Precoder I. On the other hand, γ depends on Precoder II, ρ, the channel states, and the linear equalizer, and is independent of Precoder I. Therefore, $d_{k\ell}$ will affect the coding gain, and γ will affect both the coding gain and diversity gain of the system. Due to the independence of Precoders I and II, their design will be considered separately.

First we consider the design of Precoder II. Here, $d_{k\ell}$ can be treated as a constant since it is fixed by Precoder 1. Thus, the design problem can be formulated as

$$\min_{\boldsymbol{F}} \quad : \quad \mathrm{E}Q\left(\frac{d_{k\ell}}{2}\sqrt{\gamma}\right) \tag{4a}$$

$$\text{s.t.} \quad : \quad \mathrm{tr}\left(\boldsymbol{F}^H\boldsymbol{F}\right) = MT \tag{4b}$$

where "E" denotes the expectation over the random channels, and the constraint in Eq. (4b) is such that the code matrix $\boldsymbol{F}$ maintains the power of the input signals at a constant. Solving Eq. (4), and analyzing the minimum error probability, we obtain the following result,

Theorem 1: For the proposed multi-block transmission system equipped with combined precoder/detectors, the optimal Precoder II is of *unitary trace-orthogonal* [8] structure having achievable diversity gain of $NL - M + 1$.

Proof: The optimization problem in Eq. (4) can be solved following similar derivations in [8] resulting in the optimal code matrix $\boldsymbol{F}$ being unitary trace-orthogonal. Applying this optimum code on the system described in Eq. (1), analysis shows that the diversity gain is $NL - M + 1$. □

Remark 1: Theorem 1 implies that the system diversity gain does not depend on Precoder I.

We now consider the optimum design of Precoder I. To this end, we need to maximize the minimum distance between any two distinct points in the set $\bar{\mathcal{S}}$ since the minimum distance dominates the system performance. The design problem can thus be formulated as

$$\max_{\{a_\ell,\theta_\ell,\ell=1,\cdots,L\}} \quad : \quad \min\{d_{ij}^2\}, \quad i,j \in \mathcal{K}^L, \quad i \neq j \quad (5a)$$

$$\text{s.t.} \quad : \quad \sum_{\ell=1}^{L} a_\ell = L \quad (5b)$$

The optimal a_ℓ and θ_ℓ depend on the signal constellation and L, and there is no general solution. An example in solving Eq. (5) to arrive at an optimum Precoder I design for specific cases is given in Section III.

II-C. Detection Complexity

The detection complexity of the linear equalizer employed in the proposed system is of order $\mathcal{O}(M^3)$ [8], which is the same as that for a single block system with a linear receiver. The additional complexity comes from the computational cost involved with the ML detector, i.e., $\mathcal{O}(\mathcal{K}^L)$, which is independent of M and N. Hence, for a system with a large number of transmitter/receiver antennas, multi-block transmission with a low value of L applied with the combined precoder/detector is an attractive alternative to improve the system performance.

III. DESIGN EXAMPLE

The general design criterion for Precoder I is provided in Eq. (5). The optimal Precoder I depends on the structure of the signal constellation and the value of L. We now provide an example in the design of the optimal Precoder for a 4ϕ-PSK constellation and for the conjoining of $L = 2$ blocks of time slots. Note that for an ML detector, the optimal Precoder I designed for a 4ϕ-PSK constellation is also optimum for 4-QAM, since the latter is merely a rotated version of the former.

For $L = 2$, let $\{s_i\}$ and $\{\bar{s}_i\}$ be the symbol elements in the signal set $\mathcal{S}$ and $\bar{\mathcal{S}}$ respectively. Precoder I maps the $\mathcal{S}$ into $\bar{\mathcal{S}}$ by the linear transformation: $\bar{s}_k = a_1 s_{k1} e^{j\theta_1} + a_2 s_{k2} e^{j\theta_2}$, where $a_1^2 + a_2^2 = 2$ and s_{k1} and s_{k2} are 4ϕ-PSK symbols from $\mathcal{S}$ selected to form $\bar{s}_k$. Without loss of generality, we can assume $\theta_1 = 0$, $\theta_2 = \theta$ and $a_2 \geq a_1 > 0 \;\Rightarrow\; a_2 \geq 1$. Thus, there are three design variables a_1, a_2, θ to maximize the minimum distance between any two distinct points in $\bar{\mathcal{S}}$.

The design problem hinges on the distance, i.e., the difference between two points $\{\bar{s}_j\}$ and $\{\bar{s}_k\}$ in the set $\bar{\mathcal{S}}$ which can be written as

$$\bar{s}_j - \bar{s}_k = a_1(s_{j1} - s_{k1}) + a_2 e^{j\theta}(s_{j2} - s_{k2}) \quad (6)$$

There are two terms in Eq. (6), each formed by the difference of two symbols from a 4ϕ-PSK constellation. Now, $(s_{j\ell} - s_{k\ell}), \ell = 1,2$ is the difference between signals chosen from $\{\pm 1, \pm j\}$ forming a set $\mathcal{P} = \{0, \pm 2, (\pm 1 \pm j), \pm 2j\}$ from which the first term in Eq. (6) is constructed after scaling by a_1. This set of symbols is plotted as full circular dots in Fig. 3. Similarly, the second term in Eq. (6) is selected from the set $\mathcal{Q}$ (represented by hollow circular dots in Fig. 3) which is the set $\mathcal{P}$ scaled by a_2 and rotated by an angle θ. Sets $\mathcal{P}$ and $\mathcal{Q}$ have one common point in the origin. The objective now is to $\max_{a_1,a_2,\theta} : \min\{d_{jk}^2\}$, s.t.: $a_2 > a_1 > 0, a_1^2 + a_2^2 = 2$, where $d_{jk}^2 = \|\bar{s}_j - \bar{s}_k\|^2$ for $j \neq k$. The optimal solution is provided by the following theorem:

Theorem 2: For signals of a 4ϕ-PSK or a 4-QAM constellation, when two ($L = 2$) signal blocks are combined for transmission, the optimal parameters for Precoder I are $a_1 = \sqrt{1 - \frac{1}{\sqrt{3}}}$, $a_2 = \sqrt{1 + \frac{1}{\sqrt{3}}}$, and $\theta = \frac{\pi}{12}$. The maximized minimum distance is 0.8453.

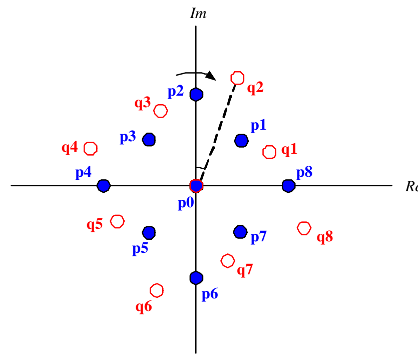

Fig. 3. Plotting of $\bar{s}_i - \bar{s}_j$

Proof: From Fig. 3, it is clear that by the symmetry of the two sets $\mathcal{P}$ and $\mathcal{Q}$, we can limit θ to the range $[0, \pi/4]$. We first obtain the minimum distance and then maximize it. From Eq. (6), the distance d_{jk} is the norm of the sum of two elements, one from each of the two sets $\mathcal{P}$ and $\mathcal{Q}$. Since the elements of both sets are symmetric about the origin, the minimum distance equals the distance between two neighboring points, one from each of the two sets. From Fig. 3, we observe that due to symmetry and $a_1 < a_2$, we only have to consider the distances between two groups of elements: $\{p_1 = \sqrt{2} a_1 e^{j\pi/4},\ p_2 = j2a_1,\ p_3 = \sqrt{2} a_1 e^{j3\pi/4}\}$; $\{q_0 = 0,\ q_2 = j2a_2 e^{-j\theta},\ q_3 = \sqrt{2} a_2 e^{j(3\pi/4-\theta)}\}$. There are thus 9 distances that should be considered: $\{d_{10}, d_{12}, d_{13}; d_{20}, d_{22}, d_{23}; d_{30}, d_{32}, d_{33}\}$ where d_{ij} denotes the distance between the two points (p_i, q_j). From Fig. 3, it is obvious that $d_{10} = d_{30} < d_{20}$; $d_{33} < d_{13}$; $d_{22} < d_{32}$. Thus, there are only 5 distances that can be the minimum, i.e., $d_{10}, d_{12}, d_{22}, d_{23}$ and d_{33}. These 5 distances can be easily calculated giving:

$$d_{10}^2 = 2a_1^2 \quad (7a)$$
$$d_{12}^2 = 4 + 2a_2^2 - 4a_1 a_2(\cos\theta - \sin\theta) \quad (7b)$$
$$d_{22}^2 = 8(1 - a_1 a_2 \cos\theta) \quad (7c)$$
$$d_{23}^2 = 4 + 2a_1^2 - 4a_1 a_2(\cos\theta + \sin\theta) \quad (7d)$$
$$d_{33}^2 = 4(1 - a_1 a_2 \cos\theta) \quad (7e)$$

From Eq. (7), clearly $d_{33}^2 \leq d_{22}^2$ and $d_{23}^2 < d_{12}^2$. Therefore, there are three possible minimum distances: d_{10}, d_{23} and d_{33}. Using the constraint $a_1^2 + a_2^2 = 2$, we can compare them such that:

$$d_{23}^2 - d_{33}^2 = 2a_1(a_1 - 2a_2 \sin\theta) \quad (8a)$$
$$d_{10}^2 - d_{33}^2 = 2a_2(2a_1 \cos\theta - a_2) \quad (8b)$$
$$d_{23}^2 - d_{10}^2 = 4(1 - \sqrt{2} a_1 a_2 \cos(\frac{\pi}{4} - \theta)) \quad (8c)$$

Those quantities in Eq. (8) can be either positive or negative in the feasible range of the variables, which means that any one d_{10}^2, d_{23}^2, or d_{33}^2 can be the minimum. Closer examination of Eqs. (7a), (7d) and (7e) reveals that increasing one distance will cause the decrease in the others. Therefore, to maximize the minimum, we should make the possible minimums all equal. If such a solution exists, then the quantities in Eq. (8) must be equal to zero. Now, we set Eqs. (8a) and (8b) to be zero (and this implies Eq. (8c) equals zero) and obtain $\sin\theta = \frac{a_1}{2a_2}$; $\cos\theta = \frac{a_2}{2a_1}$. Squaring both sides and adding, we have $\frac{a_1^2}{4a_2^2} + \frac{a_2^2}{4a_1^2} = 1$. Combined with

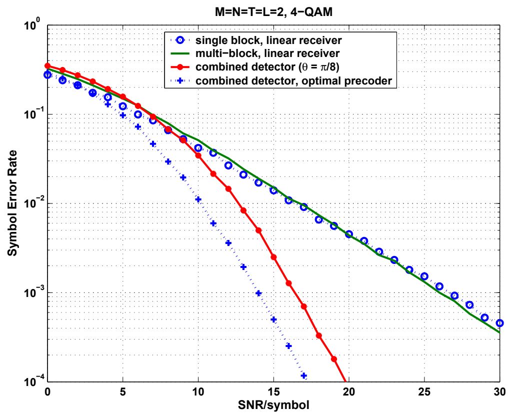

Fig. 4. Performance comparisons between single block, multi-block, and the new proposed schemes.

the condition that $a_1^2 + a_2^2 = 2$, we arrive at $a_1^2 = 1 - \frac{1}{\sqrt{3}}$ and $a_2^2 = 1 + \frac{1}{\sqrt{3}}$, and the optimal angle is $\theta = \tan^{-1} a_1^2/a_2^2 = \tan^{-1} \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\pi}{12}$. The minimum distance equals $(2 - \frac{2}{\sqrt{3}}) = 0.8453$. $\square$

IV. SIMULATION

In this section, we examine the performance of the multi-block transmission scheme with combined detectors by simulation. For the numerical experiments, we employ the optimal signal design provided in Section III in a MIMO system with $M = N = T = L = 2$. The original signals are randomly selected from a 4-QAM constellation, and the transmission (full) data rate is 4 bits per channel use (pcu). The signals are first processed by the optimal Precoder I provided in Section III, and then by the unitary trace-orthogonal precoder. The received signals pass through a linear zero-forcing (ZF) equalizer followed by an ML detector. The resulting symbol error rate (SER) verses SNR/symbol is plotted as the dotted "+" line in Fig. 4. For comparison, we also provide the SER performances of the following three transmission schemes:

1) Single block transmission. The signals are encoded by the unitary trace-orthogonal code [8] and received by a linear ZF equalizer followed by a symbol-by-symbol detector. The SER line is blue dotted with circles.
2) Multi-block transmission scheme over $L = 2$ blocks. The code employed here is the optimal multi-block STBC for a linear receiver provided in [9]. The received signals are also processed by a linear ZF receiver. The SER line is green solid in Fig. 4.
3) We perform the simulation for the same multi-block transmission with combined detector using different Precoder I. Here, in generating $\{\bar{s}\}$, we only shifted the angle, i.e., $\bar{s} = s_i + s_j e^{j\theta}$, with $\theta = \pi/8$. The resulting SER line is red solid with stars.

On Fig. 4, we have the following observations:

- The performance of the proposed scheme is much superior to those two with linear receivers only. The negative slope of the SER curve is much steeper than those for linear receivers, indicating the higher diversity. The significantly improved performance is obtained with marginally higher computational complexity ($4^2 = 16$).
- Now we compare the two SER lines for the same multi-block transmission with different Precoder I. From the analysis in Section II, we know that the design of Precoder I will affect the coding gain but will have no effect on the system diversity. This is indeed the case in Fig. 4, where both lines have the same slope at high SNR, indicating the same diversity gain. The one with the optimal Precoder I has superior performance due to the optimal coding gain.

V. CONCLUSION AND DISCUSSION

In this paper, we have proposed a combined linear and ML detector applied to a multi-block MIMO communication system. This scheme takes advantage of both the simplicity of a linear receiver, and the potential of high quality transmission of a multi-block strategy. We have shown that the diversity with this system is $NL - M + 1$, and it is achieved with a moderate increase in the complexity of detection.

The idea of combining a linear receiver and an ML receiver may be realized in various ways. The useful application to a multi-block MIMO system relies on the successful utilization of the greater degrees of freedom embedded in a multi-block system compared to that of a traditional single block transmission. In spite of the significant improvement obtained, the transmission scheme presented in this paper may be one of several ways to combine linear and ML receivers, together with multi-block transmission strategy.

VI. REFERENCES

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, pp. 744–765, Mar. 1998.

[2] O. Tirkkonen and A. Hottinen, "Square-matrix embeddable space-time codes for complex signal constellations," *IEEE Trans. Inf. Theory*, vol. 48, pp. 384–395, Feb. 2002.

[3] X.-B. Liang and X.-G. Xia, "On the nonexistence of rate-one generalized complex orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2984–2988, Nov. 2003.

[4] H. E. Gamal, G. Caire, and M. O. Damen, "Lattice coding and decoding achieve the optimal diversity–multiplexing tradeoff of MIMO channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 968–985, June 2004.

[5] P. Elia, K. Kumar, S. Pawar, P. V. Kumar, and H. Lu, "Explicit space-time codes achieving the diversity-multiplexing gain tradoff," *IEEE Trans. Inf. Theory*, vol. 52, pp. 3869–3884, Sept. 2006.

[6] F. Oggier, G. Rekaya, J.-C. Belfiore, and E. Viterbo, "Perfect space time block codes," *IEEE Trans. Inf. Theory*, vol. 52, pp. 3885–3902, Sept. 2006.

[7] V. Shashidhar, B. S. Rajan, and B. A. Sethuraman, "Information-lossless space-time block codes from crossed-product algebras," *IEEE Trans. Inf. Theory*, vol. 52, pp. 3913–3935, Sept. 2006.

[8] J. Liu, J.-K. Zhang, and K. M. Wong, "On the design of minimum BER linear space-time block codes for MIMO systems with MMSE receivers," *IEEE Trans. Signal Processing*, vol. 54, no. 8, pp. 3147–3158, Aug. 2006.

[9] J. Liu, T. Davidson, and K. M. Wong, "Diversity analysis and design of space-time multi-block codes for MIMO systems equipped with linear MMSE receivers," *IEEE Trans. Inf. Theory, submitted*, Nov. 2006.

[10] H. El Gamal and A. R. Hammons, "On the design of algebraic space-time codes for MIMO block fading channels," *IEEE Trans. Inf. Theory*, vol. 49, pp. 151–163, Jan. 2003.