KNOWLEDGE-AIDED ADAPTIVE BEAMFORMING

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ABSTRACT

In array processing, when the available snapshot number is comparable with or even smaller than the sensor number, the sample covariance matrix $\hat{\mathbf{R}}$ is a poor estimate of the true covariance matrix $\mathbf{R}$. To estimate $\mathbf{R}$ more accurately, we can make use of prior environmental knowledge, which is manifested as knowing an *a priori* covariance matrix $\mathbf{R}_0$. In this paper, we consider both modified general linear combinations (MGLC) and modified convex combinations (MCC) of the *a priori* covariance matrix $\mathbf{R}_0$, the sample covariance matrix $\hat{\mathbf{R}}$, and an identity matrix $\mathbf{I}$ to get an enhanced estimate of $\mathbf{R}$, denoted as $\tilde{\mathbf{R}}$. Numerical examples are provided to demonstrate the type of achievable performance by using $\tilde{\mathbf{R}}$ instead of $\hat{\mathbf{R}}$ in the standard Capon beamformer.

***Index Terms*—** Knowledge-Aided, Beamforming

1. INTRODUCTION

Let $\mathbf{y}(n)$ denote the nth output snapshot of an array comprising of M sensors. In practice, the true array covariance matrix $\mathbf{R}$, where $\mathbf{R} = E\{\mathbf{y}(n)\mathbf{y}^*(n)\}$, with $(\cdot)^*$ being conjugate transpose and $E(\cdot)$ denoting the expectation operation, is unknown, and so it is usually replaced by the sample covariance matrix $\hat{\mathbf{R}}$, where $\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}(n)\mathbf{y}^*(n)$, with N denoting the snapshot number. However, when N is comparable with or even smaller than M, $\hat{\mathbf{R}}$ usually is a poor estimate of $\mathbf{R}$.

To obtain an improved estimate of $\mathbf{R}$ when the snapshot number N is limited, we can make use of prior environmental knowledge. In a knowledge-aided (KA) system, we may have an initial guess of the true array covariance matrix $\mathbf{R}$, denoted as $\mathbf{R}_0$[1]. When $\mathbf{R}_0$ has full rank, we have considered two shrinkage approaches in [2], called the general linear combination (GLC) and the convex combination (CC) methods, as well as a maximum likelihood based approach in [3] to obtain an improved estimate of $\mathbf{R}$ based on $\hat{\mathbf{R}}$ and $\mathbf{R}_0$. In this paper, we consider the case of $\mathbf{R}_0$ being rank deficient. This case occurs frequently in practice when we only have prior knowledge on dominant sources or clutter discretes. We consider both modified general linear combinations (MGLC) and modified convex combinations (MCC) of the *a priori* covariance matrix $\mathbf{R}_0$, the sample covariance matrix $\hat{\mathbf{R}}$, and the identity matrix $\mathbf{I}$ to get an enhanced estimate of $\mathbf{R}$, denoted as $\tilde{\mathbf{R}}$. MGLC and MCC, respectively, are the modifications of the GLC and CC methods proposed in [2]. Both MGLC and MCC can be extended to deal with linear combinations of an arbitrary number of positive semidefinite matrices.

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2. PROBLEM FORMULATION

We consider the following modified general linear combination (MGLC) of $\mathbf{R}_0$, $\hat{\mathbf{R}}$, and $\mathbf{I}$ to get a new estimate of $\mathbf{R}$, let us call it $\tilde{\mathbf{R}}$:

$$\tilde{\mathbf{R}} = A\mathbf{R}_0 + B\hat{\mathbf{R}} + C\mathbf{I}. \qquad (1)$$

We also consider the following modified convex combination (MCC) of the three terms:

$$\tilde{\mathbf{R}} = A\mathbf{R}_0 + B\hat{\mathbf{R}} + C\mathbf{I}; \quad A + B + C = 1. \qquad (2)$$

Note that the combination weights A, B and C in (1) and (2) should be carefully chosen to guarantee that $\tilde{\mathbf{R}} \geq 0$ (positive semidefinite). We refer to the use of (1) and (2) (with optimized A, B and C, see below) to obtain an estimate of $\mathbf{R}$ as the MGLC and MCC approaches, respectively.

The first goal of this paper is to obtain optimal estimates of A, B and C that minimize the mean-squared error (MSE) of $\tilde{\mathbf{R}}$: MSE $= E\{\|\tilde{\mathbf{R}} - \mathbf{R}\|^2\}$, where $\|\cdot\|$ denotes the Frobenius norm, for both (1) and (2); the second goal of this paper is then to use $\tilde{\mathbf{R}}$ in lieu of $\hat{\mathbf{R}}$ in standard Capon beamformer (SCB)[4] to improve the array output SINR.

3. KA COVARIANCE MATRIX ESTIMATION

3.1. MGLC and MCC

We will consider the MSE minimization problem first for (1) and then for (2). For (1), using the fact that $\hat{\mathbf{R}}$ is an unbiased estimate of $\mathbf{R}$, a simple calculation yields

$$\begin{aligned}\text{MSE}(\tilde{\mathbf{R}}) \;=\; & B^2 E\{\|\hat{\mathbf{R}} - \mathbf{R}\|^2\} + (1-B)^2\|\mathbf{R}\|^2 - 2(1-B) \\ & \text{tr}\,[\mathbf{R}^*(A\mathbf{R}_0 + C\mathbf{I})] + \|A\mathbf{R}_0 + C\mathbf{I}\|^2, \end{aligned} \qquad (3)$$

where $\operatorname{tr}(\cdot)$ denotes the trace operator.

Let

$$\mathbf{b}^T = \left[\operatorname{tr}(\mathbf{R}^*\mathbf{R}_0)\ \|\mathbf{R}\|^2\ \operatorname{tr}(\mathbf{R}^*)\right], \tag{4}$$

where $(\cdot)^T$ denotes the transpose. Let $\boldsymbol{\theta} = [A\ B\ C]^T$. Then (3) can be written more compactly as

$$\operatorname{MSE}(\tilde{\mathbf{R}}) = \boldsymbol{\theta}^T\mathbf{A}\boldsymbol{\theta} - 2\mathbf{b}^T\boldsymbol{\theta} + \text{const}, \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} \|\mathbf{R}_0\|^2 & \operatorname{tr}(\mathbf{R}_0^*\mathbf{R}) & \operatorname{tr}(\mathbf{R}_0^*) \\ \operatorname{tr}(\mathbf{R}_0^*\mathbf{R}) & \|\mathbf{R}\|^2+\rho & \operatorname{tr}(\mathbf{R}^*) \\ \operatorname{tr}(\mathbf{R}_0^*) & \operatorname{tr}(\mathbf{R}^*) & \|\mathbf{I}\|^2 \end{bmatrix}, \tag{6}$$

with $\rho \triangleq E\{\|\hat{\mathbf{R}} - \mathbf{R}\|^2\}$. The minimum solution for (5) is:

$$\boldsymbol{\theta}_0 = [A_0\ B_0\ C_0]^T = \mathbf{A}^{-1}\mathbf{b}, \tag{7}$$

where $(\cdot)^{-1}$ denotes the inverse of a matrix. However, $\boldsymbol{\theta}_0$ in (7) may not guarantee that $\tilde{\mathbf{R}} \geq 0$. We consider the following MSE minimization problem with $\tilde{\mathbf{R}} \geq 0$ enforced:

$$\begin{aligned} \min_{\delta,\boldsymbol{\theta}} \quad & \delta \\ \text{s.t.} \quad & \begin{bmatrix} \delta & [\boldsymbol{\theta}-\boldsymbol{\theta}_0]^T \\ [\boldsymbol{\theta}-\boldsymbol{\theta}_0] & \mathbf{A}^{-1} \end{bmatrix} \geq 0 \\ & \tilde{\mathbf{R}}(\boldsymbol{\theta}) \geq 0. \end{aligned} \tag{8}$$

The above formulation is a Semidefinite Program (SDP) that can be efficiently solved in polynomial time using public domain software. For MCC, we only need to add the following additional constraint

$$\mathbf{u}_3^T\boldsymbol{\theta} = 1, \quad \mathbf{u}_3 = [1\ 1\ 1]^T, \tag{9}$$

to the MGLC formulation in (8). We use $\mathbf{u}_l$ to denote a vector of 1's of length l. The resulting problem is still a SDP.

In practice, $\boldsymbol{\theta}_0$ must be replaced by $\hat{\boldsymbol{\theta}}_0 = \hat{\mathbf{A}}^{-1}\hat{\mathbf{b}}$, with $\hat{\mathbf{A}}$ and $\hat{\mathbf{b}}$ being the estimates of $\mathbf{A}$ and $\mathbf{b}$, respectively. Then (8) becomes:

$$\begin{aligned} \min_{\delta,\boldsymbol{\theta}} \quad & \delta \\ \text{s.t.} \quad & \begin{bmatrix} \delta & \left[\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0\right]^T \\ \left[\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0\right] & \hat{\mathbf{A}}^{-1} \end{bmatrix} \geq 0 \\ & \tilde{\mathbf{R}}(\boldsymbol{\theta}) \geq 0. \end{aligned} \tag{10}$$

Note that $\rho + \|\mathbf{R}\|^2 = E\{\|\hat{\mathbf{R}}-\mathbf{R}\|^2\} + \|\mathbf{R}\|^2 = E\{\|\hat{\mathbf{R}}\|^2\}$, as suggested in [5],we can estimate $\rho + \|\mathbf{R}\|^2$ by $\|\hat{\mathbf{R}}\|^2$, which is an unbiased estimate, and we can estimate $\|\mathbf{R}\|^2$ by $\|\hat{\mathbf{R}}\|^2 - \hat{\rho}$, where $\hat{\rho}$ is an estimate of ρ, which can be obtained as [2]:

$$\hat{\rho} = \frac{1}{N^2}\sum_{n=1}^{N}\|\mathbf{y}(n)\|^4 - \frac{1}{N}\|\hat{\mathbf{R}}\|^2. \tag{11}$$

We also replace $\mathbf{R}$ by $\hat{\mathbf{R}}$ in $\operatorname{tr}(\mathbf{R})$ and $\operatorname{tr}(\mathbf{R}^*\mathbf{R}_0)$. We refer to the resulting SDP problem in (10) for MGLC as MGLC_1, and similarly, (10) with the constraint (9) for MCC as MCC_1.

Alternatively, we can enforce in (10) $A \geq 0$, $B \geq 0$, and $C \geq 0$. Then the constraint $\tilde{\mathbf{R}}(\boldsymbol{\theta}) \geq 0$ is trivially satisfied and (10) becomes a quadratic program (QP):

$$\begin{aligned} \min_{\boldsymbol{\theta}} \quad & \left(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0\right)^T \hat{\mathbf{A}} \left(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0\right) \\ \text{s.t.} \quad & \theta_i \geq 0, \quad i = 1,2,3, \end{aligned} \tag{12}$$

where θ_i is the ith component of $\boldsymbol{\theta}$. We denote this formulation for MGLC as MGLC_2. Similarly, adding (9) as an additional constraint to (12) yields a QP problem for MCC, which we denote as MCC_2.

3.2. Extensions

Let $\{\mathbf{R}_0^{(s)}\}_{s=1}^S$ denote S *a priori* covariance matrices. We consider the following linear combinations:

$$\tilde{\mathbf{R}} = \sum_{s=1}^{S} A^{(s)}\mathbf{R}_0^{(s)} + B\hat{\mathbf{R}} + C\mathbf{I}, \tag{13}$$

where $\{A^{(s)}\}_{s=1}^S$ are the weights applied to the *a priori* covariance matrices $\{\mathbf{R}_0^{(s)}\}_{s=1}^S$. Constraints again need to be imposed to ensure that $\tilde{\mathbf{R}} \geq 0$. Note that (1) is a special case of (13) with $S = 1$.

Matrix $\mathbf{A}$ in (6) for this extended case becomes:

$$\mathbf{A} = \begin{bmatrix} \|\mathbf{R}_0^{(1)}\|^2 & \cdots & \operatorname{tr}(\mathbf{R}_0^{(1)^*}\mathbf{R}_0^{(S)}) & \operatorname{tr}(\mathbf{R}_0^{(1)^*}\mathbf{R}) & \operatorname{tr}(\mathbf{R}_0^{(1)^*}) \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \operatorname{tr}(\mathbf{R}_0^{(1)^*}\mathbf{R}_0^{(S)}) & \cdots & \|\mathbf{R}_0^{(S)}\|^2 & \operatorname{tr}(\mathbf{R}_0^{(S)^*}\mathbf{R}) & \operatorname{tr}(\mathbf{R}_0^{(S)^*}) \\ \operatorname{tr}(\mathbf{R}_0^{(1)^*}\mathbf{R}) & \cdots & \operatorname{tr}(\mathbf{R}_0^{(S)^*}\mathbf{R}) & \|\mathbf{R}\|^2+\rho & \operatorname{tr}(\mathbf{R}^*) \\ \operatorname{tr}(\mathbf{R}_0^{(1)^*}) & \cdots & \operatorname{tr}(\mathbf{R}_0^{(S)^*}) & \operatorname{tr}(\mathbf{R}^*) & \|\mathbf{I}\|^2 \end{bmatrix}. \tag{14}$$

Similarly, $\mathbf{b}$ in (4) now has the form:

$$\mathbf{b}^T = \left[\operatorname{tr}(\mathbf{R}^*\mathbf{R}_0^{(1)})\ \cdots\ \operatorname{tr}(\mathbf{R}^*\mathbf{R}_0^{(S)})\ \|\mathbf{R}\|^2\ \operatorname{tr}(\mathbf{R}^*)\right]. \tag{15}$$

We redefine $\boldsymbol{\theta}$ as $\boldsymbol{\theta} = \left[A^{(1)}\ \cdots\ A^{(S)}\ B\ C\right]^T$. Given $\mathbf{A}$ and $\mathbf{b}$, the corresponding $\hat{\mathbf{A}}$ and $\hat{\mathbf{b}}$ can be obtained similarly as before. Consequently, an estimate of $\boldsymbol{\theta}$ can be obtained by solving (10) for extended MGLC_1, and (12) for extended MGLC_2 (with the constraints being replaced by $\theta_i \geq 0, \quad i = 1, 2, \cdots, S, S+1, S+2$). By solving (10) and (12) with the additional constraint: $\mathbf{u}_{S+2}^T\boldsymbol{\theta} = 1$, we get extended MCC_1 and MCC_2, repectively.

Let

$$\hat{\boldsymbol{\theta}}_{\text{MGLC}_i} = \left[\hat{A}^{(1)}_{\text{MGLC}_i} \ \cdots \ \hat{A}^{(S)}_{\text{MGLC}_i} \ \hat{B}_{\text{MGLC}_i} \ \hat{C}_{\text{MGLC}_i}\right]^T, \quad i = 1, 2, \tag{16}$$

be the solution to the extended MGLC$_i$ problem, $i = 1, 2$. Then the resulting $\tilde{\mathbf{R}}$ for extended MGLC$_i$, $i = 1, 2$, is:

$$\tilde{\mathbf{R}}_{\text{MGLC}_i} = \sum_{s=1}^{S} \hat{A}^{(s)}_{\text{MGLC}_i} \mathbf{R}^{(s)}_0 + \hat{B}_{\text{MGLC}_i} \hat{\mathbf{R}} + \hat{C}_{\text{MGLC}_i} \mathbf{I}, \quad i = 1, 2. \tag{17}$$

Similarly, $\hat{\boldsymbol{\theta}}_{\text{MGLC}_i}$ and $\tilde{\mathbf{R}}_{\text{MCC}_i}$ for MCC$_i$, $i = 1, 2$, can be obtained by replacing the subscript "MGL$_i$" with "MCC$_i$", $i = 1, 2$, in (16) and (17), respectively.

Remark: We note from the unconstrained solution of $\boldsymbol{\theta}$ that the value of $\hat{C}_{\text{MGLC}}$ increases as $\mathrm{tr}(\hat{\mathbf{R}})$ increases. Specifically, $\mathrm{tr}(\hat{\mathbf{R}})$ will be large in the presence of strong interferences, resulting in a very high weighting value on $\mathbf{I}$ in $\tilde{\mathbf{R}}_{\text{MGLC}}$. To avoid the said problem of MGLC, an additional constraint can be enforced in the MGLC formulations in (10) and (12):

$$\theta_{S+2} \leq \gamma \lambda_{\min}, \tag{18}$$

where $\lambda_{\min}$ is the smallest non-zero eigenvalue of $\hat{\mathbf{R}}$, and γ is a scaling factor, which is chosen to be $10^3 M/N$. The constraint in (18) is usually inactive when $\mathrm{tr}(\hat{\mathbf{R}})$ is small.

4. USING $\tilde{\mathrm{R}}$ FOR ADAPTIVE BEAMFORMING

Assume the true covariance matrix $\mathbf{R}$ of the array output has the following form: $\mathbf{R} = \sigma_0^2 \mathbf{a}_0 \mathbf{a}_0^* + \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^* + \mathbf{Q}$, where σ_0^2 and σ_k^2, respectively, are the powers of SOI and of the kth interference impinging on the array, $\mathbf{a}_0$ and $\mathbf{a}_k$ are the steering vectors, and $\mathbf{Q}$ is the noise covariance matrix.

The array weight vector $\mathbf{w}$ obtained by SCB is

$$\mathbf{w}_0 = \frac{\mathbf{R}^{-1}\mathbf{a}_0}{\mathbf{a}_0^* \mathbf{R}^{-1} \mathbf{a}_0}. \tag{19}$$

The beamformer output signal-to-interference-plus-noise ratio (SINR) can be expressed as

$$\text{SINR} = \frac{\sigma_0^2 \left|\mathbf{w}_0^* \mathbf{a}_0\right|^2}{\mathbf{w}_0^* (\sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^* + \mathbf{Q}) \mathbf{w}_0}. \tag{20}$$

By inserting (19) in (20) and using the matrix inversion lemma, we get the optimal array output SINR: $\text{SINR}_{\text{opt}} = \sigma_0^2 \mathbf{a}_0^* \left(\sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^* + \mathbf{Q}\right)^{-1} \mathbf{a}_0$. The weight vectors for SCB, MGLC and MCC can be obtained by replacing $\mathbf{R}$ in (19) with $\hat{\mathbf{R}}$, $\{\tilde{\mathbf{R}}_{\text{MGLC}_i}\}_{i=1,2}$ and $\{\tilde{\mathbf{R}}_{\text{MCC}_i}\}_{i=1,2}$, respectively. The corresponding SINR values can then be obtained by using (20) with the $\mathbf{w}_0$ in (20) replaced by the SCB, MGLC and MCC weight vectors.

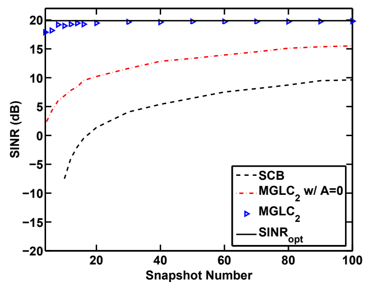
5. NUMERICAL EXAMPLES

In this section, we compare the performance of SCB and MGLC. The performance is also compared with that obtained by setting A's to zero in MGLC, resulting in a diagonal loading approach. The performance of MCC was inferior to that of MGLC and MGLC$_2$ gave overall the best performance in all of our examples and hence only the MGLC$_2$ results are presented hereafter. We consider a uniform linear array (ULA) with $M = 10$ sensors and half-wavelength spacing between adjacent elements. Assume a spatially white Gaussian noise and $\mathbf{Q} = \mathbf{I}$. We assume that the direction-of-arrival (DOA) of the signal of interest (SOI) relative to the array normal is $\theta_0 = 0°$ and that there are $K = 2$ interferences whose DOAs are $\theta_1 = -40°$, $\theta_2 = 20°$. The powers of the SOI and the two interferences are $\sigma_0^2 = 10$ dB, $\sigma_1^2 = 60$ dB and $\sigma_2^2 = 50$ dB, respectively. Also, we assume knowledge of the steering vector $\mathbf{a}_0$. Unlike SCB, MGLC allows N to be less than M.

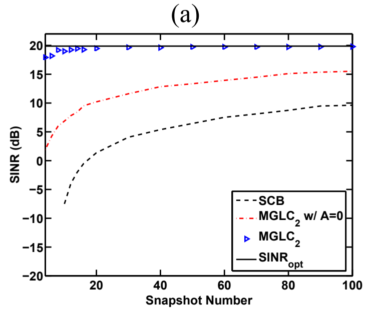
We consider the following six cases (we use $\mathbf{R}_{0i}$ to denote the *a priori* covariance matrix for the ith case, $i = 1, \cdots, 6$):
(i). *Accurate a priori knowledge*, i.e., $\mathbf{R}_{01} = \mathbf{R} - \mathbf{Q} = \sigma_0^2 \mathbf{a}_0 \mathbf{a}_0^* + \sum_{k=1}^{2} \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^*$.
(ii). *Accurate a priori knowledge of the interferences*, i.e., $\mathbf{R}_{02} = \sum_{k=1}^{2} \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^*$.
(iii). *Only the DOA of the first interference is accurately known*: $\mathbf{R}_{03} = \mathbf{a}_1 \mathbf{a}_1^*$.
(iv). *Inaccurate* a priori *knowledge.* We consider the case where the *a priori* knowledge on the DOAs of the interferences is wrong, i.e., $\mathbf{R}_{04} = \sigma_3^2 \mathbf{a}_3 \mathbf{a}_3^* + \sigma_4^2 \mathbf{a}_4 \mathbf{a}_4^*$, where $\sigma_3^2 = \sigma_4^2 = 10$ dB and $\mathbf{a}_3$ and $\mathbf{a}_4$ are the steering vectors for two uncorrelated signals impinging on the array from -55° and 60°.
(v). *The DOAs of the interferences are accurately known*: $\mathbf{R}_{05}^{(1)} = \mathbf{a}_1 \mathbf{a}_1^*$, and $\mathbf{R}_{05}^{(2)} = \mathbf{a}_2 \mathbf{a}_2^*$.
(vi). *The DOA of the first interference is accurately known, but we assume wrongly that the DOA of the second interference is* 60°: $\mathbf{R}_{06}^{(1)} = \mathbf{a}_1 \mathbf{a}_1^*$ and $\mathbf{R}_{06}^{(2)} = \mathbf{a}_4 \mathbf{a}_4^*$.
Figures 1(a) - 1(d) show the averaged array output SINR versus the snapshot number N for Cases (i) - (iv). As shown in 1(a) - 1(c), with (partially) accurate *a priori* knowledge, MGLC$_2$ significantly outperforms MGLC$_2$ with $A = 0$. When the *a priori* knowledge is inaccurate, the performance of MGLC$_2$ is similar to that of MGLC$_2$ with $A = 0$ (see Figure 1(d)). The SINR results for Cases (v) and (vi) are displayed in Figure 2. Again, with (partially) accurate *a prior* knowledge, MGLC$_2$ outperforms MGLC$_2$ with $A^{(1)} = A^{(2)} = 0$.

As we can see from Figures 1 and 2, providing (partially) accurate *a priori* knowledge can give great SINR improvement in the presence of strong interferences.

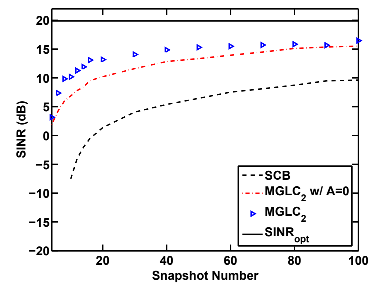
Finally, we comment that our numerical examples (not presented herein) show that, as $N \to \infty$, all SINR curves (including those of MGLC$_1$) approach the optimal SINR curve, as expected. Moreover, the SINR curves of MGLC$_2$ always stay above those of MGLC with $A = 0$ as well as those of SCB.



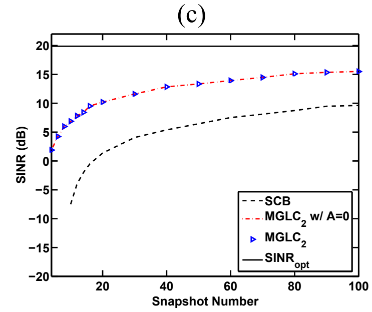
(a)



(b)



(c)



(d)

Fig. 1. SINR versus N for (a) Case (i), (b) Case (ii), (c) Case (iii), and (d) Case (iv).

6. CONCLUSIONS

In this paper, we have presented two fully *automatic* methods, namely MGLC and MCC, for combining the sample covariance matrix $\hat{\mathbf{R}}$ with the *a priori* covariance matrix $\mathbf{R}_0$ (obtained from prior knowledge) and the identity matrix $\mathbf{I}$ to get an enhanced estimate of $\mathbf{R}$ in the optimal mean squared error sense. We have shown that providing accurate or partially accurate *a priori* knowledge can significantly improve the performance of the adaptive beamformers.

7. REFERENCES

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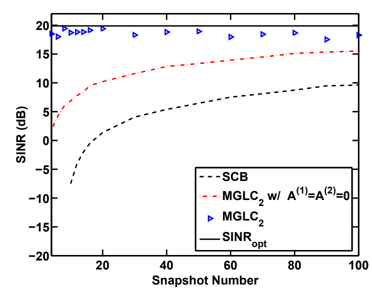
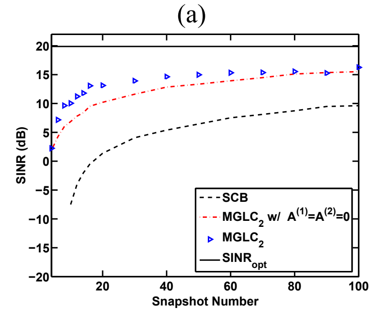


(a)



(b)

Fig. 2. SINR versus N for (a) Case (v), and (b) Case (vi).