BEAMSPACE SLOW-TIME MIMO RADAR FOR MULTIPATH CLUTTER MITIGATION

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ABSTRACT

This paper concerns beamspace multiple-input multiple-output (MIMO) space-time adaptive processing (STAP) to mitigate radar clutter subject to multipath propagation between transmit and receive arrays. Transmit beams are phase-coded to be orthogonal after Doppler processing at the receiver (in "slow-time"). This permits separation and coherent recombination of transmit beams to form virtual transmit nulls in directions that would otherwise result in multipath clutter returns in the receive mainlobe. Compared to element-space methods, beamspace MIMO is particularly advantageous because physical transmit beams can be designed to efficiently illuminate a sector of interest as well as maintain a low transmit voltage standing wave ratio (VSWR).

Index Terms— Radar signal processing, array signal processing, MIMO systems.

1. INTRODUCTION

In this paper, a simple space-time coding framework for multiple-input-multiple-output (MIMO) radar is presented with the goal of mitigating radar clutter subject to multipath propagation between transmit and receive arrays. Multipath clutter occurs when ground backscatter returns arrive at the receive elements via multiple propagation paths. Generally, each path has its own Doppler frequency and wavenumber spreading. Of particular interest is the problem of multipath clutter mitigation for skywave HF over-the-horizon radar (OTHR) [1]. In this application, multiple ionospheric propagation paths can cause ground returns in transmitter sidelobe directions to return in the receive mainlobe with different Doppler shifts, potentially masking targets of interest. In such cases, conventional radar processing techniques cannot mitigate Doppler spread clutter without also suppressing the target. Similar multipath clutter scenarios can occur in other settings, such as ground moving target indicator (GMTI) radars operating in complex terrain. Previous implementations of slow-time MIMO radar [2] can potentially radiate large amounts of energy into non-propagating wavefronts when the transmit array elements are separated by less than a half-wavelength [1]. Though previous work in [3] partially corrects this radiation efficiency issue, this paper presents a slow-time MIMO weight design scheme that naturally handles spatially compact beamspace transmit designs while achieving low VSWR values.

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2. SLOW-TIME MIMO TRANSMIT MODEL

2.1. Transmit Coding

The design presented here will be used in a pulsed Doppler radar system emitting M pulses with a constant pulse repetition interval (PRI), T_r, and a slow-time Doppler channel width (sub-PRF) , f_c, that satisfy $T_r f_c = C/M$, where M/C is an integer. Define an L-by-M matrix $\mathbf{W}$ that contains all of the complex transmitter weights for one coherent processing interval (CPI). The L rows of $\mathbf{W}$ correspond to L transmitter elements. Generally, the columns of $\mathbf{W}$ represent time-domain samples that can be defined differently for various applications. In the case of fast-time MIMO radar, these samples are the sub-PRI waveform samples. For the slow-time MIMO radar herein, there are M columns corresponding to the slow-time dimension (sub-CPI) that provides Doppler information. Each row of $\mathbf{W}$ corresponds to a slow-time weighting progression for a single transmit element. One simple approach to the design of $\mathbf{W}$ for C slow-time channels involves decomposing $\mathbf{W}$ into three components, namely

$$\mathbf{W} = \mathbf{B}\mathbf{P}\mathbf{D}^H. \tag{1}$$

The matrix $\mathbf{B}$ in (1) is L-by-B, where each of the columns of $\mathbf{B}$ contains L element-space complex weights necessary to a transmit beam. The B-by-C matrix $\mathbf{P}$ gives the mapping of the beams in the columns of $\mathbf{B}$ to the slow-time channels in the columns of $\mathbf{D}$. The $(b, c)^{th}$ element of $\mathbf{P}$ gives the contribution of the b^{th} beam in the c^{th} channel. These two matrices are used to project element-space samples $\mathbf{z}_{\text{es}}$ into beamspace samples $\mathbf{z}_{\text{bs}}$ by

$$\mathbf{z}_{\text{bs}} = (\mathbf{BP})^H \mathbf{z}_{\text{es}}. \tag{2}$$

Each of the columns of matrix $\mathbf{D}$ contains M slow-time phase terms that establish C channels in the Doppler domain. Denote the C columns as follows: $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_C]$, where

$$\mathbf{d}_i = \left[1, \ldots, e^{-j2\pi(C-1-2i)\frac{f_c}{2}T_r(M-1)}\right]^T. \tag{3}$$

This is the method used in [2, 3], where $\mathbf{d}_i$ is a sinusoidal Doppler frequency vector containing a unique harmonic of f_c. Thus, the i^{th} channel is associated with a Doppler shift of $-f_c(C-1-2i)/2$.

2.2. Target Response

In this section we derive an expression for a far-field target response after pulse compression at one range gate to provide insight into the design of $\mathbf{B}$ and $\mathbf{D}$ in (1). Consider a target with an L-by-1 steering vector $\mathbf{v}_t$ in the transmit element domain and an N-by-1 steering vector $\mathbf{v}_r$ in the receive element domain. For uniform linear arrays these two vectors are defined as $\mathbf{v}_t = [v_{t1}, \ldots, v_{tL}]^T = \left[1, \ldots, e^{-j\frac{2\pi}{\lambda_0}(L-1)d\sin\phi_t}\right]^T$, and $\mathbf{v}_r = [v_{r1}, \ldots, v_{rN}]^T = \left[1, \ldots, e^{j\frac{2\pi}{\lambda_0}(N-1)d\sin\phi_r}\right]^T$, where d is the interelement spacing of the array elements, and ϕ_t and ϕ_r are the transmit and receive directions to the target, respectively. Letting f_t represent the two-way Doppler shift of the target relative to both transmit and receive arrays, define a Doppler frequency vector $\mathbf{b}(f_tT_r) = \left[1, \ldots, e^{j2\pi(M-1)f_tT_r}\right]^T$.

To derive the response to a target with Doppler frequency f_t, let $\mathbf{x}_1$ represent the M-by-1 response of the target *at a single receive element*, expressed as

$$\begin{aligned} \mathbf{x}_1 &= \alpha_t \boldsymbol{\Gamma}_{\mathbf{b}} \mathbf{W}^H \mathbf{v}_t v_{r1} \\ &= \alpha_t \left[\boldsymbol{\Gamma}_{\mathbf{b}} \mathbf{D}\right] \left[(\mathbf{BP})^H \mathbf{v}_t\right] v_{r1}, \end{aligned} \tag{4}$$

suppressing vector arguments and using $\boldsymbol{\Gamma}_{\mathbf{z}}$ to denote a diagonal matrix with the values contained in vector $\mathbf{z}$ along the diagonal and where α_t is a random complex number that accounts for target scattering characteristics. The term in the left set of brackets in (4) corresponds to a two-stage modulation: first by the Doppler frequencies in the columns of $\mathbf{D}$ and then by the target Doppler in $\boldsymbol{\Gamma}_{\mathbf{b}}$. The right set of brackets is a projection of the element-space $\mathbf{v}_t$ into the beamspace defined by $\mathbf{BP}$.

Multiple receivers are handled by stacking the N sensor responses into the columns of a matrix $\mathbf{X}_r$

$$\begin{aligned} \mathbf{X}_r &= [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N] \\ &= \boldsymbol{\Gamma}_{\mathbf{b}} \mathbf{D}\, (\mathbf{BP})^H\, \mathbf{v}_t \mathbf{1}_N^T \boldsymbol{\Gamma}_{\mathbf{v}_r}. \end{aligned} \tag{5}$$

Matrix $\mathbf{X}_r$ is the M-by-N data matrix that is the fundamental processing unit in the STAP literature [4].

2.3. Slow-Time MIMO Processing

The first step in the signal processing chain is to apply a time-domain matched filter to demodulate each of the channels established by the columns of $\mathbf{D}$. Note that the product $\boldsymbol{\Gamma}^*_{\mathbf{d}_i} \boldsymbol{\Gamma}_{\mathbf{b}} \mathbf{D}$ is a modulation of each column of $\mathbf{D}$ by a Doppler frequency equal to $f_c(C-1-2i)/2 + f_t$, thus shifting the i^{th} column of $\mathbf{D}$ to baseband in Doppler. Denote the single-target demodulated response for channel i in (5) as $\mathbf{X}_i, \forall\, i = 1, \ldots, C$

$$\begin{aligned} \mathbf{X}_i &= \mathbf{F}\, \boldsymbol{\Gamma}^*_{\mathbf{d}_i} \mathbf{X}_r \\ &= \mathbf{F}\, \boldsymbol{\Gamma}^*_{\mathbf{d}_i} \boldsymbol{\Gamma}_{\mathbf{b}} \mathbf{D}\, (\mathbf{BP})^H\, \mathbf{v}_t \mathbf{1}_N^T \boldsymbol{\Gamma}_{\mathbf{v}_r} \end{aligned} \tag{6}$$

where $\mathbf{F}$ is a M/C-by-M matrix representing the operations of lowpass filtering with cutoff frequency f_c and decimation by a factor of C to retain the unique channel frequency content. Transmit domain beamspace processing can continue from this point forward by combining the channels of (6) using a series of weights $\mathbf{w}_c$ to obtain a transmit-beamformed output matrix $\mathbf{Y}$ of size M/C-by-L:

$$\mathbf{Y} = \sum_{i=1}^{C} w_{ci} \mathbf{X}_i. \tag{7}$$

The elements of $\mathbf{w}_c$ affect the transmit beampattern and can be designed adaptively or non-adaptively. Non-adaptive methods involve using $(\mathbf{BP})^H$ to project element domain vectors into beamspace vectors for processing.

The channel separation in (6) motivates the use of orthogonal sinusoids in $\{\mathbf{d}_i\}_{i=1}^C$, as opposed to a set of arbitrary orthogonal functions. In the presence of white noise $\mathbf{n}$, consider representing K targets and clutter sources as a sum of terms in (5), each with different Doppler shifts, transmit directions, and receive directions

$$\mathbf{X}_i = \mathbf{F}\, \boldsymbol{\Gamma}^*_{\mathbf{d}_i} \sum_{k=1}^{K} \boldsymbol{\Gamma}_{\mathbf{b}_k} \mathbf{D}\, (\mathbf{BP})^H\, \mathbf{v}_{tk} \mathbf{1}_N^T \boldsymbol{\Gamma}_{\mathbf{v}_{rk}} + \mathbf{n}. \tag{8}$$

In order for channel separation in (8) to occur, the set $\{\mathbf{d}_i\}_{i=1}^C$ must retain orthogonality after the modulation imparted by the target terms in $\boldsymbol{\Gamma}_{\mathbf{b}_k}$ and the summation over the K sources. Complex sinusoids will suffice as long as the sub-PRF width f_c is larger than the largest Doppler shift in the received data.

2.4. Other Practicalities Involving W

Layout of the transmit weights in a matrix like $\mathbf{W}$ is very useful in determining the transmit directivity and Doppler modulation that is achieved. Transmit beampatterns at each pulse are computed by operating on the columns of $\mathbf{W}$. For a CPI that uses a pulse train with constant PRIs, the DFT along each row of $\mathbf{W}$ illustrates the Doppler frequency signature associated with each transmit element.

The decomposition of $\mathbf{W}$ in (1) can also be applied to other radar transmit weighting schemes. For example, single-input multiple-output (SIMO) radar is a special case of (1).

$$\mathbf{W}_{(\text{SIMO})} = \mathbf{w}_t \;\rho\; \mathbf{1}_M^T$$

where $\mathbf{w}_t$ is an L-by-1 column vector of complex transmit weights and ρ represents a constant. In this case, the transmit beampattern is constant from pulse-to-pulse and is completely defined by $\mathbf{w}_t$. The Doppler domain response of the SIMO weighting scheme is a delta function at zero Doppler.

Element-space MIMO with a single channel on each transmit element as in [2] falls into (1) by choosing $C = B = L$

$$\mathbf{W}_{(\text{ES-MIMO})} = \mathbf{w}_t \mathbf{1}_L^T \;\mathbf{I}_L\; \mathbf{D}$$

where the columns of $\mathbf{D}$ are given by (3). The beampattern of this type of slow-time MIMO weighting scheme has a beampattern with a sweeping mainlobe caused by the distinct channel phase variation found on each element. This magnitude of the elements is constant across each row of $\mathbf{W}$ indicating that this scheme can be implemented as a phase-only modification to a SIMO radar. Sub-arrayed MIMO techniques found in [3] are easily implemented using a non-square $\mathbf{P}$ in (1), but will not be explored any further here.

Beamspace slow-time MIMO, which is a new method presented in Section 3, requires slow-time control of each transmit element's amplitude and phase. For this version of MIMO, each of the columns of $\mathbf{B}$ corresponds to a transmit beam. The columns of $\mathbf{D}$ are sinusoids in slow-time like (3) that establish sub-PRF channels. This gives a one-to-one mapping of a beam to a slow-time Doppler channel. For the simulations that follow in Section 3, $\mathbf{P}$ will be a diagonal C-by-C square matrix.

Arranging the transmit weights in matrix $\mathbf{W}$ is also useful when characterizing the radiation efficiency of the array. One such metric is the voltage standing wave ratio (VSWR). Using a scattering parameter matrix $\mathbf{S}$ for the transmit antenna's operating frequency, the VSWR can be simply calculated

$$\text{VSWR} = \frac{|\mathbf{W}| + |\mathbf{SW}|}{|\mathbf{W}| - |\mathbf{SW}|}. \quad (9)$$

3. SIMULATIONS

To illustrate the benefit of the slow-time MIMO weight design of (1), consider a beamspace implementation employing $L = 16$ transmitter and $N = 127$ receiver element ULAs placed concentrically along the same axis. The operating wavelength is $\lambda_0 = 10.7$ m with array elements spaced at $d = \lambda_0/3$.

Consider a far-field direct path target placed at $\phi_t = 11°$ with an associated Doppler shift of $f_t = -1.94$ Hz (10.3 m/s) and SNR = 25 dB relative to the background white noise level. Direct path ground clutter appears at all azimuth angles at a Doppler of 0 Hz with a clutter-to-noise ratio (CNR) of 40 dB. A strong multipath clutter patch existed between $\phi_t = [27°, 30°]$ with a CNR on transmission of 75 dB. On reception, the energy is spread into other directions and Doppler frequencies according to the model of [5], including a complete masking of the target. The received data is Taylor windowed in the temporal domain and receiver spatial domain.

A SIMO radar is employed using $M = 32$ slow-time pulses and $T_r = 1/7.5$ s, giving a CPI of 4.27 seconds. Returns are only expected in the range of Doppler frequencies between ± 3.75 Hz corresponding to a maximum unambiguous radial speed of 20 m/s. This choice of operating parameters might be appropriate for over-the-horizon (OTH) detection of surface targets such as ships or ground vehicles. The resulting azimuth-Doppler spectrum from traditional SIMO operation over the Doppler frequencies of interest appears in Figure 1. The direct path target at normalized wavenumber (to $2\pi/\lambda_0$) appears at $\sin(\phi_t) = \sin(\phi_r) = 0.19$ and $f_t = -1.94$ Hz in Doppler, and is masked by the Doppler spread clutter.

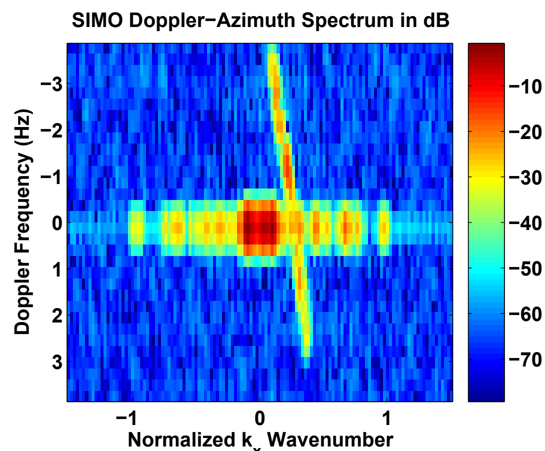
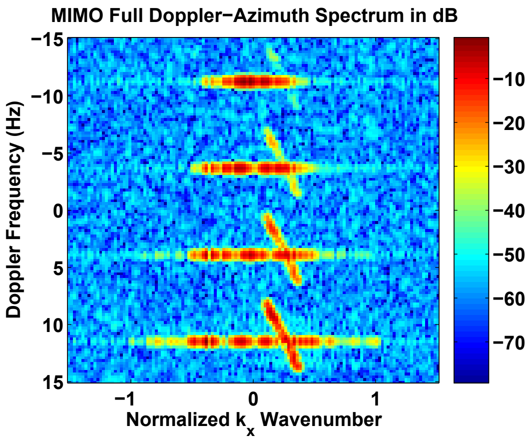
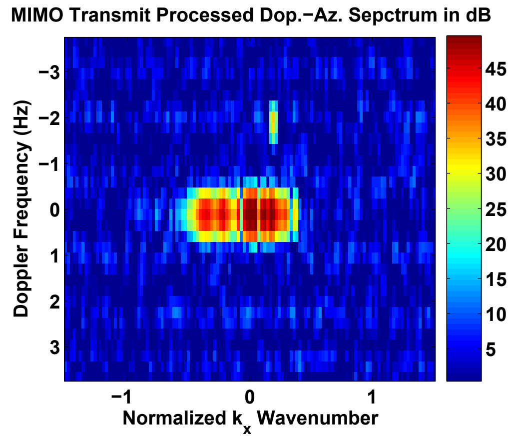


Fig. 1. SIMO radar Doppler-azimuth spectrum where target is obscured by multipath clutter.

Next, consider a $C = 4$ channel MIMO radar system utilizing $M = 128$ slow-time pulses with $T_r = 1/30$ s, also giving a CPI of 4.27 seconds and maintaining an identical Doppler bin-width to the SIMO result from Figure 1. Again, returns are only expected in the range of Doppler frequencies between ± 3.75 Hz. Matrix $\mathbf{B}$ is formed to contain $B = C = 4$ beams taken from the discrete prolate spheroidal sequences (DPSS) that maximally fill the wavenumber spectrum between $\phi_t = \pm 30°$, also used in [1]. In order to create a one-to-one correspondence between beam and channel, $\mathbf{D}$ is a C-by-C diagonal matrix containing the eigenvalues corresponding to the DPSS beams. Doppler channels are established using slow-time sinusoids as in (3). Thus, each spatial beam is 'tagged' with a unique Doppler harmonic separated by $f_c = 7.5$ Hz. The non-adaptively Doppler processed and receive beamformed result of the data in the form of (5) appears in Figure 2(a). The transmitted energy is spatially compact, as evidenced by the 4 strong ground clutter



(a) Full Doppler-azimuth spectrum at receive array for beamspace slow-time MIMO radar.



(b) Result of combining beams to null clutter transmit directions.

Fig. 2. beamspace slow-time MIMO simulation results.

returns between normalized wavenumbers $\pm 1/2$. The resulting Doppler-azimuth spectrum following demodulation and combination of the slow-time channels as described in (7) is shown in Figure 2(b). Here, $\mathbf{w}_c$ is designed in the element-space to place $C - 1 = 3$ nulls between $\phi_t = 27°$ and $\phi_t = 30°$ while maintaining a distortionless response at $\phi_t = 11°$ and then projected into beamspace using $(\mathbf{BP})^H$ as in (2), thus nulling the multipath clutter on transmit while preserving the target response. Observe that the target is unmasked in Figure 2(b) at $\sin \phi_t = 0.19$ and $f_t = -1.94$.

To explore the transmit radiation efficiency, the VSWR in (9) is calculated for both the SIMO and slow-time beamspace MIMO radars, where matrix $\mathbf{S}$ is obtained from a field-tested OTH radar array. VSWR values at each pulse for half of the transmit elements appear in Figure 3, and are a representative samples of the other transmit elements. Note that the values of VSWR for the beamspace slow-time MIMO do not deviate greatly from the SIMO radar values. This indicates that both systems have comparable radiation efficiencies.

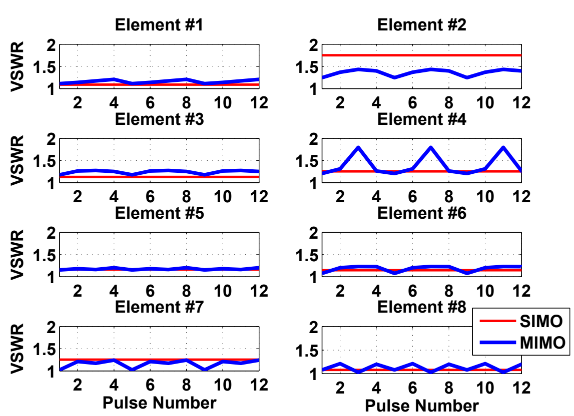


Fig. 3. VSWR for 8 of the 16 transmit elements under SIMO and slow-time beamspace MIMO operation.

4. CONCLUSIONS

A beamspace MIMO STAP approach has been presented and demonstrated for multipath clutter mitigation in OTHR. Beamspace slow-time MIMO has advantages in both transmit efficiency and its relative ease of implementation, particularly on legacy systems which have neither arbitrary function generators on each transmit element nor digital receivers on each antenna. Further work is required to evaluate ambiguity and cross-ambiguity functions for practical transmit waveform design.

5. REFERENCES

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