Cooperative Relay for Decentralized Detection

Hao Chen, Pramod K. Varshney and Biao Chen
Department of EECS
335 Link Hall, Syracuse Univerisity
Syracuse, NY 13244
hchen21{varshney,bichen}@syr.edu

***Abstract*—We consider decentralized detection for resource-constrained wireless sensor networks where local sensor decisions need to go through a multi-hop relay network before reaching the fusion center. Our objective is to collectively design sensor decision rules and relay rules for optimum detection performance. Under the Bayesian criterion, we establish the necessary conditions for an optimal system and derive the form of the optimal fusion rule at the fusion center. Under some conditional independence assumptions, we derive the forms of the optimal local decision rules and the optimal relay rules and show that the optimal set of decision rules for the entire system can be specified by a set of parameters. We demonstrate the advantages of our proposed systematic approach against more conventional design approaches through a numerical example.**

***Index Terms*—Distributed Detection, Cooperative Relay, Wireless Sensor Network**

I. Introduction

Analysis and optimization of the distributed detection system including the design of local decision rules and the fusion center decision rule (fusion rule) under different criteria and constraints are the essential problems of distributed detection systems [1]–[3]. For a binary hypotheses detection problem, the optimal fusion rule can be obtained by applying the likelihood-ratio test (LRT) at the fusion center [4], [5]. Under the assumptions that local observations are conditionally independent given the hypothesis and the fusion center receives the local sensor outputs without any loss, the optimality of the LRT for local sensor decision rules under the Bayesian criterion and the Neyman-Pearson (NP) criterion has been proved in [6] and [7]. For the case where the channels between local sensors and the fusion center are non-ideal, the optimality of local LRTs was proved in [8], [9] and [10] for independent channels and a general multiple access channel, respectively.

In many wireless sensor network (WSN) applications where sensors are operating on a small, irreplaceable power supply, the transmission range of sensors is often limited. Therefore, instead of direct connections, local sensor decisions reach the fusion center through through a relay network consisting of one or multiple hops of relay nodes. The overall detection performance at the fusion center depends not only on the local sensor rules but also on the relay rules. For the case where the relay rules are fixed, the link between local sensors and the fusion center can be modeled equivalently as a fixed channel and the previous approaches proposed in [2] and [10] can be applied to obtain the optimal detection performance. When the relay rules are adjustable, i.e., we can design the "channel" between the local sensors and the fusion center under certain constraints, the fixed channel assumption is no longer valid and such approaches cannot be applied directly. In [11], the decision fusion problem is considered for a binary hypotheses distributed detection system involving a relay network consisting of several parallel independent multi-hop relay paths. Several decision fusion rules were examined with each relay node forwarding the sign of its received signal to the next node.

In this paper, we consider the multiple hypotheses detection problem for distributed detection systems where the local sensor decisions are sent to the fusion center via one or several layers of relay nodes. Unlike the model in [11] where one-to-one independent connections between relay nodes were assumed, here, each relay node may receive signals from one or several nodes in the previous layer of nodes and may send signals to one or several nodes in the next layer. Our goal is to jointly and optimally design all decision rules (local sensor decision rules, relay rules and the fusion rule) to achieve the minimum Bayesian risk. The overall detection performance is obtained as a function of the decision rules and the underlying distributions of the hypotheses. Noticing the similarity between the communication problem and the distributed detection problem, our approach can also be employed for a wireless communication system to improve its performance by reducing the error probability at the destination node.

II. Problem Formulation

As shown in Fig. 1, we consider an M-ary distributed detection system consisting of three types of nodes: local sensors, N layers of relay nodes and the fusion center. To detect the phenomenon $H = \{H_0, H_1, \cdots, H_{M-1}\}$, local decisions are made at the local sensors and transmitted through channels and sets of relay nodes to a fusion center where the final decision U_0 is made. Unlike traditional distributed detection systems where either a perfect lossless channel [1] or a fixed noisy channel [2], [10] between the local sensors and the fusion center is assumed, in this paper, the connection between the local sensors and the fusion center is changeable by adjusting the relay rules at the relay nodes. The system model illustrated in Fig. 1 can be described in detail below.

1) **Local Sensor Layer**. Upon receiving its observation X_k, the kth sensor makes its decision based on its local decision rule $\gamma_k(\cdot)$ such that $U_k = \gamma_k(X_k)$, $k = 1, 2, \cdots, K$. Without loss of generality, we assume that U_k belongs to

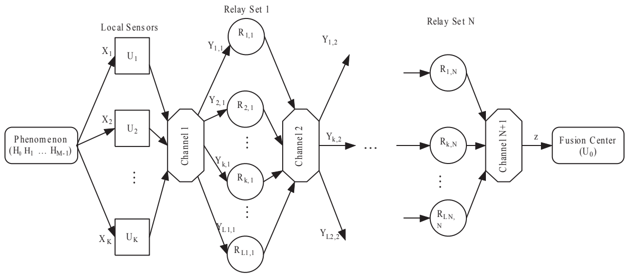

Fig. 1. The canonical parallel wireless distribution fusion system.

a finite-alphabet (FA) set $F_k = \{0, 1, \cdots, V_k - 1\}$ where V_k is the number of possible outputs at the kth local sensor. The local sensors make their decisions based on their own observations only without communicating with others. Let $\mathbf{x} = [X_1, X_2, \cdots, X_K]^T$ and $\mathbf{u} = [U_1, U_2, \cdots, U_K]^T \in F$ be the set of local observations and associated decisions, where $F = F_1 \times F_2 \times \cdots \times F_K$ is the Cartesian product of sets $F_1, F_2, \cdots, F_K$. We have

$$P(\mathbf{u} = [u_1, u_2, \cdots, u_K]^T | \mathbf{x}) = \prod_{k=1}^{K} P(U_k = u_k | X_k). \tag{1}$$

The local sensor decisions are sent to the fusion center through N layers of relay nodes.

2) **Fusion Center**. At the fusion center, a final decision U_0 is made based on its input $\mathbf{z}$ and the fusion rule $\gamma_0(\cdot)$ such that

$$U_0 = \gamma_0(\mathbf{z}). \tag{2}$$

Without loss of generality, U_0 is assumed to be drawn from the set $\{0, 1, 2, \cdots, M-1\}$ where $U_0 = i$ means H_i is accepted, $i = 0, 1, \cdots, M-1$. Note that the similarities between the local sensor, the relay node layers and the fusion center layer, the local sensors can also be considered as the 0th layer of relay nodes, and the fusion center can be considered as the $N+1$th layer of relay nodes.

3) **Relay Nodes**. A total of N layers of relay nodes relay the decisions from the local sensors to the fusion center. At the ith relay node of the jth set, upon receiving its observation $Y_{i,j}$, based on its relay rule $\gamma_{i,j}(\cdot)$, the relay signal $R_{i,j} = \gamma_{i,j}(Y_{i,j})$ is selected from a FA set $G_{i,j} = \{0, 1, \cdots, D_{i,j} - 1\}$ for $1 \leq i \leq L_j, 1 \leq j \leq N$ where L_j is the number of relay nodes in the jth set. Let $\mathbf{y}_j = [Y_{1,j}, Y_{2,j}, \cdots, Y_{L_j,j}]^T$ and $\mathbf{r}_j = [R_{1,j}, R_{2,j}, \cdots, R_{L_j,j}]^T \in G_j$, be the set of observations and decisions for the jth relay set, where $G_j = G_{1,j} \times G_{2,j} \times \cdots \times G_{L_j,j}$ is the Cartesian product of sets $G_{1,j}, G_{2,j}, \cdots, G_{L_j,j}$. we have

$$P(\mathbf{r}_j | \mathbf{y}_j) = \prod_{i=1}^{L_j} P_{i,j}(R_{i,j} = r_{i,j} | Y_{i,j}), \tag{3}$$

where $P_{i,j}(R_{i,j} | Y_{i,j})$ is determined by $\gamma_{i,j}$.
The channel statistics of the jth channel between the j-1th layer of relay nodes and the jth relay nodes is described by its transmission matrix $p_j(\mathbf{y}_j | \mathbf{r}_{j-1})$ where the channel input $\mathbf{r}_{j-1}$ is the relay signal from the $j-1$th layer of relay nodes and the channel output $\mathbf{y}_j$ is the input signal to the jth layer of relay nodes, $j = 1, 2, \cdots, N+1$.

It can be shown that $H \rightarrow \mathbf{x} \rightarrow \mathbf{u} \rightarrow \mathbf{y}_1 \rightarrow \mathbf{r}_1 \rightarrow \mathbf{y}_2 \rightarrow \mathbf{r}_2 \rightarrow \cdots \rightarrow \mathbf{r}_N \rightarrow \mathbf{z} \rightarrow \mathbf{U}_0$ form a Markov chain.

For this M-ary hypotheses detection problem, the Bayesian criterion is adopted where the goal is to minimize the Bayesian cost C given by

$$\begin{aligned} C &= \sum_{0 \leq a,b \leq M-1} c_{a,b} P(U_0 = a, H_b) \\ &= \sum_{0 \leq a,b \leq M-1} c_{a,b} P(U_0 = a | H_b) \pi_b, \end{aligned} \tag{4}$$

where $c_{a,b}$ is the cost of global decision being a when H_b is present, $\pi_b = P(H_b)$ is the prior probability of hypothesis H_b, $b = 0, 1, \cdots, M-1$. For the special case where the cost is the probability of error, we have $c_{a,b} = 0, a = b$ and 1 otherwise. The overall optimization problem can be formalized as follows.

In a M-ary distributed detection system as shown in Fig. 1, given the following:

- the prior probabilities of the hypotheses $\pi_b = P(H_b)$;
- the underlying distribution $\mathbf{x}$ given each hypothesis at local sensors $p(\mathbf{x}|H_b)$, $b = 0, 1, \cdots, M-1$;
- the channel statistics for the $N+1$ channels $p_j(\cdot|\cdot)$, $j = 1, 2, \cdots, N+1$;

design the local sensor decision rules γ_k, $k = 1, 2, \cdots, K$, the relay rules $\gamma_{i,j}$, $j = 1, 2, \cdots N, i = 1, 2, \cdots, L_j$, and the fusion rule γ_0 subject to (1) and (3), respectively such that the Bayesian cost (4) is minimized. In some practical applications, one or several rules may be fixed. In those cases, the goal is to optimally design those rules that are variable to obtain the best achievable performance.

III. Optimum Design of the Distributed Detection System

For this joint optimization problem where three sets of rules are to be determined, the optimum detection performance is obtained when all the rules are optimized at the same time at all nodes. In order to find an optimum set of rules, we first find the necessary conditions for an optimum solution by solving the optimization problem for one node while keeping all other decision rules fixed, i.e., using the person-by-person optimization (PBPO) approach. The optimum fusion rule γ_0 can be obtained in a relatively straightforward manner. Given the set of local sensor decision rules, relay rules and the channel statistics, the Bayesian cost C is given by

$$C = \int_{\mathbf{z}} \sum_{0 \leq a \leq M-1} P(U_0 = a|\mathbf{z}) f_0(\mathbf{z}, U_0 = a) d\mathbf{z}, \tag{5}$$

where

$$\begin{aligned} f_0(\mathbf{z}, U_0 = a) &= \sum_{\mathbf{r}_N} \sum_{0 \leq b \leq M-1} c_{a,b} P(\mathbf{r}_N | H_b) p_{N+1}(\mathbf{z}|\mathbf{r}_N) \pi_b \\ &= \sum_{\mathbf{r}_N} \kappa_0(a, \mathbf{r}_N) p_{N+1}(\mathbf{z}|\mathbf{r}_N) \end{aligned} \tag{6}$$

is the Bayesian cost density function (BCDF) at the fusion center with input $\mathbf{z}$ and decision $U_0 = a$, where

$$\kappa_0(a, \mathbf{r}_N) = \sum_{0 \le b \le M-1} c_{a,b} P(\mathbf{r}_N | H_b) \pi_b$$

is independent of $\mathbf{z}$ and is a function of $\mathbf{r}_N$, output of the Nth relay set and the final decision $U_0 = a$. In order to minimize the cost C, from (5), the γ_0 here amounts to the maximum *a posteriori* probability (MAP) decision, i.e., upon receiving its observation $\mathbf{z}$, the optimal fusion rule γ_0 is to decide the hypothesis with the minimum cost such that

$$\gamma_0(\mathbf{z}) = u_0^m = \arg \min_{a \in \{0,1,\cdots,M-1\}} f_0(\mathbf{z}, U_0 = a). \quad (7)$$

Since the $N+1$th channel statistics $p_{N+1}(\mathbf{z}|\mathbf{r}_N)$ is fixed and known, the form of $f_0(\mathbf{z}, U_0)$ is also fixed, i.e., $f_0(\mathbf{z}, U_0)$ is always a *linear combination of $p_{N+1}(\mathbf{z}|\mathbf{r}_N)$ terms, only the set of coefficients $\kappa(U_0, \mathbf{r}_N)$ is subject to change for different local sensor rules and relay rules*.

Next, we determine the optimal local sensor rules and relay rules. It can be shown that the optimization problem is NP-hard in general. However, under some independence assumptions, forms of the optimal local sensor decision rules and relay rules can be determined. First, let us consider the case where the observations at the local sensors $X_1, X_2, \cdots, X_K$ are assumed to be conditionally independent under any given hypothesis such that $p(\mathbf{x}|H) = \prod_{i=1}^{K} p(X_k|H)$. Let

$$\mathbf{u}^k = [U_1, U_2, \cdots, U_{k-1}, U_{k+1}, \cdots, U_K]^T$$

and

$$\mathbf{x}^k = [X_1, X_2, \cdots, X_{k-1}, X_{k+1}, \cdots, X_K]^T.$$

At the kth local sensor, given all other rules are fixed, the detection performance is given by

$$C = \int_{X_k} \sum_{U_k} P(U_k|X_k) f_k(X_k, U_k) dX_k, \quad (8)$$

with

$$f_k(X_k, U_k) = \sum_{0 \le b \le M-1} \kappa_k(H_b, U_k) p(X_k|H_b), \quad (9)$$

where

$$\begin{aligned}\kappa_k(H_b, U_k) &= \int_{\mathbf{x}^k} \sum_{\mathbf{u}^k} \sum_{0 \le a \le M-1} c_{a,b} \\ &\quad P(\mathbf{u}^k|\mathbf{x}^k) P(U_0 = a|\mathbf{u}^k, U_k) p(\mathbf{x}^k|H_b) \pi_b d\mathbf{x}^k \\ &= \sum_{\mathbf{u}^k} \sum_{0 \le a \le M-1} c_{a,b} P(U_0 = a|\mathbf{u}^k, U_k) \\ &\quad P(\mathbf{u}^k|H_b) \end{aligned} \quad (10)$$

is independent of X_k and is a function of the underlying hypothesis H_b, the local decision U_k and all other decision rules. Thus, the optimal $f_k(X_k, U_k)$ is always *a linear combination of functions $p(X_k|H_b)$ with suitable coefficients under all circumstances*. Thus, the optimal kth local sensor rule γ_k to minimize the overall Bayesian cost C is given by

$$\gamma_k(X_k) = u_k^m = \arg \min_{u_k \in \{0,1,\cdots,V_k-1\}} f_k(X_k, u_k). \quad (11)$$

We next consider the case where $[Y_{1,j}, Y_{2,j}, \cdots, Y_{L_j,j}]$, the inputs at the jth relay set are assumed to be independent given $\mathbf{r}_{j-1}$, output of the previous relay set, i.e.,

$$p_j(\mathbf{y}_j|\mathbf{r}_{j-1}) = \prod_{i=1}^{L_j} p_{i,j}(Y_{i,j}|\mathbf{r}_{j-1}), \quad (12)$$

$j = 1, 2, \cdots, N$. By fixing all rules but $\gamma_{i,j}$, we have

$$C = \int_{Y_{i,j}} \sum_{R_{i,j}} P(R_{i,j}|Y_{i,j}) f_{i,j}(Y_{i,j}, R_{i,j}) dY_{i,j} \quad (13)$$

with

$$f_{i,j}(Y_{i,j}, R_{i,j}) = \sum_{\mathbf{r}_{j-1}} \kappa_{i,j}(\mathbf{r}_{j-1}, R_{i,j}) P(Y_{i,j}|\mathbf{r}_{j-1}) \quad (14)$$

where

$$\begin{aligned}\kappa_{i,j}(\mathbf{r}_{j-1}, R_{i,j}) &= \sum_{0 \le a,b \le M-1} c_{a,b} \sum_{\mathbf{r}_j^i} P(U_0 = a|\mathbf{r}_j^i, R_{i,j}) \\ &\quad P(\mathbf{r}_j^i|\mathbf{r}_{j-1}) P(\mathbf{r}_{j-1}|H_b) \end{aligned} \quad (15)$$

is independent of $Y_{i,j}$ and is a function of previous relay set output $\mathbf{r}_{j-1}$, current relay node decision $R_{i,j}$ and all decision rules except $\gamma_{i,j}$. Thus, the optimal $f_{i,j}(Y_{i,j}, R_{i,j})$ is always *a linear combination of channel statistics $P(Y_{i,j}|\mathbf{r}_{j-1})$ with suitable coefficients under all circumstances*. The optimal relay rule $\gamma_{i,j}$ is to decide the one with the minimum cost, i.e.,

$$\gamma_{i,j}(Y_{i,j}) = r_{i,j}^m = \arg \min_{r_{i,j} \in \{1,2,\cdots,D_{i,j}\}} f_{i,j}(Y_{i,j}, R_{i,j}), \quad (16)$$

where $r_{i,j}^m$ is the relay output.

IV. A Detection Example

Here, we consider a two-sensor and two-relay distributed detection system where the goal is to detect a known signal in additive Gaussian noise using two sensors such that

$$\begin{cases} H_0\colon X_k = N_k \\ H_1\colon X_k = A + N_k \end{cases}, \quad (17)$$

for $k = 1, 2$, where N_1 and N_2 are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance σ^2. Without loss of generality, we assume $A = 1$ and $\sigma^2 = 1$. We also assume that the prior probability $\pi_0 = P(H_0) = P(H_1) = \pi_1 = 0.5$. Each sensor makes a binary decision U_k based on its observation X_k such that $U_k = \gamma_k(X_k)$. $U_k = 1$ if H_1 is decided and $U_k = 0$ if H_0 is decided. In this experiment, U_1 and U_2 are sent to the relay nodes $R_{1,1}$ and $R_{2,1}$ via a Gaussian interference channel (channel 1) such that

$$\begin{aligned} Y_{1,1} &= U_1 + c_{21} U_2 + W_1, \\ Y_{2,1} &= U_2 + c_{12} U_1 + W_2, \end{aligned} \quad (18)$$

where c_{12} and c_{21} are real numbers, and W_1 and W_2 are i.i.d. Gaussian random variables with zero mean and unit variance. $Y_{1,1}$ and $Y_{2,1}$ are conditionally independent given the local sensor outputs $[U_1, U_2]$ and the transmission matrix $P(Y_{1,1}, Y_{2,1}|U_1, U_2)$ is given by

$$P(Y_{1,1}, Y_{2,1}|U_1, U_2) = \phi(Y_{1,1} - U_1 - c_{21}U_2) \cdot \phi(Y_{2,1} - U_2 - c_{12}U_1), \quad (19)$$

where $\phi(\cdot) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$ is the probability density function (pdf) for the standard Gaussian distribution. Note that for the special case where $c_{12} = c_{21} = 0$, the relay channel consists of two i.i.d. Gaussian channels. One bit quantization is assumed at relay sensors, i.e., $R_{i,1} = 0, 1$ depending on its received signal, $i = 1, 2$. The relay signal $\mathbf{r}_1 = [R_{1,1}, R_{2,1}]^T$ is transmitted to the fusion center via channel 2 where the final decision $U_0 = 0$ or 1 is made. In this example, channel 2 consists of two perfect lossless channels such that

$$Z_1 = R_{1,1}$$
$$Z_2 = R_{2,1} \quad (20)$$

Since both conditional independence assumptions are satisfied, the form of optimal local sensor rules and relay rules are given by Eqns. (11) and (16), respectively.

The performance of our approach is compared to a "fixed relay rules" strategy where the output of each relay node is the sign of its received signal, i.e., $R_{i,j} = 1$ when $Y_{i,j} > 0$ and 0 otherwise. For both strategies, we optimize the local sensor rules and fusion rules to achieve the best achievable detection performance in terms of P_E. Here, the prior probabilities of both hypotheses are assumed to be equal such that $\pi_0 = \pi_1 = 0.5$. We vary the channel interference $c_{12} = c_{21} = \rho$ and evaluate the detection performance of both strategies, $0 \leq \rho \leq 1$. P_E as a function of ρ for both strategies is shown in Fig. 2. The proposed approach clearly outperforms the "fixed relay rules" approach for all $\rho > 0$. The performance gain is more significant as $\rho \to 1$. Another interesting observation is that the detection performance for this distributed detection system gets better when the interference between two channels gets larger. Because of the existence of the Gaussian noises W_1 and W_2, the side information/disturbance from the other sensor helps the relay node to better determine the correct underlying hypothesis.

V. Concluding Remarks

We considered the system design problem for a wireless sensor network where the local information is sent to the destination via a relay network. The necessary conditions for the optimal system were established. The form of the optimal fusion rule was shown to be a linear combination of the channel statistics. Similar forms of the optimal local sensor rules and relay rules were also established under certain independence assumptions. Future investigation such as the robust design of decision rules against channel uncertainty is underway. Details of this work can be found in [12].

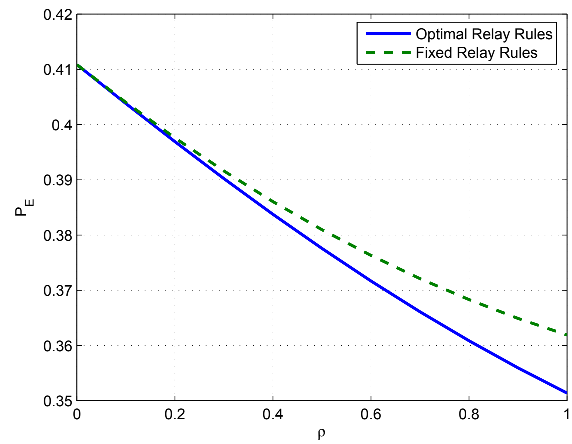

Fig. 2. Detection performance comparison between the optimally designed system and the fixed relay system.

References

[1] Pramod K. Varshney, *Distributed Detection and Data Fusion*, Springer, 1997.

[2] Biao Chen, Lang Tong, and Pramod K. Varshney, "Channel-aware distributed detection in wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 23, no. 4, pp. 16–26, July 2006.

[3] Mujdat Cetin, Lei Chen, John W. Fisher III, Alexander T. Ihler, Randolph L. Moses, Martin J. Wainwright, and Alan S. Willsky, "Distributed fusion in sensor networks," *IEEE Signal Processing Magazine*, vol. 23, no. 4, pp. 42–55, July 2006.

[4] Z. Chair and Pramod K. Varshney, "Optimal data fusion in multiple sensor detection systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-22, pp. 98–101, 1986.

[5] E.Drakopoulos and C. C. Lee, "Optimum multisensor fusion of correlated local decisions," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 27, no. 4, pp. 593 – 606, July 1991.

[6] I. Y. Hoballah and Pramod K. Varshney, "Distributed bayesian signal detection," *IEEE Transactions on Information Theory*, vol. 35, no. 5, pp. 995–1000, Sept. 1989.

[7] S. C. A Thomopoulos, R. Viswanathan, and D. K. Bougoulias, "Optimal distributed decision fusion," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 25, no. 5, pp. 761–765, Sept. 1989.

[8] Biao Chen and Peter K. Willett, "On the optimality of the likelihood-ratio test for local sensor decision rules in the presence of nonideal channels," *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 693–698, Feb. 2005.

[9] Akshay Kashyap, "Comments on "on the optimality of the likelihood-ratio test for local sensor decision rules in the presence of nonideal channels"," *IEEE Transactions on Information Theory*, vol. 52, no. 3, pp. 1274–1275, Mar. 2006.

[10] Hao Chen, Biao Chen, and Pramod K. Varshney, "Further results on on the optimality of the likelihood-ratio test for local sensor decision rules in the presence of nonideal channels," *Submitted to IEEE Transactions on Information Theory*, 2006.

[11] Ying Lin, Biao Chen, and Pramod K. Varshney, "Decision fusion rules in multi-hop wireless sensor networks," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, no. 2, pp. 475–488, 2005.

[12] Hao Chen, Pramod K. Varshney, and Biao Chen, "On the optimal relay design in wireless sensor networks," *under preparation*, 2007.