DISTRIBUTED DETECTION IN UWB WIRELESS SENSOR NETWORKS

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ABSTRACT

In this paper we consider distributed detection in ultra-wideband (UWB) wireless sensor networks with asynchronous transmissions over frequency-selective channels. Three amplify-and-forward schemes with different requirements on channel state information (CSI) are investigated. Performances are studied and compared by using the large deviation principle and simulations.

Index Terms— Distributed detection, UWB, large deviation, sensor networks

1. INTRODUCTION

Distributed detection is an important application for wireless sensor networks. In a typical distributed detection scenario, the information of interest is collected locally by a large number of low-cost sensors. Each sensor then delivers a summary of its own observation to a remote fusion center where the received data are processed to decide one of several hypothesis. In such systems, the primary issue is joint optimal design of local transmission strategies at the sensors and a decision rule at the fusion center, which is known to be difficult to optimize in general, and has been studied for more than two decades (see [1, 2, 3]). However, most existing works ignore the effect of fading and interferences in the channel, and/or assume transmissions from local sensors are perfectly synchronized. Until recently, there has only been limited work on distributed sensing under practical physical layer models ([4, 5, 6]).

Impulse-radio UWB has been considered as a promising candidate for wireless sensor networks due to its low complexity and low cost. In this paper, we consider distributed detection with UWB as underlying physical layer in the presence of practical power, fading and synchronization constraints. Unlike [5], we allow for noise in the sensing model and do not assume that all sensors agree on the same message before transmission. A significant and surprising result of distributed sensing problem is that in a Gaussian system with AWGN channel, a simple amplify and forward analog scheme along with coherent combining outperforms any separate source and channel coding scheme and achieves an asymptotically optimal scaling law [7]. This is because that the goal of distributed sensing is to obtain some common information instead of recovering separate data from every sensor. Since the received signals may cancel out each other if the channel fading is zero mean or in the presence of random synchronization errors, the channel state information (CSI) has to be fed back to each sensor to compensate for the fading and synchronize the signal. However, the UWB signal experiences a frequency-selective channel and has an extremely narrow pulse duration. It is neither practical to feed back full CSI to all sensors, nor synchronize at the pulse level at the receiver.

In this paper, we study the detection performance of UWB systems with amplify-and-forward. In particular, we are interested in the following questions: (i) what is the tradeoff between the detection performance and the feedback overhead? (ii) How can the asymptotically optimal performance be achieved under the practical limits? (iii) How is the asymptotical optimality affected by the system bandwidth and power? To reveal some answers for the above questions, we investigate three schemes with different requirements on CSI, and compare their performances using large deviation analysis. We first derive the performance of a log-likelihood ratio detector when no CSI is available at the sensors. Then, we show that if each sensor knows the sum of its own multipath gain, a coherent combining scheme can achieve asymptotically optimal performance. To reduce the feedback further, we also propose a scheme requiring only 1 bit feedback and having a maximal error exponent with a factor of $2/\pi$ loss compared to the best possible one.

2. SYSTEM MODEL

We study a binary hypothesis distributed detection problem. Let H_0 and H_1 denote the null and alternative hypothesis with prior probabilities $P(H_0) = P(H_1) = 0.5$. M sensors are deployed in the event area and make independent observations U_i, for $i = 1, \cdots, M$, in each observation period. To be specified, we assume

H_0: The ith sensor observes $U_i = w_i$;

H_1: The ith sensor observes $U_i = \theta + w_i$, where $\theta > 0$ is a known constant and w_i is i.i.d Gaussian observation noise with zero mean and variance σ_w^2. Note that our analysis below can be generalized to multiple hypothesis test problems and correlated Gaussian observation noise cases.

We consider an analog transmission strategy where each sensor amplifies its observation and then forwards it to the fusion center using UWB signalling. The transmitted signal from the ith sensor is given by

$$s_i(t) = \sum_{j=0}^{N_f-1} g_i U_i c_i^{DS}(j) p\big(t - jT_f - c_i^{TH}(j)T_c\big), \quad (1)$$

where T_f is the frame duration; N_f is the number of frames in one observation period; $c_i^{DS}(j) = \pm 1$ and $c_i^{TH}(j) = 1, \cdots, L_c$ are the pseudo-random direct sequence and time hopping codes, respectively, in order to smooth the spectrum; T_c is the chip duration; $p(t)$ represents the monocycle waveform with pulse energy normalized to 1 and pulse duration $T_p \ll T_f$; and g_i are the amplification coefficients which must satisfy a power constraint P_{tot}, i.e.

$$\sum_{i=1}^{M} \mathrm{E}(s_i^2(t)) = N_f \sum_{i=1}^{M} \mathrm{E}(g_i^2)(\sigma_w^2 + 0.5\theta^2) \le P_{tot}. \quad (2)$$

The frequency selective channel of UWB systems is modeled by a tapped-delay-line model [8]

$$h_i(t) = \sum_{l=0}^{L-1} \alpha_{li} \delta(t - \tau_{0i} - lT_c), \quad (3)$$

where L is the number of multipath components, τ_{0i} is the delay of the ith sensor's first arrival multipath, and α_{li} is the lth multipath gain of the ith sensor. Since the pulse duration is ultra small and the number of sensors is large, it is unrealistic to assume that each sensor knows τ_{0i} so as to synchronize on a pulse level. We assume that τ_{0i} is unknown to all sensors such that signals arrive at the fusion center unsynchronized. The composite received signal at the fusion center is given by

$$r(t) = \sum_{i=1}^{M} \sum_{l=0}^{L-1} \alpha_{li} s_i(t - \tau_{0i} - lT_c) + n(t), \quad (4)$$

where $n(t)$ is the received white Gaussian noise. The fusion center decides on H_0 or H_1 based on $r(t)$. To simplify the analysis, we have made the following assumptions.

1. We assume $T_p = T_c = 1/W$, where W is the signal bandwidth. Let $N = \lfloor T_f/T_p \rfloor$ denotes the number of resolvable bins in one frame, and $L = \lfloor T_m/T_p \rfloor$, where T_m is multipath delay spread.
2. We assume that the differences of sensor'signal arrival time are limited within T_f, i.e., we assume that $k_i := \tau_{0i}/T_p + c_i^{TH}$, for $i = 1, \cdots, M$, is an integer random number uniformly distributed within $[1, N - L]$.
3. We assume $\hat{\alpha}_{li} := c_i^{DS} \alpha_{li}$ are i.i.d Gaussian random variables with zero mean and normalized variance $1/L$.
4. Since the signals are periodic with period T_f. Without loss of generality, we assume $N_f = 1$.

Under the above assumptions, the received signal can be expanded by N orthogonal samples $\mathbf{r} = [r_1, \cdots, r_N]^T$, and

$$r_j = \sum_{i=1}^{M} g_i U_i h_{ji} + n_j, \quad (5)$$

where T denotes matrix transpose, $n_j \sim \mathcal{N}(0, \sigma_n^2)$ is i.i.d received Gaussian noise, and h_{ji} is the effective channel coefficient, given by

$$h_{ji} = \begin{cases} \hat{\alpha}_{(j-k_i),i} & \text{if } k_i \le j < k_i + L; \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

The received signal in matrix form is given by $\mathbf{r} = \mathbf{HX} + \mathbf{n}$, where $\mathbf{H} = \{h_{ji}\}_{N \times M}$, $\mathbf{X} = [g_1 U_1, \cdots, g_M U_M]^T$ and $\mathbf{n} = [n_1, \cdots, n_N]^T$. In the following, we focus on the performance analysis for Bayesian test, although similar results can be generalized for Neyman-Pearson test.

3. NO CSI AT TRANSMITTERS

If sensors do not have any CSI, given the identical sensor assumption, the amplification coefficients should be the same

$$g = g_1 = \cdots = g_M = \sqrt{\frac{P_{tot}}{M\delta}}, \quad (7)$$

where $\delta = \sigma_w^2 + 0.5\theta^2$. If $\mathbf{H}$ is known by the fusion center, it can be easily verified that $\mathbf{r}$ is Gaussian distributed under both hypothesis,i.e.,

H_0: $\mathbf{r} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$;

H_1: $\mathbf{r} \sim \mathcal{N}(\theta g \mathbf{H1}, \mathbf{\Sigma})$,

where $\mathbf{0} = [0, \cdots, 0]_N^T$, $\mathbf{1} = [1, \cdots, 1]_N^T$, and $\mathbf{\Sigma} = g^2\sigma_w^2 \mathbf{HH}^T + \sigma_n^2 \mathbf{I}_N$ and $\mathbf{I}$ is identity matrix. It is well known that the optimum detector is the log-likelihood ratio test. In the above problem, the decision rule is

$$T_1(\mathbf{r}) := \theta g \mathbf{r}^T \mathbf{\Sigma}^{-1} \mathbf{H1} \gtrless_{H_0}^{H_1} \frac{1}{2}\theta^2 g^2 \mathbf{1}^T \mathbf{H}^T \mathbf{\Sigma}^{-1} \mathbf{H1}. \quad (8)$$

The decision variable $T_1(\mathbf{r})$ is Gaussian random variable with $\mathrm{E}(T_1(\mathbf{r})|H_0) = 0$ and $\mathrm{E}(T_1(\mathbf{r})|H_1) = \mathrm{var}(T_1(\mathbf{r})|H_0) = \mathrm{var}(T_1(\mathbf{r})|H_1) = \theta^2 g^2 \mathbf{1}^T \mathbf{H}^T \mathbf{\Sigma}^{-1} \mathbf{H1}$. Therefore, the error probability of the detector is given by

$$P_{e,1} = Q\left(\frac{1}{2}\sqrt{\theta^2 g^2 \mathbf{1}^T \mathbf{H}^T \mathbf{\Sigma}^{-1} \mathbf{H1}}\right) \quad (9)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$. Although the above scheme does not require any feedback, the performance is generally not optimal as observed from the numerical and simulated results in Sec. 5, since the signals at the same bin from different sensors may cancel out each other.

4. PARTIAL CSI AT TRANSMITTERS

To achieve the optimal performance, both the sensors and the fusion center need to fully exploit the CSI. However, for UWB sensor networks usually with a larger number of sensors and frequency selective channels, it is impractical to feed back the full CSI to all sensors. In this section, we propose two simple schemes which only require partial CSI at the sensors.

Consider the following simple test

$$T_2(\mathbf{r}) := \sum_{j=1}^{N} r_j \gtrless_{H_0}^{H_1} \frac{1}{2}\theta \sum_{i=1}^{M} g_i \sum_{l=0}^{L} \hat{\alpha}_{li}. \qquad (10)$$

Our idea of this test is to mimic the coherent combining method in the AWGN channel proposed in [7]. Assume H_1 is true, we have

$$T_2(\mathbf{r}|H_1) = \theta \sum_{i=1}^{M} g_i f_i + \sum_{i=1}^{M} g_i f_i w_i + \sum_{j=1}^{N} n_j, \qquad (11)$$

where $f_i = \sum_{l=0}^{L} \hat{\alpha}_{li}$. Thus, $T_2(\mathbf{r})$ is Gaussian random variable under both hypothesis, and the detector performance is given by

$$P_{e,2} = Q\left(\frac{1}{2}\sqrt{\frac{(\theta \sum_{i=1}^{M} g_i f_i)^2}{\sigma_w^2 \sum_{i=1}^{M} g_i^2 f_i^2 + N\sigma_n^2}}\right). \qquad (12)$$

We are interested in the asymptotical performance when M goes to infinity. Assume that $g_i = g(f_i)$ for all sensors where $g(\cdot)$ is some function. Using the Cramér's large deviation theorem, we have

$$\lim_{M\to\infty} -\frac{\log P_{e,2}}{M} = \lim_{M\to\infty} \frac{(\theta \sum_{i=1}^{M} g_i f_i)^2}{8M(\sigma_w^2 \sum_{i=1}^{M} g_i^2 f_i^2 + N\sigma_n^2)} = \lim_{M\to\infty} \frac{\theta^2 E^2[g(f_i) f_i]}{8(\sigma_w^2 E[g^2(f_i) f_i^2] + \frac{N}{M}\sigma_n^2)}, \qquad (13)$$

where the second equality is due to the law of large numbers. Since $\mathrm{E}^2[g(f_i)f_i] \leq \mathrm{E}[g^2(f_i)f_i^2]$, eq. (13) is maximized when $\mathrm{var}[g(f_i)f_i] = 0$. Note that the amplification coefficients g_i have to satisfy the power constraint given by (2). To solve this problem, let

$$g(f) = \begin{cases} 1/f, & \text{if } |f| \geq \xi; \\ 0, & \text{if } |f| < \xi. \end{cases} \qquad (14)$$

where ξ is the positive solution of $\int_{\xi}^{\infty} \frac{1}{f^2} b(f) df = \frac{P_{tot}}{2M\delta}$, and $b(f)$ denotes the probability density function of f. Let p_o denote the probability of $|f| \geq \xi$. This scheme requires the fusion center to feed back f_i to each sensor. We call it the sum-feedback scheme.Substituting (14) into (13), we have

$$\lim_{M\to\infty} -\frac{1}{M}\log P_{e,2} = \lim_{M\to\infty} \frac{\theta^2 p_o}{8(\sigma_w^2 + \frac{N}{M p_o}\sigma_n^2)}. \qquad (15)$$

Note that $\theta^2/(8\sigma_w^2)$ is actually the error exponent rate when the sensors are connected to the fusion center with a perfect channel. This is because, if the fusion center has the knowledge of all U_i directly, applying the log-likelihood ratio test to $U_1, \cdots, U_M$, the error exponent is given by $\theta^2/(8\sigma_w^2)$. Therefore, the detector $T_2(\mathbf{r})$ along with g_i given by (14) approaches asymptotically optimal if $p_o \to 1$ and $\lim_{M\to\infty} \frac{N}{M} = 0$. Note that $p_o \to 1$ whenever $\lim_{M\to\infty} \frac{P_{tot}}{M} = \infty$. To get more insight, we consider the following cases for the sum-feedback scheme.

1. If $\lim_{M\to\infty} \frac{P_{tot}}{M} = \infty$ and $\lim_{M\to\infty} \frac{N}{M} = 0$, the sum-feedback scheme has the best possible error exponent.
2. If $\lim_{M\to\infty} \frac{P_{tot}}{M} = 0$ or $\lim_{M\to\infty} \frac{N}{M} = \infty$, the error exponent equals to 0, indicating non-exponential, slow error probability decay rate.
3. Otherwise, the sum-feedback scheme has a sub-optimal error exponent.

From (15), we can see that as N increases the error exponent decreases. Note that our proposed scheme is the counterpart for the frequency-selective channel of the analog scheme with coherent combining in [7]. Since $N = \lfloor T_f W \rfloor$, this shows that with a frequency-selective channel, the performance of the analog scheme with coherent combining may decrease as the bandwidth or the asynchronous interval increases.

To further reduce the overhead of the system, we propose another scheme called the 1-bit-feedback scheme. In such a scheme, the fusion center feeds back only 1 bit information to each sensor, which is described as: if $f_i/|f_i| = 1$, feed back 1 to the ith sensor; otherwise, feed back 0. The amplification coefficient at the ith sensor is thus given by

$$g_i = \frac{f_i}{|f_i|}\sqrt{\frac{P_{tot}}{M\delta}} \qquad (16)$$

The decision rule is still give by (10) and the error probability $P_{e,3}$ is characterized by (12). The error exponent is

$$\lim_{M\to\infty} -\frac{1}{M}\log P_{e,3} = \lim_{M\to\infty} \frac{\theta^2}{4\pi(\sigma_w^2 + \frac{N}{P_{tot}}\sigma_n^2 \delta)}, \qquad (17)$$

Thus, when $\lim_{M\to\infty} \frac{P_{tot}}{N} = \infty$, the error exponent of the 1-bit-feedback scheme achieves its maximum, which has a factor of $2/\pi$ loss compared to the optimal error exponent. When $\lim_{M\to\infty} \frac{P_{tot}}{N} = 0$, the error exponent equals to 0. Similarly, the performance decreases as N increases.

5. SIMULATION RESULTS AND COMPARISONS

In this section, we compare the above three schemes using numerical methods and simulations. The parameters we use are: $L = 10, \theta = 1, \sigma_w^2 = \sigma_n^2 = 1$. Fig. 1 shows the probabilities of error of the three schemes as a function of

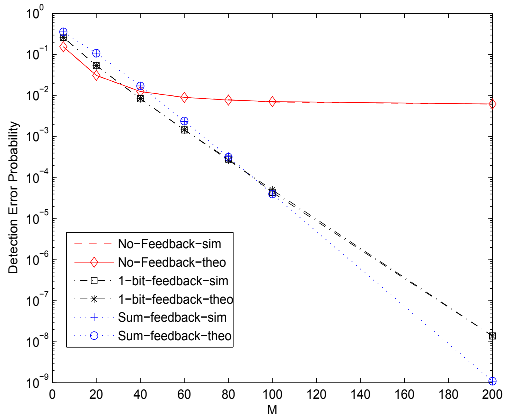

Fig. 1. Performance comparison when $N = 20$, $P_{tot}/M = 10$.

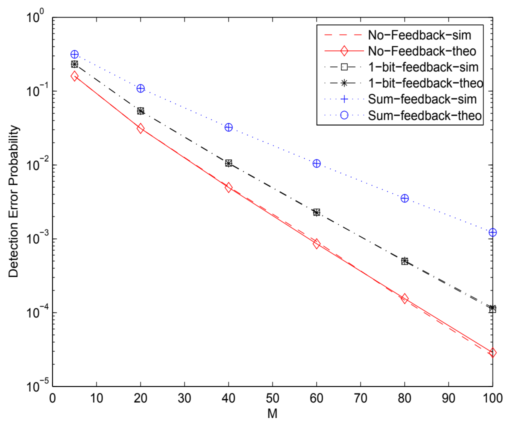

Fig. 2. Performance comparison when $N/M = 1$, $P_{tot}/M = 10$.

M when N is fixed to 20 and P_{tot} increases linearly with M. Note that the Y axis of the figure is in logarithm scale. It can be seen that as M becomes large, the probabilities of error decease very fast for the two feedback schemes. However, since N is fixed, the no-feedback scheme decreases slowly due to sensors cancelling each other's signal. Fig. 2 compares the performances when both N and P_{tot} increase linearly with M. In this case, the no-feedback scheme achieves exponentially decreasing rate and outperforms the two feedback schemes. This is because as N increases with M, the pseudo-random time-hopping code and direct sequence along with the asynchronous transmission enable orthogonal multiple access for the no-feedback scheme. It is noticeable that the sum-feedback scheme outperforms the 1-bit-feedback scheme only when M is very large and N is fixed. Considering the impact of feedback overhead in large networks, the 1-bit-feedback scheme is suggested over the sum-feedback scheme.

6. CONCLUSIONS

We investigate three amplify-and-forward schemes with different CSI requirements for distributed detection in UWB wireless sensor networks. If no CSI is available at the sensors, the best detector can only achieve an exponential error decay rate when both the total power and the time-bandwidth product increase linearly with the number of users. If each sensor knows the sum of its own multipath gain, a coherent combining scheme is proposed to achieve asymptotically optimal performance. To reduce the feedback further, a scheme requiring only 1 bit feedback is also proposed, and it is shown that its maximal error exponent has only a factor of $2/\pi$ loss compared to the best possible one. We also reveal that for the coherent combining approach such as the two feedback schemes, the performance generally deteriorates as the time-bandwidth product increases.

7. REFERENCES

[1] J. N. Tsitsiklis, "Decentralized detection," in *Advances in Signal Processing*, H. V. Poor and J. B. Thomas, Eds. JAI Press, 1993, vol. 2, pp. 297–344.

[2] R. R. Tenney and J. Sandell, N. R., "Detection with distributed sensors," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 17, pp. 501–510, Aug 1981.

[3] J.-F. Chamberland and V. Veeravalli, "Asymptotic results for decentralized detection in power constrained wireless sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1007 – 1015, 2004.

[4] S. Jayaweera, "Large sensor system performance of decentralized detection in noisy, bandlimited channels," in *Proc. Vehicular Technology Conference, 2005*, vol. 2, May 2005, pp. 1096– 1100.

[5] T. Quek, M. Win, and M. Chiani, "Distributed diversity in ultrawide bandwidth wireless sensor networks," in *Proc. Vehicular Technology Conference, 2005*, vol. 2, May 2005, pp. 1355– 1359.

[6] S. Wei, "Spreading sequence-based non-coherent sensor fusion and its resulting large deviation exponents," in *Proc. Acoustics, Speech and Signal Processing, 2007*, vol. 3, 2007, pp. III–177–III–180.

[7] M. Gastpar and M. Vetterli, "Source-channel communication in sensor networks," in *2nd International Workshop on Information Processing in Sensor Networks (IPSN'03)*, L. J. Guibas and F. Zhao, Eds. New York: Springer, April 2003, vol. 2634, pp. 162–177.

[8] D. Cassioli, M. Win, and A. Molisch, "The ultra-wide bandwidth indoor channel: from statistical model to simulations," *IEEE Journal on Selected Areas in Communications*, vol. 20, pp. 1247–1257, Aug. 2002.