ANTI-JAM DISTRIBUTED MIMO DECODING USING WIRELESS SENSOR NETWORKS

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ABSTRACT

Consider a set of sensors that wish to consent on the message broadcasted by a multi-antenna transmitter in the presence of white-noise jamming. The jammer's interference introduces correlation across receivers and destroys the decomposable form of the maximum-likelihood decoder, thus preventing direct application of known distributed detection algorithms. This paper develops distributed detectors that circumvent this problem. Treating the jammer signal as deterministic, we develop two distributed estimation-decoding algorithms. The first algorithm relies on the generalized likelihood ratio test, whereas the second algorithm relies on semi-definite relaxation techniques and is suitable for large alphabet sizes. Both algorithms feature: (i) distributed implementation requiring only single-hop communications; (ii) no constraints on the network topology so long as it is connected; and (iii) performance close to the optimum centralized detector in the presence of severe jamming.

***Index Terms*—** consensus, jamming, generalized likelihood ratio test, semi-definite relaxation, distributed algorithm

1. INTRODUCTION

Consider the scenario depicted in Fig.1, where a multi-antenna access point (AP) broadcasts a common message to a set of single-antenna receivers in the presence of a white-noise jammer. Each receiver forms an estimate of the broadcasted message by iteratively processing its local observations and exchanging messages with one-hop neighboring receivers. The objective is to have all receivers reach an agreement (consensus) on the decoded message while at the same time mitigating the jammer's effect by collecting the available spatial diversity.

The jammer's interference introduces spatial correlation among noise terms at different sensors and renders distributed decoding much more challenging. The difficulty arises because state-of-the-art distributed detection and estimation algorithms apply to objective functions that can be decomposed as a sum of terms, with each term available only at a specific receiver [3, 5, 6]. The common noise terms introduced by the jammer destroy the decomposable structure of the objective function and prevents direct application of known distributed consensus algorithms [3, 5, 9]. Discarding the correlations to allow for distributed designs causes severe error performance degradation, as the jammer interference can be much stronger than the white ambient noise.

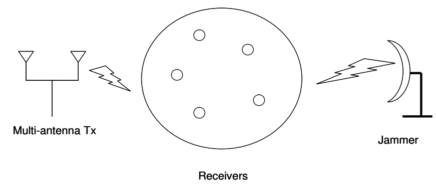

Fig. 1. System setup

We will devise two distributed decoders that exploit these correlations. The main idea is to treat jamming and ambient noises separately. While the uncorrelated ambient noise is used to form the probability density function (pdf) as in maximum-likelihood (ML) detection, the jammer's signal is treated as deterministic but unknown and common to all receivers. This novel approach accommodates non-Gaussian jammers as well. Based on this idea, the first algorithm implements a generalized likelihood ratio test (GLRT), while the second algorithm builds on the semi-definite relaxation (SDR) decoding approach of [7, 8]. The novel SDR aims to reduce the communication overhead that the GLRT based one incurs for large constellations, as SDR features inter-sensor exchanges of polynomial order compared to the exponential overhead of GLRT. The proposed SDR is a modified version of [9] that accounts for jamming effects. Relative to ML, both distributed decoders feature distributed implementation at the cost of sub-optimum performance. However, the degradation is shown to be small for a strong jammer (e.g., 10 dB stronger than ambient noise).

2. PROBLEM FORMULATION

Consider the AP is equipped with M transmit antennas and broadcasts a data vector $\mathbf{b}$ to a network of K single-antenna sensors. The jammer generates noise signal J. The received signal at the k-th sensor is $y_k := \mathbf{h}_k^T\mathbf{b} + g_k J + w_k$, where $\mathbf{h}_k$ is the transmitter-receiver channel gain vector, g_k is the jammer-receiver channel gain, and w_k denotes the receiver noise, which is uncorrelated from sensor to sensor. It is assumed that the k-th sensor knows its own channel gains $\mathbf{h}_k$ and g_k. Notice that the time index is omitted for notational simplicity. Stacking the

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received signals from all sensors in $y := [y_1, \ldots, y_K]^T$, the input/output (I/O) relationship becomes

$$\mathbf{y} = \mathbf{H}\mathbf{b} + J\mathbf{g} + \mathbf{w} \quad (1)$$

where $\mathbf{g} := [g_1, \ldots, g_K]^T$, $\mathbf{w} = [w_1, \ldots, w_K]^T$, and $\mathbf{H} := [\mathbf{h}_1, \ldots, \mathbf{h}_K]^T$. For simplicity, and without loss of generality, we pick $\mathbf{b}$ from a real-valued alphabet $\mathcal{A}_b$ with cardinality $L := |\mathcal{A}_b|$. Accordingly, (1) is real-valued with $J \sim \mathcal{N}(0, \sigma_J^2)$ and $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. From (1), the aggregate noise covariance matrix is

$$\mathbf{C} := \mathrm{E}\left[(\mathbf{w} + J\mathbf{g})(\mathbf{w} + J\mathbf{g})^T\right] = \mathbf{I} + \sigma_J^2 \mathbf{g}\mathbf{g}^T. \quad (2)$$

Applying the matrix inversion lemma [2, pp. 534] to simplify $\mathbf{C}^{-1}$, the ML detector of $\mathbf{b}$ in (1) becomes

$$\hat{\mathbf{b}}_{ML} := \arg \min_{i=\{1,\ldots,L\}} \left\{ \left(\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)}\right)^T \left(\mathbf{I} - \frac{\sigma_J^2 \mathbf{g}\mathbf{g}^T}{1 + \sigma_J^2 \|\mathbf{g}\|^2}\right) \left(\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)}\right) \right\} \quad (3)$$

where $\mathbf{b}^{(i)}$ denotes the i-th candidate data block and i is associated with a corresponding hypothesis. Since $\mathbf{C}^{-1}$ is not diagonal, terms such as $(y_k - \mathbf{h}_k^T \mathbf{b})(y_j - \mathbf{h}_j^T \mathbf{b})$, with $k \neq j$, appear in the objective and hinder the distributed implementation of (3).

3. DISTRIBUTED GLRT DECODER

To circumvent the problem, we treat J as a deterministic parameter and remove it from the covariance matrix in (2). The new covariance $\mathbf{C}'^{-1} := (\mathrm{E}[\mathbf{w}\mathbf{w}^T])^{-1} = \mathbf{I}$ is diagonal and renders the objective decomposable. On the other hand, the pdf of $\mathbf{y}$ is now parameterized by the unknown J, and thus the detector becomes a composite hypotheses test. To solve it, we resort to a generalized like-likelihood ratio test. The GLRT detector yields [2, Section 6.4]

$$\hat{\mathbf{b}}_{GLRT} := \arg \min_{i=0,\ldots,L-1} \|\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)} - \hat{J}^{(i)}\mathbf{g}\|^2 \quad (4)$$

$$\hat{J}^{(i)} := \arg \min_J \|\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)} - J\mathbf{g}\|^2. \quad (5)$$

We can solve (5) for $\hat{J}^{(i)}$ in closed form and plug it back into (4) to find

$$\hat{\mathbf{b}}_{GLRT} := \arg \min_{i=1,\ldots,L} \left\{ \left(\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)}\right)^T \left(\mathbf{I} - \frac{\mathbf{g}\mathbf{g}^T}{\|\mathbf{g}\|^2}\right) \left(\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)}\right) \right\}. \quad (6)$$

One observes that if σ_J^2 is large, (3) and (6) yield essentially the same solution. On the other hand, (6) allows for an easy distributed implementation while (3) does not.

3.1. Estimation-Decoding Algorithms

For a fixed value of i in (4) and (5), we implement (5) using the alternating-direction method of multipliers (AD-MoM) [6]. First, we replace the optimization variable J in (5) with a set of local variables J_k and rewrite (5) as

$$\begin{cases} \min\limits_{\{(J_k),(\bar{J}_k)\}_{k=1}^K} & \sum\limits_{k=1}^K (y_k - \mathbf{h}_k^T \mathbf{b}^{(i)} - g_k J_k)^2 \\ \text{subject to} & J_k = \bar{J}_n, \ \forall k = 1, \ldots, K, \ \forall n \in \mathcal{N}(k) \end{cases} \quad (7)$$

where we have introduced the consensus variables $\bar{J}_n$, and $\mathcal{N}(k)$ denotes the set of one-hop neighbors of node k. Note that (7) and (5) yield the same solution, as proved in [6]. Next, we form the augmented Lagrangian of (7) as

$$\mathcal{L}\left(\left\{J_k, \bar{J}_k, \{\lambda_k^n\}_{n\in\mathcal{N}(k)}\right\}_{k=1}^K\right) := \sum_{k=1}^K \left[\frac{1}{2}\left(y_k - \mathbf{h}_k^T \mathbf{b}^{(i)} - g_k J_k\right)^2 + \sum_{n\in\mathcal{N}(k)} \lambda_k^n (J_k - \bar{J}_n) + \sum_{n\in\mathcal{N}(k)} \frac{c_k}{2}(J_k - \bar{J}_n)^2\right] \quad (8)$$

where $c_k > 0$ are arbitrary penalty factors, λ_k^n are Lagrange multipliers and the factor $(1/2)$ is for mathematical convenience. Notice that the k-th summand inside the bracket is only a function of the local observations at the k-th node, as well as its neighboring consensus variables (which can be communicated). We invoke AD-MoM to iteratively optimize (8) over J_k's, $\bar{J}_n$'s, and λ_k^n's in a distributed fashion. The algorithm is summarized below (cf. [1, Sec. 3.4] and [6]):

Algorithm 1.1 Set $\bar{J}_n(0) = \lambda_k^n(0) = 0$ for all k, n. For iterations $t = 0, 1, \ldots, T-1$ (T denotes maximum number of iterations) repeat the following steps:

Step 1. For each $k = 1, \ldots, K$ update J_k as

$$J_k(t+1) = \frac{g_k(y_k - \mathbf{h}_k^T \mathbf{b}^{(i)}) + \sum_{n\in\mathcal{N}(k)} \left[c_k \bar{J}_n(t) - \lambda_k^n(t)\right]}{g_k^2 + |\mathcal{N}(k)| c_k}$$

and communicate $c_k J_k(t+1) + \lambda_k^n(t)$ to neighbors in $\mathcal{N}(k)$.

Step 2. For each $n = 1, \ldots, K$, update $\bar{J}_n$ as

$$\bar{J}_n(t+1) = \frac{\sum_{k\in\mathcal{N}(n)} \left[c_k J_k(t+1) + \lambda_k^n(t)\right]}{\sum_{k\in\mathcal{N}(n)} c_k}$$

and communicate $\bar{J}_n(t+1)$ to neighbors in $\mathcal{N}(n)$.

Step 3. For each $k = 1, \ldots, K$, update λ_k^n as

$$\lambda_k^n(t+1) = \lambda_k^n(t) + c_k \left(J_k(t+1) - \bar{J}_n(t+1)\right), \forall n \in \mathcal{N}(k).$$

For T sufficiently large, $\bar{J}_k(T) \approx \hat{J}^{(i)}$ for all $k = 1, \ldots, K$ [1, Sec. 3.4], [6]. Having now the estimate of $\hat{J}^{(i)}$ available at each node, we turn our attention back to (4) and observe that

$$\|\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)} - \hat{J}^{(i)}\mathbf{g}\|^2 = \sum_{k=1}^K \left(y_k - \mathbf{h}_k^T \mathbf{b}^{(i)} - g_k \hat{J}^{(i)}\right)^2. \quad (9)$$

After normalizing by K^{-1}, equation (9) can be seen as a consensus averaging cost. To find its value, we rely on an existing algorithm [4]:

Algorithm 1.2 Define $p_k(0) := (y_k - \mathbf{h}_k^T \mathbf{b}^{(i)} - g_k \hat{J}^{(i)})^2 \ \forall k = 1, \ldots, K$. For iterations $t = 0, 1, \ldots, T-1$ update p_k for $k = 1, \ldots, K$ as

$$p_k(t+1) = p_k(t) + \mu \sum_{n\in\mathcal{N}(k)} (p_n(t) - p_k(t))$$

where μ is a constant step size that should be chosen sufficiently small to ensure convergence [3].

Provided T is large enough, $p_k(T) \approx \|\mathbf{y} - \mathbf{H}\mathbf{b}^{(i)} - \hat{J}^{(i)}\mathbf{g}\|^2$ for all k. For each value of i in (4) and (5), Algorithms 1.1 and 1.2 are run consecutively. Afterwards, the $\mathbf{b}^{(i)}$ that yields the smallest p_k over all L hypotheses is chosen as the GLRT estimate. Since all nodes reach consensus on p_k, they will reach consensus on $\hat{\mathbf{b}}_{GLRT}$ as well.

To obtain this estimate, $2TL$ iterations are required and each node has to communicate $3TL$ scalars. Since L grows exponentially with M and the constellation size, this distributed GLRT may have prohibitively high complexity as the number of hypotheses increases. This motivates the distributed SDR decoder described next.

4. DISTRIBUTED SDR DECODER

ML decoding incurs high complexity because the integer constraint on $\mathbf{b}$ renders it non-convex. Since the bi-dual problem (dual of the dual) of ML optimal decoding is convex, it can be efficiently solved and its solution can be used to approximate the original ML solution. This is the well-known Lagrangian relaxation method in optimization. For 16-QAM and smaller constellations, SDR essentially solves the ML bi-dual problem [8]. For higher-order constellations, SDR is not ML bi-dual and has to be further modified to ensure convexity; nonetheless, its performance remains satisfactory [7]. A distributed SDR algorithm was recently proposed [9], but it requires decomposability of the ML detector. We will derive a modified version of this detector that incorporates the jammer's effect. We treat J as deterministic but unknown and seek to jointly estimate $\mathbf{b}$ and J in (1) using the ML criterion. For that matter, notice that

$$\|\mathbf{y} - \mathbf{H}\mathbf{b} - J\mathbf{g}\|^2 = \sum_{k=1}^{K} \left(y_k - \mathbf{h}_k^T\mathbf{b} - g_k J\right)^2$$

$$= \sum_{k=1}^{K} \begin{bmatrix} \mathbf{b}^T & J & 1 \end{bmatrix}^T \begin{bmatrix} \mathbf{h}_k\mathbf{h}_k^T & g_k\mathbf{h}_k & -y_k\mathbf{h}_k \\ g_k\mathbf{h}_k^T & g_k^2 & -y_k g_k \\ -y_k\mathbf{h}_k^T & -y_k g_k & y_k^2 \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ J \\ 1 \end{bmatrix}$$

$$:= \sum_{k=1}^{K} \mathbf{x}^T\mathbf{Q}_k\mathbf{x} = \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{X}\mathbf{Q}_k), \qquad \mathbf{X} := \mathbf{x}\mathbf{x}^T \tag{10}$$

where $\mathbf{x}, \mathbf{Q}_k$ in the third line are defined from the second line and $\mathbf{Q}_k$ depends on the k-th sensor local observations only. Since $\mathbf{X} := \mathbf{x}\mathbf{x}^T$, it satisfies $\mathbf{X} \geq \mathbf{0}$ (in the semi-definite sense), rank$(\mathbf{X}) = 1$ and $\mathbf{X}_{i,i} = 1$, for $i = 1, \ldots, M, M+2$, where for simplicity a binary constellation $\mathbf{b} \in \{\pm 1\}^M$ is considered (see also [7, 8] for available generalizations). With these definitions, the optimization problem becomes

$$\begin{cases} \min\limits_{\{\mathbf{X}\}} & \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{Q}_k\mathbf{X}) \\ \text{subject to} & \mathbf{X} \in \mathcal{P}, \quad \text{rank}(\mathbf{X}) = \mathbf{1} \end{cases} \tag{11}$$

where the set $\mathcal{P}$ is defined as

$$\mathcal{P} = \{\mathbf{X} | \mathbf{X} \geq \mathbf{0}, \quad \mathbf{X}_{i,i} = 1, \ i = 1, \ldots, M, M+2\}.$$

Notice that now $\mathbf{X}$ carries $\mathbf{b}$ and J, and the jammer has increased SDR matrix-vector dimensions by one compared to [9]. Using a similar argument that lead us from (5) to (7), the SDR problem which minimizes (10) over $\mathbf{X}$ can be written in the equivalent form

$$\begin{cases} \min\limits_{\{\mathbf{X}_k, \bar{\mathbf{X}}_k\}_{k=1}^{K}} & \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{Q}_k\mathbf{X}_k) \\ \text{subject to} & \mathbf{X}_k = \bar{\mathbf{X}}_n, \ \forall n \in \mathcal{N}(k), \ \mathbf{X}_k \in \mathcal{P} \end{cases} \tag{12}$$

where the rank constraint is dropped to convexify the problem. The augmented Lagrangian for (12) is formed by

$$\mathcal{L}\left(\left\{\mathbf{X}_k, \bar{\mathbf{X}}_k, \{\mathbf{\Lambda}_k^n\}_{n\in\mathcal{N}(k)}\right\}_{k=1}^{K}\right) := \sum_{k=1}^{K} \Bigg[\mathrm{Tr}(\mathbf{X}_k\mathbf{Q}_k)$$

$$+ \sum_{n\in\mathcal{N}(k)} \mathrm{Tr}\left(\mathbf{\Lambda}_k^{nT}(\mathbf{X}_k - \bar{\mathbf{X}}_n)\right) + \sum_{n\in\mathcal{N}(k)} \frac{c_k}{2}\|\mathbf{X}_k - \bar{\mathbf{X}}_n\|_F^2\Bigg]$$

where F denotes the Frobenius norm of a matrix and $\mathbf{\Lambda}_k^n$ are Lagrange multiplier matrices. The AD-MoM algorithm for $\mathcal{L}(.)$ is derived as follows (cf. [9]):

Algorithm 2. Set $\bar{\mathbf{X}}_k(0) = \mathbf{\Lambda}_k^n(0) = \mathbf{0}$ for all k, n. For iterations $t = 0, 1, \ldots, T-1$ repeat the following steps:

Step 1. For each $k = 1, \ldots, K$, update $\mathbf{X}_k$ as

$$\mathbf{X}_k(t+1) = \arg\min_{\mathbf{X}\in\mathcal{P}} \Bigg[\mathrm{Tr}(\mathbf{X}\mathbf{Q}_k) + \sum_{n\in\mathcal{N}(k)}$$

$$\mathrm{Tr}\left(\mathbf{\Lambda}_k^n(t)^T(\mathbf{X} - \bar{\mathbf{X}}_n(t))\right) + \sum_{n\in\mathcal{N}(k)} c_k\|\mathbf{X} - \bar{\mathbf{X}}_n(t)\|_F^2\Bigg]$$

and communicate $c_k\mathbf{X}_k(t+1) + \mathbf{\Lambda}_k^n(t)$ to neighbors in $\mathcal{N}(k)$.

Step 2. For each $n = 1, \ldots, K$, update $\bar{\mathbf{X}}_n$ as

$$\bar{\mathbf{X}}_n(t+1) = \frac{\sum_{k\in\mathcal{N}(n)} [c_k\mathbf{X}_k(t+1) + \mathbf{\Lambda}_k^n(t)]}{\sum_{k\in\mathcal{N}(n)} c_k}$$

and communicate $\bar{\mathbf{X}}_n(t+1)$ to neighbors in $\mathcal{N}(n)$.

Step 3. For each $k = 1, \ldots, K$, update $\mathbf{\Lambda}_k^n$ as

$$\mathbf{\Lambda}_k^n(t+1) = \mathbf{\Lambda}_k^n(t) + c_k\left(\mathbf{X}_k(t+1) - \bar{\mathbf{X}}_n(t+1)\right), \forall n \in \mathcal{N}(k).$$

If T is sufficiently large, $\bar{\mathbf{X}}_k(T)$ approximates the solution of (12) $\forall\ k = 1, \ldots, K$. To recover the estimates $\hat{\mathbf{b}}_{SDP}$ from $\bar{\mathbf{X}}_k(T)$ we will follow the eigenvalue decomposition approach of [8]. As for the communication overhead, a total of T iterations are needed irrespective of M and constellation size. For one algorithm run, each node has to communicate $T(M+2)(M+3)$ scalars which grows as a polynomial function of M.

It is important to remark that the solution of (11) is not equivalent to the centralized SDR problem. We derived (11) from (1) after treating J as a deterministic parameter; while the centralized SDR is derived directly from (3), where J is incorporated into the pdf (see [7, 8]). The performance gap between the two will be assessed through the simulations.

5. SIMULATIONS

In this section, we depict the bit-error rate (BER) performance of distributed GLRT and SDR algorithms derived in Sections 3 and 4 as a function of the signal-to-noise ratio (SNR). We consider a setup with a jammer 10 dB stronger than the ambient noise. We use a randomly generated connected graph to determine the one-hop communication links among receivers. Receiver's one-hop neighbors are assumed to be sufficiently close to justify error-free message exchanges with error-control coded communications.

Fig. 2 depicts BER performance of the distributed GLRT algorithm for $T = 3, 6, 15$ iterations using $M = 3$ transmit-antennas and $K = 6$ receivers. The centralized ML is also plotted as a benchmark. Note that the performance is satisfactory even with $T = 3$ iterations. We also include the performance of the ML detector with correlations discarded. The BER performance is considerably degraded as no collaborative jamming mitigation is possible in this case.

Fig. 3 depicts BER performance of the distributed SDR algorithm as a function of SNR for $M = 3$ transmit-antennas and $K = 6$ receivers for a 16-QAM constellation. The centralized SDR is also plotted for comparison, along with the distributed SDR in [9], where the correlations are discarded. Note that the proposed detector considerably outperforms [9] with a performance gap of more than 4 dB. In addition, the performance of the proposed method is almost the same as the centralized SDR.

6. CONCLUSIONS

The problem of distributed decoding under a white noise jamming attack was investigated. Jammer's interference introduces correlation across receivers and destroys the decomposable structure of the objective required for distributed implementation. Treating the jammer as a deterministic unknown parameter to maintain the objective's decomposable structure, two distributed decoding algorithms, namely GLRT and SDR, were developed. Simulations determined that their performance remains close to their optimum centralized counterparts when the jammer interference is much stronger than the ambient noise.

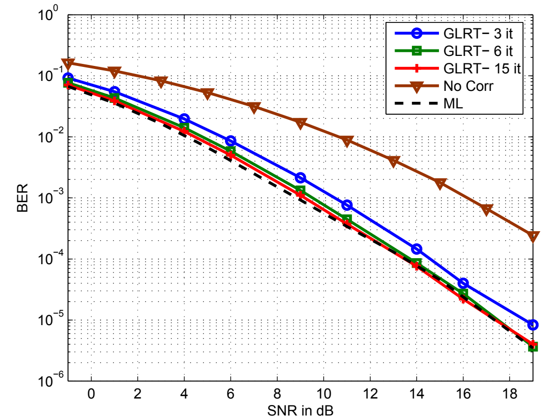

Fig. 2. Distributed GLRT performance

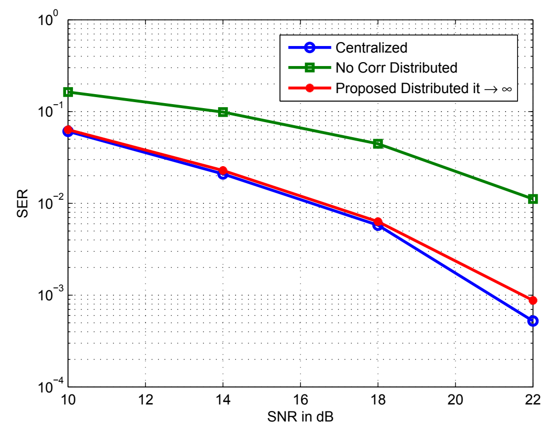

Fig. 3. Distributed SDR performance

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