PERFORMANCE OF DISTRIBUTED ESTIMATION OVER MULTIPLE ACCESS FADING CHANNELS WITH PARTIAL FEEDBACK

Mahesh K. Banavar, Cihan Tepedelenlioğlu, Andreas Spanias

Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287. USA.
Email: {maheshkb, cihan, spanias}@asu.edu

ABSTRACT

We consider a wireless sensor network for distributed estimation over Rayleigh fading channels. The sensors transmit their observations over fading channels to a fusion center, where a source parameter is estimated. Since the sensor transmissions add incoherently over a multiple access channel, we consider partial channel knowledge at the sensors to improve performance. We calculate the variance of the estimate when the channel phase is quantized uniformly and fed back to the sensors. We show that as few as 3 bits of feedback is sufficient for a loss in performance of about 5%. We also show that the performance is robust in the presence of feedback errors.

Index Terms— Distributed estimation, Fading channels, Feedback, Quantization, Sensor Networks

1. INTRODUCTION

Distributed estimation with multiple sensors transmitting data over wireless channels is used in areas such as environmental monitoring and remote sensing. In the most general case, sensors observe data, and after compression and channel coding, transmit their observations to a fusion center over wireless channels. The fusion center then processes the data and provides an estimate of the parameter being observed. We consider the case where the sensor data is transmitted over multiple access fading channels to a fusion center as shown in Figure 1. Amplify and forward with a total power constraint is used, owing to its asymptotic optimality [1].

It is shown in [2] that if the channels between the sensors and the fusion center are zero-mean and sensors have no channel knowledge, the performance of the estimator is poor when the histogram of a finite alphabet source is being estimated. A solution to this problem is to provide channel information to the sensors with feedback from the fusion center. In [3], the authors consider deterministic and non-fading channels, where the sensor gains are selected to minimize the variance of the estimator under perfect channel feedback. In [4], we investigated the performance of the system over Rayleigh fading channels for a large number of sensors, with different choices of feedback from the fusion center to the sensors. We showed that the variance with only channel phase feedback

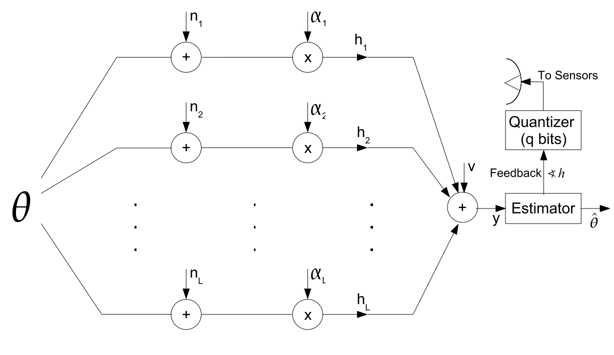

Fig. 1. System Model.

instead of full channel feedback incurs a performance loss of less than a factor of $4/\pi$ in the variance, compared to the optimal case.

Since the feedback channel has a limited capacity, it is natural to characterize the effect of limited feedback on performance. We show that the feedback of only channel phase, even when quantized, leads to a surprisingly small performance loss. We also characterize the effect of errors in feedback on the performance.

2. SYSTEM MODEL

Figure 1 shows our wireless sensor network setup. The fusion center aggregates data from the L sensors in the network and estimates the random parameter, θ, observed by the sensors in additive Gaussian noise. The sensors transmit their observations to the fusion center over L independent Rayleigh fading channels. The estimate obtained at the fusion center is represented by $\hat{\theta}$. It is assumed that the fusion center has complete knowledge of the channels and sensor gains but only statistical information about the noise sources.

The gains on the channel between the l^{th} sensor and the fusion center, represented by h_l, are i.i.d., and distributed as $\mathcal{CN}(0, 1)$. The observation noise added between the parameter and the sensors is given by $n_l \sim \mathcal{CN}(0, \sigma_n^2),\ l = 1, \dots, L$ and the noise added on the multiple-access channel is represented by $v \sim \mathcal{CN}(0, \sigma_v^2)$. Each sensor contributes a gain of α_i to the received signal, before forwarding the data to the fusion center using the amplify and forward scheme. The random parameter being estimated, θ, has variance σ_θ^2. All the noise sources, the channels and the random parameters are statistically independent of each other. The received signal at the fusion center is given by

$$y = \sum_{i=1}^{L} (\theta + n_i)\alpha_i h_i + v, \tag{1}$$

where the sensor gains are constrained in terms of the total power P_T by

$$P = \sum_{l=1}^{L} |\alpha_l|^2 = \frac{P_T}{\sigma_\theta^2 + \sigma_n^2}. \tag{2}$$

The estimator for $\hat{\theta}$ at the fusion center is linear and unbiased as below:

$$\hat{\theta} = \frac{y}{\sum_{i=1}^{L} \alpha_i h_i} = \theta + \frac{\sum_{i=1}^{L} n_i \alpha_i h_i}{\sum_{i=1}^{L} \alpha_i h_i} + \frac{v}{\sum_{i=1}^{L} \alpha_i h_i}. \tag{3}$$

2.1. Performance Evaluation

Before we address the finite-rate feedback case, we summarize our results in [4]. It has been shown in [4], that the variance of $\hat{\theta}$ in (3) is given by

$$\text{var}\left(\hat{\theta}\right) = \frac{\sigma_n^2 \sum_{i=1}^{L} \left(|\alpha_i|^2 |h_i|^2 \right) + \sigma_v^2}{\left| \sum_{i=1}^{L} \alpha_i h_i \right|^2}. \tag{4}$$

When the channels are fading, the h_l's are random and consequently, the variance is random. In the cases that are of interest to us, the amplification factor α_i depends on h_i which means that there is (partial) channel state information at the sensors (CSIS). In these cases, the variance in [4] goes to zero in such a way that

$$\lim_{L \to \infty} L\text{var}\left(\hat{\theta}\right) = C, \tag{5}$$

where C is a deterministic constant. We will use different values of C for different choices of partial feedback to quantify the performance of the system. Clearly, a smaller value of C is preferred.

Since the performance over the AWGN channel is expected to behave better than over fading channels, we can obtain a lower bound on C by setting $h_l = 1 \, \forall l$. To respect the power constraint, we set $\alpha_l = \sqrt{P/L}$. Substituting the values of α_l and h_l in (4), we obtain

$$C_{AWGN} = \frac{\sigma_n^2 P + \sigma_v^2}{P}. \tag{6}$$

2.2. Performance over Rayleigh Fading Channels

When there is no channel state information at the sensors (CSIS), the sensor gains are all set to equal constants, $\alpha_l = \sqrt{P/L} \, \forall l$. It can be seen that the denominator in (4), properly normalized, converges to zero, which indicates poor performance with large variance. When non-zero mean channels such as Rician or Nakagami channels are used, even with no CSIS, the denominator converges to a non-zero constant. The variance of $\hat{\theta}$ now exists, and decays to zero in a way that satisfies (5), as shown in [4].

A solution to the zero mean channel problem is to provide some channel information as feedback from the fusion center to the sensors in order to make the effective channel ($\alpha_l h_l$) non-zero mean. The best performance for fading channels is obtained when the sensors receive complete channel information from the fusion center and optimize the sensor gains to minimize the variance of $\hat{\theta}$ subject to the power constraint [4]. In this case, (5) becomes

$$C_{OPT} = \frac{\sigma_n^4 P^2 - e^{\frac{\sigma_v^2}{P^2 \sigma_n^2}} \sigma_v^2 E_1\left(\frac{\sigma_v^2}{P^2 \sigma_n^2}\right)}{\sigma_n^4 P^2}, \tag{7}$$

where

$$E_1(x) = \int_1^\infty \frac{e^{-tx} dt}{t} = \int_x^\infty \frac{e^{-u} du}{u}.$$

The computational complexity required to calculate the optimal sensor gains is large. Also, the values for the α_l's have a very large dynamic range across sensors and across time. Alternatively, we can set equal magnitude gains to satisfy the power constraint on the sensors, and the phases of the sensors are set to cancel the phase of the channels, i.e., $\alpha_i = \sqrt{P/L} e^{-j\angle h_i}$. Using (4) and (5), the performance in this case can be quantified [4] as

$$C_{PO} = \frac{\sigma_n^2 P + \sigma_v^2}{P} \frac{4}{\pi}. \tag{8}$$

Clearly, $C_{PO} \geq C_{OPT} \geq C_{AWGN}$ and in [4], we showed that $C_{PO} = (4/\pi) C_{AWGN}$. It can be seen that C_{AWGN} can be viewed as a simple benchmark, and C_{PO} is only a factor of $4/\pi$ worse that C_{AWGN}. However, C_{AWGN} is not necessarily achievable over fading channels for some choice of $\{\alpha_l\}_{l=1}^{L}$, whereas C_{OPT} represents an achievable benchmark [4].

3. FINITE RATE FEEDBACK

If we feedback both the channel gain and phase, the performance will be better than if we feedback only the channel phase. However, for a fading channel, the magnitude of the channel gain has a very large dynamic range. Developing a robust scheme to quantize channel gain in this scenario is non-trivial. In what follows, we describe quantizing the phase-only feedback and analyze its performance for large L using (4) and (5).

Let the number of bits used for phase-only feedback be q. The phase of each channel coefficient will be divided into 2^q sectors and from each sector, a representative angle is selected. When the channel angle falls within a particular sector, the phase that is fed-back to the sensor with that channel is the representative angle of that particular sector. Each sector spans an angle of $2\pi/2^q$. The worst possible error occurs when the actual phase is on the sector boundary, and is $\pi/2^q$. To send the appropriate phase feedback, each sector is mapped to a q-bit sequence. This sector-to-bit sequence mapping is similar to decision regions for M-PSK.

Since we have no channel magnitude information, we set the magnitudes of all sensor gains to $\sqrt{P/L}$. Therefore,

$$\alpha_{l,q} = \sqrt{\frac{P}{L}} e^{-jf_q(h_l)}, \tag{9}$$

where $f_q(h_l)$ is the representative angle of the specific sector. With $f_q(h_l) \in \left\{ e^{j\frac{2\pi k}{2^q}} \right\}_{k=0}^{2^q-1}$, each quantization point, $f_q(h_l)$, is chosen as:

$$f_q(h_l) = \min_k \left| \angle h_l - \frac{2\pi k}{2^q} \right|. \tag{10}$$

Substituting this in (4), we have

$$\text{var}\left(\hat{\theta}\right) = \frac{\sigma_n^2 P \frac{1}{L} \sum_{i=1}^{L} |h_i|^2 + \sigma_v^2}{PL \left[\frac{1}{L} \sum_{l=1}^{L} |h_l| \, e^{-j\left[f_q(h_l) - \angle h_l\right]} \right]^2}. \tag{11}$$

Using law of large numbers and the fact that the variance is a continuous function of the denominator which is positive, we obtain

$$C_{PO}(q) = \lim_{L\to\infty} L\text{var}\left(\hat{\theta}\right) = \frac{\frac{\sigma_n^2 P + \sigma_v^2}{P} \frac{4}{\pi}}{\left[E\left(e^{-j\left\{ f_q(h_l) - \angle h_l \right\}} \right) \right]^2}. \tag{12}$$

To calculate (12), let $\phi = (f_q(h_l) - \angle h_l)$. Since this is the error in phase and because $\angle h_l$ is uniformly distributed, ϕ is uniform between $-\pi/2^q$ and $\pi/2^q$. From this:

$$E\left[e^{-j\phi}\right] = \frac{2^{q-1}}{\pi} \int_{-\frac{\pi}{2^q}}^{\frac{\pi}{2^q}} e^{-j\phi} d\phi = \frac{2^q}{\pi} \sin\left(\frac{\pi}{2^q}\right). \tag{13}$$

Substituting (13) in (12), it follows that

$$C_{PO}(q) = \left[\text{sinc}(2^{-q})\right]^{-2} C_{PO}, \tag{14}$$

where C_{PO} is as in (8) and $\text{sinc}(x) = (\sin(\pi x))/(\pi x)$.

Loss in performance due to quantization is $C_{PO}(q)/C_{PO}$, which takes the value of 2.4674 for $q = 1$ and is 1 as $q \to \infty$. It is interesting that due to our large sensor analysis, we can simply relate the performances for the AWGN benchmark, phase-only, and quantized phase-only cases using the following:

$$C_{PO}(q) = \underbrace{C_{AWGN}\left(\frac{4}{\pi}\right)}_{C_{PO}} \left[\text{sinc}(2^{-q})\right]^{-2}. \tag{15}$$

q	1	2	3	4	5
$\frac{C_{PO}(q)}{C_{PO}}$	2.4674	1.2337	1.0530	1.0130	1.0032

Table 1. Degree of deterioration due to quantization.

Table 1 contains the values of $C_{PO}(q)/C_{PO}$ for different values of q. It can be seen that by using three bits of quantization, there is an increase in variance of about 5% and for five bits, the deterioration is less than a percent.

Figure 2 shows the effect of quantization on the performance of the system at low P. The best performance is seen for C_{OPT} described in (7). The next best performance is obtained for C_{PO} in (8). Between C_{PO} and C_{OPT}, there is a loss of less than a dB caused by not feeding back channel magnitudes. For two bits of quantization, there is a loss in performance by a factor of about 1 dB compared to C_{PO}. Comparing C_{PO} with when we use four bits of quantization, we find that the loss incurred by quantization is negligible.

3.1. Error on the Feedback Channel

When the feedback channel is noisy, it induces errors in the feedback bit sequence. If the probability of one bit being toggled is p, the probability of having one bit error in the bit sequence is $p(1-p)^{q-1}$. The probability of making a two bit error is $p^2(1-p)^{q-2}$ which is negligible compared to $p(1-p)^{q-1}$ when $p \ll 1$. The probability of making two or more bit errors is very low and we can approximate the performance by analyzing when there is only a one bit error. Since flipping one bit in the feedback sequence will move the phase from the sector representing the actual channel phase to some other sector, we need to ensure that the region-to-bit mapping is least sensitive to one bit errors. In Gray coding for M-PSK, the goal is to have a single bit error for the most likely symbol errors. Conversely, in our set-up, we would like every one bit error to cause as little phase-error as possible. Interestingly, Gray coding is a good choice for this problem as well. The Gray codes are generated as detailed in [5]. In addition, the structure and symmetry provided by the Gray code greatly simplifies analysis.

We enumerate the sectors staring from 0 to $2^q - 1$, and represent them using the respective Gray codes. We can have q possible sequences as a result of a one-bit error. This will cause the feedback to be misinterpreted to be one of q incorrect phases. With Gray mapping, two of the wrong phases lie in adjoining sectors, and cause minimum error.

In order to evaluate the performance of the system, calculate (12) in the presence of feedback errors. The only factor in

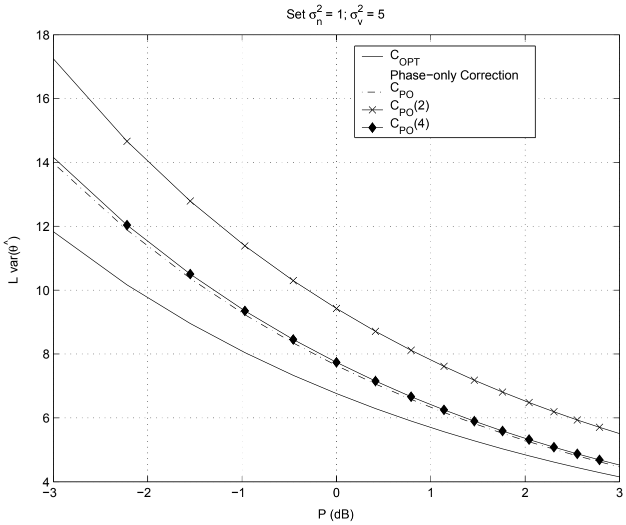

Fig. 2. Variance for different q.

the equation affected by the errors in feedback is the expected value in the denominator. We recompute it as follows:

$$
\begin{aligned}
E\left[e^{-j\phi}\right] =& Pr(\text{no error}) E\left[e^{-j\phi}|\text{no error}\right] \\
&+ Pr(\text{one bit error}) E\left[e^{-j\phi}|\text{one bit error}\right] \\
&+ o(p) \\
\approx& (1-p)^{q-1} \frac{2^q}{\pi}\left[\sin\left(\frac{\pi}{2^q}\right)\right] \\
&\times \left[(1-p) + p\left\{e^{-j\frac{2\pi}{2^q}} + \sum_{k=1}^{q-1} e^{j\frac{2(2^k-1)\pi}{2^q}}\right\}\right],
\end{aligned}
\tag{16}
$$

where $o(p)$ represents terms that go to zero faster than p as $p \to 0$.

Therefore, for large values of L, from (5) and (12), the performance due to error in feedback can be defined as

$$
\begin{aligned}
C_{PO}(q,p) =& (1-p)^{-2(q-1)} C_{PO}(q) \\
&\times \left|(1-p) + p\left\{e^{-j\frac{2\pi}{2^q}} + \sum_{k=1}^{q-1} e^{j\frac{2(2^k-1)\pi}{2^q}}\right\}\right|^{-2}.
\end{aligned}
\tag{17}
$$

It can be verified that $C_{PO}(q,0) = C_{PO}(q)$. Table 2 shows the effect of errors on the feedback channel. It can be seen that even with using only a few bits ($q = 5$) and a large probability of error ($p = 0.1$), the deterioration from C_{PO} is only a factor of about 1.5.

q	$\frac{C_{PO}(q,0)}{C_{PO}}$	$\frac{C_{PO}(q,0.01)}{C_{PO}}$	$\frac{C_{PO}(q,0.1)}{C_{PO}}$
1	2.4674	2.5691	3.8553
2	1.2337	1.2843	1.8804
3	1.05300	1.1026	1.6943
4	1.01295	1.0691	1.6640
5	1.003219	1.0653	1.5204

Table 2. Degree of deterioration due to feedback errors.

4. CONCLUSIONS

For the wireless sensor network considered, the best performance is obtained when the sensors have complete channel information and we optimize their gains to minimize the variance of the estimate, subject to a power constraint. Instead, with channel phase information only, all the sensors exhibit a constant magnitude gain and adjust only their respective phases. This is better suited for practical reasons, and we have shown that the loss in performance is low, making it preferable for implementation in sensor networks.

Since in most cases, there is a large bandwidth constraint, the feedback signals are quantized. This quantization causes a further performance loss. When using more than 4 bits, this loss in performance is contained to within 1%. When the feedback channel is noisy, the case of one bit error is considered. In that case, when 5 bits of quantization are used and with a probability of error of 0.01, there is only a 4% loss in performance compared to the no-quantization case.

The performance with quantization causes a very small loss in performance and is robust to error. Therefore, the proposed scheme of quantization and phase to bit mapping is computationally simple and easy to implement.

5. REFERENCES

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