MULTIPLE DESCRIPTION CODING OVER CORRELATED MULTIPATH ERASURE CHANNELS

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ABSTRACT

The problem of optimal rate allocation to streams constituting multiple description coding (MDC) has largely been addressed under the assumption that the multiple paths available for transmission are uncorrelated. In this paper, we model the transmission paths as correlated erasure channels and then examine the problem of allocating a given coding rate to the multiple descriptions in order to minimize the average distortion of the reconstructed message. We also investigate the relationship between the optimal average distortion and correlation of channels and prove that the bound on the average distortion will decrease as the correlation increases. Furthermore, we derive a closed-form solution for the contour which determines the region where multiple description coding (MDC) or single description coding (SDC) yields the minimal bound on the average distortion. Relying on this contour, we present a heuristic of the optimal rate allocation problem to determine the number of descriptions and relative rates.

Index Terms— MDC, Optimal rate allocation, Correlated erasure channels, Multipath, KKT

1. INTRODUCTION

Multiple description coding (MDC) offers a mechanism for providing error resilience in information transmission in scenarios where multiple paths are available but having a severe delay constraint and a high probability of packet loss. MDC generates multiple coded streams to represent the source information, and any subset of these streams can be used to reconstruct the source message with a corresponding level of distortion. Specifically, when all the descriptions are available at the receiver, a high quality reconstruction of the source is possible. However, in the absence of some of the descriptions at the receiver, the quality of reconstruction should still be acceptable.

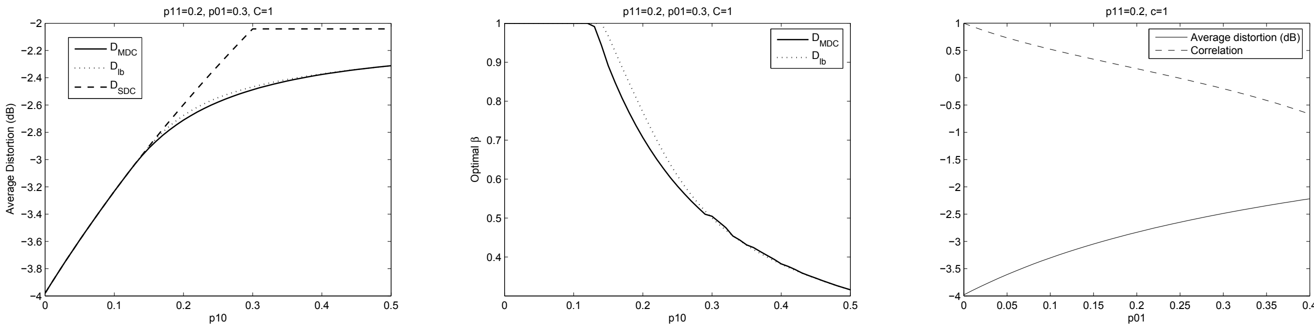
MDC relies on the descriptions available at the receiver to recover signals with reduced but acceptable quality even when some of the descriptions have been corrupted. This approach is fundamentally different from the use of Layered Coding (LC), where a specific enhancement layer can be used to improve the quality of the decoded signal provided that the base layer and all lower-level enhancement layers have been recovered by the receiver. It has been shown that MDC is more effective than LC in high-error probability channels in [1]. Furthermore, when used in conjunction with network diversity, MDC increases the tolerance to packet loss and delay constraints

Batllo et al. [2] studied the optimal bit allocation problem in order to minimize the central distortion given constraints on the side distortion and the total coding rate. Coward et at. [3] considered a symmetric bit allocation and studied the effect of channel codes parameters on the the erasure probabilities and on the average distortion. In [4], we demonstrated that MDC outperforms SDC only when the channel code correction capabilities are poor [3] and adapt the descriptions' rate according to the channel erasure probability. However, all the results are based on the multiple independent channels. In practice, the channels could be correlated. Thus, in this paper, we extend the work that we presented in [4] to correlated channels and investigate the relationship between channel correlation to the optimal average distortion at the decoder. We prove that the bound on the average distortion will decrease as the correlation of the channels increases. We also derive a closed-form expression for the contour which determines the operating region where MDC or SDC minimizes a bound on the average distortion. Furthermore, we use the contour derived to propose a heuristic which characterizes the optimal solution to the rate allocation problem.

The remainder of the paper is organized as follows: In Section 2, we describe the system model. The main results of the optimal rate allocation problem in the correlated channels are presented in Section 3. Finally, we conclude with a brief summary and discussion of our results in Section 4.

2. SYSTEM MODEL

The simplest model for the MDC problem is the case of two channels and three receivers. The MDC encoder generates two descriptions. Each individual description provides an approximation to the original message, and multiple descriptions can refine each other, to produce a better approximation than that attainable by any single one alone. If both descriptions are received, then the decoder can reconstruct the source at some small distortion value D_0 (the central distortion), but if either one is lost, the decoder can still reconstruct



(a) (b) (c)

Fig. 1. (a) Average distortion for D_{MDC},D_{lb},D_{SDC}, (b) Optimal β for D_{MDC},D_{lb}, (c) Correlation and optimal average distortion *v.s.* p_{01}

the source at some higher distortion D_1 or D_2 (the side distortions). Practical MDC designs appear in [5].

The characterization of the rate-distortion region for generic source and generic distortion measure is an open problem. The achievable rate-distortion region is completely known for a memoryless unit-variance Gaussian source with mean-squared error distortion [6]. In this case, the set of achievable rates is

$$D_i \geq 2^{-2R_i}, i = 1, 2 \tag{1}$$

$$D_0 \geq 2^{-2(R_1+R_2)}\gamma(D_1, D_2, R_1, R_2) \tag{2}$$

where

$$\frac{1}{\gamma} = 1 - \left(\sqrt{(1-D_1)(1-D_2)} - \sqrt{D_1 D_2 - 2^{-2(R_1+R_2)}}\right)^2$$

for $D_1 + D_2 < 1 + 2^{-2(R_1+R_2)}$ and $\gamma = 1$ otherwise.

We assume to transmit the MDC coded i.i.d. unit-variance Gaussian source over two *correlated erasure channels*. Let E_i, $i \in \{1, 2\}$, be the random variable that indicates whether the packet on channel i has been erased ($E_i = 1$) or not ($E_i = 0$). Let $p_{ij} = \Pr[E_1 = i, E_2 = j]$, $i \in \{0, 1\}$ and $j \in \{0, 1\}$, The correlation among the erasures on the two channels is

$$\rho = \frac{p_{11} - p_{e1}p_{e2}}{\sqrt{p_{e1}(1-p_{e1})\,p_{e2}(1-p_{e2})}} \tag{3}$$

where $p_{e1} = p_{11} + p_{10} = E[E_1]$ and $p_{e2} = p_{11} + p_{01} = E[E_2]$ are the erasure rates on channel 1 and 2, respectively.

If a packet in one description is erased, the appropriate side decoder is used. If both descriptions are lost, the mean value of the source is output, which results in a distortion equal to the source variance. Thus, the average distortion [3] is

$$D_{\text{ave}} = p_{11} + p_{01}D_1 + p_{10}D_2 + p_{00}D_0 \tag{4}$$

In order to allow sensible comparisons between MDC and single description code (SDC), we further require that the total source coding rate satisfies $R_1 + R_2 = C$, where C is a positive constant representing the total number of bits per source sample. The total rate constraint is introduced to ensure that we do not give any advantage to MDC over SDC. Furthermore, this restriction has practical significance when the costs of network services depend on transmission rates as well as in the design of multiplexing systems for transmission of multiple streams.

Our goal is to find the optimal rate allocation policy $\beta \in [0, 1]$ such that $R_1 = \beta C$ and $R_2 = (1-\beta)C$ minimize the average distortion in (4) within the rate-distortion region specified by (1)-(2). We denote with D_{MDC} the optimal value of D_{ave}. With the introduction of the parameter β, the sum of the side distortions always satisfy

$$e^{-\beta c} + e^{-(1-\beta)c} \leq D_1 + D_2 < 1 + e^{-c}$$

for all $0 < \beta < 1$, where $c = 2\ln(2)C$ is the total rate expressed in nats per channel use. This means that the region $D_1 + D_2 \geq 1 + e^{-c}$, for which $\gamma = 1$, corresponds to SDC, i.e., β is either 0 or 1.

3. MAIN RESULTS

3.1. Bounds on D_{MDC}

We can bound D_{MDC} as $D_{\text{MDC}} \leq D_{\text{lb}} \leq D_{\text{SDC}}$, where D_{lb} is the bound obtained by equating the side distortions to their minimum possible value in (1), that is,

$$D_{\text{lb}} = p_{11} + \min_{\beta\in[0,1]}\{p_{01}e^{-\beta c} + p_{10}e^{-(1-\beta)c} + p_{00}\frac{e^{-c}}{e^{-\beta c} + e^{-(1-\beta)c} - e^{-c}}\} = p_{11} + \min_{\beta\in[0,1]} F(\beta)$$

Indeed, in general, it is not possible to achieve equality simultaneously in the three equations in (1)-(2) since two individually good descriptions tend to be similar to each other (thus, the second description will contribute very little to improve the quality of the first one when both descriptions are received), while two descriptions which are complementary cannot be individually good (thus, the quality when only one description is received tends to be poor) [5]. D_{SDC} which is the optimal distortion subject to the SDC constraint is

$$D_{\mathrm{SDC}} = \min\{p_{e1}, p_{e2}\} + (1 - \min\{p_{e1}, p_{e2}\})e^{-c}$$

that amounts to sending an SDC code over the channel with lowest erasure rate, regardless of the value of c.

Fig. 1(a) 1(b) plots simulation for $p_{11} = 0.2$, $p_{01} = 0.3$, and $C = 1$. We see that the optimal average distortion and the optimal β for D_{MDC} and D_{lb} are very close. Hence, we can simplify our problem by considering the relationship between optimal average distortion and correlation of channels in the case of D_{lb} instead of general D_{MDC}.

3.2. Relationship between optimal average distortion and correlation

Given C, we chose different sets of p_{00}, p_{01}, p_{10}, and p_{11} for a fixed correlation, and computed the optimal average distortion. We were not able to identify any specific (monotonic) behavior of D_{DMC} as a function of the correlation besides for the some special cases, such as $p_{01} = p_{10}$, that is, equal erasure rates on both channels.

Fig. 1(c) shows that the correlation coefficient and the average distortion for $p_{11} = 0.2$, and $C = 1$, vs. $p_{01} = p_{10}$. The correlation decrease while the optimal average distortion increase with $p_{01} = p_{10}$. Hence, the optimal average distortion decrease when the correlation increases. This conclusion does not hold in general. We can prove it analytically by using the bound D_{lb} (referred as "with three lower bounds" in the following.)

1) The correlation for fixed p_{11} and $p_{01} = p_{10}$ is given by $\rho = \frac{p_{11} - {p_e}^2}{p_e(1-p_e)}$, $\quad p_e = p_{11} + p_{01}$, which is a decreasing function of p_e. Indeed,

$$\frac{d\rho}{dp_e} = \frac{-(p_e - p_{11})^2 - p_{11}(1 - p_{11})}{{p_e}^2(1 - p_e)^2} \le 0$$

2) The optimal average distortion D_{lb} increases with p_{01} for fixed p_{11} and $p_{01} = p_{10}$. Indeed, let

$$y = D_1 + D_2 - e^{-c} = e^{-\beta c} + e^{-(1-\beta)c} - e^{-c}.$$

Then D_{lb} can be re-written as

$$D_{\mathrm{lb}} = \min_y \left\{ p_{11} + p_{10}(y + e^{-c}) + p_{00}\frac{e^{-c}}{y} \right\}$$

By differentiating with y we get that the optimal y must satisfies

$$y = \sqrt{\frac{p_{00}}{p_{10}}e^{-c}} \in [2e^{-c/2} - e^{-c}, 1]$$

where the range for y is determined by the constraints in defining the rate distortion region. Hence: if

$$\sqrt{\frac{p_{00}}{p_{10}}e^{-c}} \ge 1 \Leftrightarrow y^{(\mathrm{opt})} = 1 \Leftrightarrow \beta \in \{0, 1\};$$

if

$$\sqrt{\tfrac{p_{00}}{p_{10}}e^{-c}} \le 2e^{-c/2} - e^{-c} \Leftrightarrow y^{(\mathrm{opt})} = 2e^{-c/2} - e^{-c} \Leftrightarrow \beta = \tfrac{1}{2};$$

else

$$y^{(\mathrm{opt})} = \sqrt{\tfrac{p_{00}}{p_{10}}e^{-c}} \Leftrightarrow \beta = \tfrac{1}{2} - \tfrac{1}{c}\cosh^{-1}\left(\frac{\sqrt{\frac{p_{00}}{p_{10}}} - e^{-c/2}}{2}\right)$$

We can then show: when $\sqrt{\frac{p_{00}}{p_{10}}e^{-c}} \ge 1$, then

$$D_{\mathrm{lb}} = p_{11} + p_{10}(1 + e^{-c}) + (1 - p_{11} - 2p_{10})e^{-c}$$

which is clearly increasing in p_{10}; when $\sqrt{\frac{p_{00}}{p_{10}}} \le 2 - e^{-c/2}$ then

$$D_{\mathrm{lb}} = p_{11} + 2p_{10}e^{-c/2} + (1 - p_{11} - 2p_{10})\frac{e^{-c}}{2e^{-c/2} - e^{-c}}$$

whose derivative with p_{10} is

$$e^{-c/2} - \frac{e^{-c/2}}{2 - e^{-c/2}} > 0 \Leftrightarrow e^{-c/2} < 1$$

hence D_{lb} is increasing with p_{10}; else $y^{(\mathrm{opt})} = \sqrt{\frac{p_{00}}{p_{10}}e^{-c}}$ and

$$D_{\mathrm{lb}} = p_{11} + p_{10}\left(\sqrt{\frac{p_{00}}{p_{10}}e^{-c}} + e^{-c}\right) + (1 - p_{11} - 2p_{10})\frac{e^{-c}}{\sqrt{\frac{p_{00}}{p_{10}}e^{-c}}}$$

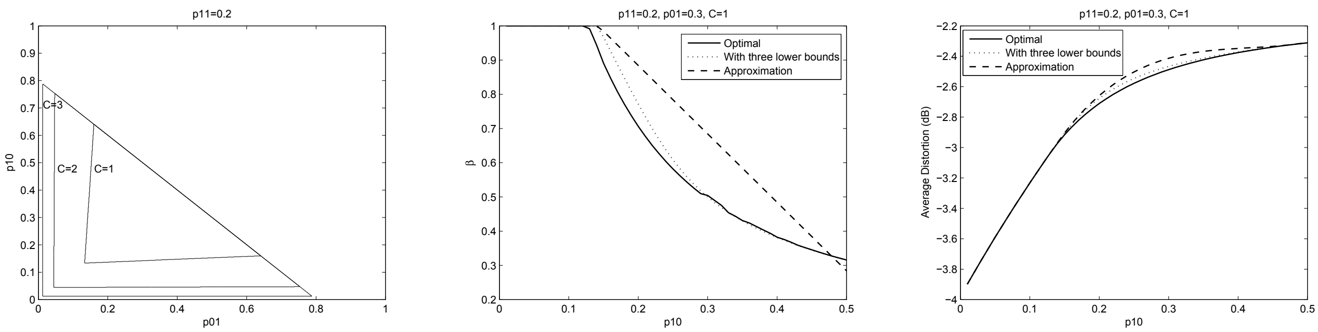
whose derivative with p_{10} is

$$\frac{dD_{\mathrm{ave}}}{dq} = e^{-c} + e^{-c/2}\frac{1 - p_{11} - 4p_{10}}{\sqrt{p_{00}p_{10}}} > 0$$

$$\Leftrightarrow e^{-c/2}\sqrt{\frac{p_{00}}{p_{10}}} \ge \left(2 - \left(\sqrt{\frac{p_{00}}{p_{10}}}\right)^2\right) \Leftrightarrow 1 > e^{-c/2}$$

hence also in this last case D_{lb} is increasing with p_{10}.

We analytically show that D_{lb} decreases when p_{01} increases in the case of fixed p_{11}, $p_{01} = p_{10}$. In this situation, let's take a look at two special cases of correlation 1 and correlation -1. For correlation 1, we will have $p_{10} = p_{01} = 0$. Hence, $D_{\mathrm{lb}} = p_{11} + p_{00}D_0 = p_{11} + (1 - p_{11})D_0 = D_{\mathrm{SDC}}$. Then in this case, the packet will both arrive or drop. We can infer that in this case SDC is optimal, since both packets either arrive together or both drop. For correlation -1, we have $p_{10} = p_{01} = 1/2$ and $p_{00} = p_{11} = 0$. Hence, $D_{\mathrm{lb}} = \frac{1}{2}(D_1 + D_2) = e^{-c/2}$. Then in this case, each time only one of the two packets will arrive. We can infer that in this case, the two descriptions could be the same. Clearly, $e^{-c/2}$ is greater than e^{-c}, which is consistent with our conclusion that the optimal average distortion will decrease as correlation increases in this situation.



(a) (b) (c)

Fig. 2. (a) Boundary of MDC or SDC in p_{01}, p_{10} plane, (b) approximate β, (c) approximate average distortion for $p_{01} = 0.3$, $p_{11} = 0.2, C = 1$

3.3. Contour of MDC or SDC

Our analysis does not lead to the closed-form solution of optimal β for the rate allocation problem even in the case with three lower bounds. However, as in [4], using the KKT optimization conditions, we can find the closed-form conditions for MDC (i.e., β strictly in (0,1)) in the case of three lower bounds, which is the region as follows:

$$\frac{dF(\beta)}{d\beta}|_{\beta=0} = -p_{01} + p_{10}e^{-c} + p_{00}e^{-c}(1 - e^{-c}) < 0$$
$$\frac{dF(\beta)}{d\beta}|_{\beta=1} = -p_{01}e^{-c} + p_{10} - p_{00}e^{-c}(1 - e^{-c}) > 0$$

The above two conditions with another additional condition ($p_{01} + p_{10} + p_{00} + p_{11} = 1$) define the contour of MDC region. When $p_{11} = 0.2$, the simulation results are shown in Fig. 2(a) for different C. We can see that MDC regions enlarge when C increases. We can utilize these contours to approximate the optimal β with a linear function. For example, with regard to the contour for $C = 1, p_{11} = 0.2$, with $p_{01} = 0.3$, the optimal β as a function of p_{10} is approximated as

$$\beta = \begin{cases} 0 & p_{10} \in [0, a),\ a = \frac{(1-p_{11})e^c(1-e^c)+p_{01}e^{2c}}{1+e^c(1-e^c)} \\ \frac{p_{10}-a}{b} & p_{10} \in [a, b],\ b = 1 - p_{01} - p_{11} \end{cases}$$

Fig. 2(b) and Fig. 2(c) show how the proposed approximation compare with the optimal solution and "three lower bounds" approximation. Notice that the optimal β inside the contour is in general non-linear, which causes the approximation of optimal β to deviate from the other two cases. Nonetheless, the average distortion curves are almost superimposed. Hence, we can use this approximation to directly determine the optimal β in practice.

4. CONCLUSION AND FUTURE WORK

In this paper, we studied the optimal rate allocation problem for MDC in order to attain the minimum average distortion over multiple correlated erasure channels under the constraint that the total coding rate is fixed. We analytically show that the optimal average distortion will decrease as the correlation increases in the case of "three lower bounds". Moreover, we attain the closed-form conditions for the contour of using MDC or SDC. Also, utilizing the contour, we proposed a heuristic to directly determine the optimal rate allocation for MDC over multiple erasure channels. In the future, we plan to investigate the optimal rate allocation for MDC when we consider the relation between the data rate and the probability of erasure.

5. REFERENCES

[1] Yen-Chi Lee, J. Kim, Y. Altunbasak, and R.M. Mersereau, "Performance comparisons of layered and multiple description coded video streaming over error-prone networks," in *ICC*. IEEE, 2003.

[2] J.C. Batllo and V.A. Vaishampayan, "Asymptotic performance of multiple description transform codes," *IEEE Trans. Inform. Theory*, vol. 43, pp. 703–707, Mar. 1997.

[3] H. Coward, R. Knopp, and S.D. Servetto, "On the performance of multiple description codes over bit error channels," in *Int. Symp. Information Theory*, 2001, p. 240.

[4] S. Zhao, D. Tuninetti, R. Ansari, and D. Schonfeld, "Multiple description coding over erasure channels," in *IEEE Int. Conf. on Commun.*, 2008, submitted.

[5] V.K. Goyal and J. Kovačević, "Generalized multiple description coding with correlating transforms," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2199–2224, Sep. 2001.

[6] L. Ozarow, "On a source-coding problem with two channels and three receivers," in *Bell Syst. Tech. J.*, 1980, pp. 1909–1921.