

OPTIMAL MEDIUM ACCESS CONTROL IN COGNITIVE RADIOS: A SEQUENTIAL DESIGN APPROACH

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ABSTRACT

The design of medium access control protocols for a cognitive user wishing to opportunistically exploit frequency bands within parts of the radio spectrum having multiple bands is considered. In the scenario under consideration, the availability probability of each channel is unknown *a priori* to the cognitive user. Hence efficient medium access strategies must strike a balance between exploring the availability of channels and exploiting the opportunities identified thus far. Using a sequential design approach, an optimal medium access strategy is derived. To avoid the prohibitive computational complexity of this optimal strategy, a low complexity asymptotically optimal strategy is also developed. The proposed strategy does not require any prior statistical knowledge about the traffic pattern on the different channels.

Index Terms— Cognitive radio, bandit problem, medium access control.

1. INTRODUCTION

As a promising technique to increase spectral efficiency of overcrowded parts of the radio spectrum, the opportunistic spectrum access problem has been the focus of significant research activities [1]. The underlying idea is to allow unlicensed users (i.e., cognitive users) to access the available spectrum when the licensed users (i.e., primary users) are not active. The presence of high priority primary users and the requirement that the cognitive users should not interfere with them introduce new challenges for protocol design. The overarching goal of the current work is to develop a unified framework for the design of efficient, and low complexity, cognitive medium access protocols.

The spectral opportunities available to cognitive users are by their nature time-varying. To avoid interfering with the primary network, cognitive users must first probe to determine whether there are primary activities before transmission. Under the assumption that each cognitive user cannot access all of the available channels simultaneously, the main task of the medium access protocol is to distributively choose which channels each cognitive user should attempt to use in different time slots, in order to fully (or maximally) utilize the spectral

opportunities. This decision process can be enhanced by taking into account any available statistical information about the primary traffic. For example, with a single cognitive user capable of accessing (sensing) only one channel at a time, the problem becomes trivial if the probability that each channel is free is known *a priori*. In this case, the optimal rule is for the cognitive user to access the channel with the highest probability of being free in all time slots. However, such time-varying traffic information is typically not available to the cognitive users *a priori*. The need to learn this information on-line creates a fundamental tradeoff between exploitation and exploration. Exploitation refers to the short-term gain resulting from accessing the channel with the estimated highest probability of being free (based on the results of previous sensing decisions) whereas exploration is the process by which a cognitive user learns the statistical behavior of the primary traffic (by choosing possibly different channels to probe across time slots). In the presence of multiple cognitive users, the medium access algorithm must also account for the competition between different users over the same channel.

In this paper, we develop a unified framework for the design and analysis of cognitive medium access protocols in the presence of a single cognitive user who can access a single channel in each time slot. As argued in the sequel, this framework allows for the construction of strategies that strike an optimal balance between exploration and exploitation. We derive an optimal sensing rule that maximizes the expected throughput obtained by the cognitive user. Compared with a genie-aided scheme, in which the cognitive user knows *a priori* the primary network traffic information, there is a throughput loss suffered by any medium access strategy. We obtain a lower bound on this loss and further construct a linear complexity single index protocol that achieves this lower bound asymptotically (when the primary traffic behavior changes slowly). Similar approaches have been considered in [3] and [4], but with different emphases.

We have also extended our study to networks with multiple cognitive users and networks with more capable cognitive users, and have developed optimal strategies for these scenarios. However, due to space limitations, we do not discuss these results here. We also omit the proofs of results presented in this paper. Interested readers can refer to [5] for details.

The rest of this paper is organized as follows. Our network model is detailed in Section 2. Section 3 develops and analyzes an optimal strategy for the single cognitive user sce-

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nario. Finally, Section 4 summarizes our conclusions.

2. NETWORK MODEL

Figure 1 shows the channel model of interest. We consider a primary network consisting of N non-overlapping channels, $\mathcal{N} = \{1, \dots, N\}$, each with bandwidth B . The users in the primary network are operated in a synchronous time-slotted fashion. We assume that at each time slot, channel i is free with probability θ_i . Let $Z_i(j)$ be a random variable that equals 1 if channel i is free at time slot j and equals 0 otherwise. Hence, given θ_i , $Z_i(j)$ is a Bernoulli random variable with distribution $h_{\theta_i}(z_i(j)) = \theta_i\delta(1) + (1 - \theta_i)\delta(0)$, where $\delta(\cdot)$ is a delta function. Furthermore, for a given $\theta = (\theta_1, \dots, \theta_N)$, the $Z_i(j)$ are independent for each i and j . We consider a block varying model in which the value of θ is fixed for a block of T time slots and then randomly changes at the beginning of the next block according to a joint probability density function (pdf) $f(\theta)$.

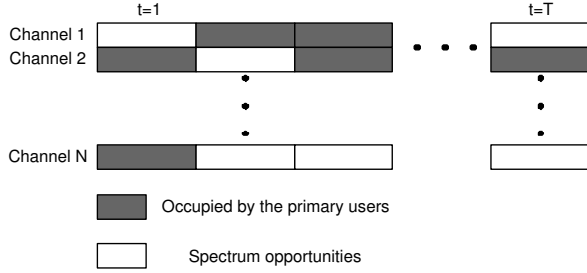


Fig. 1. Channel model.

In our model, the cognitive users attempt to exploit the availability of free channels in the primary network by sensing the activity at the beginning of each time slot. Our work seeks to characterize efficient strategies for choosing which channels to sense (access). The challenge here stems from the fact that the cognitive users are assumed to be unaware of θ *a priori*. We consider two cases in which a cognitive user either has or does not have prior information about the pdf of θ , i.e., $f(\theta)$. In the scenario presented in this paper, at time slot j , a single cognitive user selects one channel $S(j) \in \mathcal{N}$ to access. If the sensing result shows that channel $S(j)$ is free, i.e., $Z_{S(j)}(j) = 1$, the cognitive user can send B bits over this channel; otherwise, the cognitive user will wait until the next time slot and pick a possibly different channel to access. Therefore, the total number of bits that the cognitive user is able to send over one block (of T time slots) is

$$W = \sum_{j=1}^T BZ_{S(j)}(j).$$

It is clear that W is a random variable that depends on the traffic in the primary network and, more importantly for us, the medium access protocols employed by the cognitive user. Therefore, the overarching goal of this paper is to construct low complexity medium access protocols that maximize $\mathbb{E}\{W\}$.

Intuitively, the cognitive user would like to select the channel with the highest probability of being free in order to obtain more transmission opportunities. If θ is known then this problem is trivial: the cognitive user should choose the channel $i^* = \arg \max_{i \in \mathcal{N}} \theta_i$ to sense. The uncertainty in θ imposes a fundamental tradeoff between exploration, in order to learn θ , and exploitation, by accessing the channel with the highest estimated free probability based on current available information, as detailed in the following section.

3. OPTIMAL MEDIUM ACCESS PROTOCOLS

We start by developing the optimal solution under the idealized assumption that $f(\theta)$ is known *a priori* by the cognitive user. As we will see, this optimal medium access algorithm suffers from a prohibitive computational complexity that grows exponentially with the block length T . This motivates the design of low complexity asymptotically optimal approaches, which we also consider.

Our cognitive medium access problem belongs to the class of bandit problems. In this setting, the decision maker must sequentially choose one process to observe from $N \geq 2$ stochastic processes. These processes usually have parameters that are unknown to the decision maker and, associated with each observation is a utility function. The objective of the decision maker is to maximize the sum or discounted sum of the utilities via a strategy that specifies which process to observe for every possible history of selections and observations. A comprehensive treatment covering different variants of bandit problems can be found in [2].

We are now ready to rigorously formulate our problem. The cognitive user employs a medium access strategy Γ , which will select channel $S(j) \in \mathcal{N}$ to sense at time slot j for any possible causal information pattern obtained through the previous $j - 1$ observations: $\Psi(j) = \{s(1), z_{s(1)}(1), \dots, s(j-1), z_{s(j-1)}(j-1)\}$, $j \geq 2$, i.e. $s(j) = \Gamma(f, \Psi(j))$. Notice that $z_{s(j)}(j)$ is the sensing outcome of the j^{th} time slot, in which $s(j)$ is the channel being accessed. If $j = 1$, there is no accumulated information, and thus $\Psi(1) = \phi$ and $s(1) = \Gamma(f)$. The utility that the cognitive user obtains by making decision $S(j)$ at time slot j is the number of bits it can transmit at time slot j , which is $BZ_{S(j)}(j)$. We denote the expected value of the payoff obtained by a cognitive user who uses strategy Γ as

$$W_\Gamma = \mathbb{E}_f \left\{ \sum_{j=1}^T BZ_{S(j)}(j) \right\}. \quad (1)$$

We further denote $V^*(f, T) = \sup_{\Gamma} W_\Gamma$, which is the largest throughput that the cognitive user could obtain when the spectral opportunities are governed by $f(\theta)$ and the exact value of each realization of θ is not known *a priori* by the user.

Each medium access decision made by the cognitive user has two effects. The first one is the short-term gain, i.e., an immediate transmission opportunity if the chosen channel is found free. The second one is the long-term gain, i.e., the updated statistical information about $f(\theta)$. This information

will help the cognitive user in making better decisions in future stages. There is an interesting tradeoff between the short and long-term gains. If we only want to maximize the short-term gain, we can choose the channel with the highest availability probability to sense, based on the current information. This myopic strategy maximally exploits the existing information. On the other hand, by choosing other channels to sense, we gain statistical information about $f(\theta)$ which can effectively guide future decisions. This process is typically referred to as exploration, as noted previously.

More specifically, let $f^j(\theta)$ be the updated pdf after making $j - 1$ observations. We begin with $f^1(\theta) = f(\theta)$. After observing $z_{s(j)}(j)$, we update the pdf using the following Bayesian formula.

1. If $z_{s(j)}(j) = 1$, $f^{j+1}(\theta) = \frac{\theta_{s(j)} f^j(\theta)}{\int \theta_{s(j)} f^j(\theta) d\theta}$,
2. If $z_{s(j)}(j) = 0$, $f^{j+1}(\theta) = \frac{(1 - \theta_{s(j)}) f^j(\theta)}{\int (1 - \theta_{s(j)}) f^j(\theta) d\theta}$.

The following result characterizes the optimal strategy that maximizes the average throughput the cognitive user obtains from the network.

Lemma 1 *For any prior pdf f , there exists an optimal strategy Γ^* to the channel selection problem (1), and $V^*(f, T)$ is achievable. Moreover, V^* satisfies the following condition:*

$$V^*(f, T) = \max_{s(1) \in \mathcal{N}} \mathbb{E}_f \{ BZ_{s(1)} + V^*(f_{Z_{s(1)}}, T - 1) \}, \quad (2)$$

where $f_{Z_{s(1)}}$ is the conditional distribution updated using the Bayesian rule described above, as if the cognitive user chooses $s(1)$ and observes $Z_{s(1)}$. Also, $V^*(f_{Z_{s(1)}}, T - 1)$ is the value of a bandit problem with prior information $f_{Z_{s(1)}}$ and $T - 1$ sequential observations.

In principle, Lemma 1 provides the solution to problem (1). Effectively, it decouples the calculation at each stage, and hence, allows the use of dynamic programming to solve the problem. The idea is to solve the channel selection problem with a smaller dimension first and then use backward deduction to obtain the optimal solution for a problem with a larger dimension. Starting with $T = 1$, the second term inside the expectation in (2) is 0, since $T - 1 = 0$. Hence, the optimal solution is to choose the channel i having the largest $\mathbb{E}_f \{ BZ_i \}$, which can be calculated as $\mathbb{E}_f \{ BZ_i \} = B \int \theta_i f(\theta) d\theta$. And $V^*(f, 1) = \max_{i \in \mathcal{N}} \mathbb{E}_f \{ BZ_i \}$. With the solution for $T = 1$ at hand, we can now solve the $T = 2$ case using (2). At first, for every possible choice of $s(1)$ and possible observation $z_{s(1)}$, we calculate the updated distribution $f_{z_{s(1)}}$ using the Bayesian formula. Next, we calculate $V^*(f_{z_{s(1)}}, 1)$ (which is equivalent to the $T = 1$ problem described above). Finally, applying (2), we have the following equation for the channel selection problems with $T = 2$:

$$V^*(f, 2) = \max_{i \in \mathcal{N}} \int [B\theta_i + \theta_i V^*(f_{z_i=1}, 1) + (1 - \theta_i) V^*(f_{z_i=0}, 1)] f(\theta) d\theta.$$

Hence, in the first step, the cognitive user should choose $i^*(1) = \arg \max_{i \in \mathcal{N}} V^*(f, 2)$ to sense. After observing $z_{i^*(1)}$, the cognitive user has $\Psi(1) = \{z_{i^*(1)}\}$, and it should choose $i^*(2) =$

$\arg \max_{i \in \mathcal{N}} V^*(f_{z_{i^*(1)}}, 1)$. Similarly, after solving the $T = 2$ problem, one can proceed to solve the $T = 3$ case. Using this procedure recursively, we can solve the problem with $T - 1$ observations. Finally, our original problem with T observations is solved as follows.

$$V^*(f, T) = \max_{i \in \mathcal{N}} \int [B\theta_i + \theta_i V^*(f_{z_i=1}, T - 1) + (1 - \theta_i) V^*(f_{z_i=0}, T - 1)] f(\theta) d\theta.$$

The optimal solution developed above suffers from a prohibitive computational complexity. In particular, the dimensionality of our search dimension grows exponentially with the block length T . Moreover, one can envision many practical scenarios in which it would be difficult for the cognitive user to obtain the prior information $f(\theta)$. This motivates our pursuit of low complexity non-parametric protocols which maintain certain optimality properties and do not depend on $f(\theta)$ explicitly. Hence, in the following, we aim to develop strategies that depend only on the information obtained through observations Ψ .

For a given strategy Γ , the expected number of bits the cognitive user is able to transmit through a block with given parameters θ is

$$\mathbb{E} \left\{ \sum_{j=1}^T BZ_{S(j)}(j) \right\} = \sum_{j=1}^T B \sum_{i=1}^N \theta_i \Pr \{ \Gamma(\Psi(j)) = i \}.$$

Recall that $\Gamma(\Psi(j)) = i$ means that, following strategy Γ , the cognitive user should choose channel i in time slot j , based on the available information $\Psi(j)$. Here $\Pr \{ \Gamma(\Psi(j)) = i \}$ is the probability that the cognitive user will choose channel i at time slot j , following the strategy Γ .

Compared with the idealistic case where the exact value of θ is known, in which the optimal strategy for the cognitive user is to always choose the channel with the largest availability probability, the loss incurred by Γ is given by

$$L(\theta; \Gamma) = \sum_{j=1}^T B\theta_{i^*} - \sum_{j=1}^T B \sum_{i=1}^N \theta_i \Pr \{ \Gamma(\Psi(j)) = i \},$$

where $\theta_{i^*} = \max\{\theta_1, \dots, \theta_N\}$. We say that a strategy Γ is consistent if, for any $\theta \in [0, 1]^N$, there exists $\beta < 1$ such that $L(\theta; \Gamma)$ scales as $O(T^\beta)$. In the sequel, we use the following notations 1) $g_1(N) = \omega(g_2(N))$ means that $\forall c > 0, \exists N_0$, such that $\forall N > N_0, g_2(N) < c g_1(N)$; 2) $g_1(n) = O(g_2(N))$ means that $\exists c_1, c_2 > 0$ and N_0 , such that $\forall N > N_0, c_1 g_2(N) \leq g_1(N) \leq c_2 g_2(N)$. For example, consider a loyal scheme in which the cognitive user selects channel i at the beginning of a block and sticks to it. If θ_i is the largest one among θ , $L(\theta; \Gamma) = 0$. On the other hand, if θ_i is not the largest one, $L(\theta; \Gamma) \sim O(T)$. Hence, this loyal scheme is not consistent. The following lemma characterizes the fundamental limits of any consistent scheme.

Lemma 2 *For any θ and any consistent strategy Γ , we have*

$$\liminf_{T \rightarrow \infty} \frac{L(\theta; \Gamma)}{\ln T} \geq B \sum_{i \in \mathcal{N} \setminus \{i^*\}} \frac{\theta_{i^*} - \theta_i}{D(\theta_i || \theta_{i^*})}, \quad (3)$$

where $D(\theta_i||\theta_l)$ denotes the Kullback-Leibler divergence between the two Bernoulli random variables with parameters θ_i and θ_l respectively: $D(\theta_i||\theta_l) = \theta_i \ln \left(\frac{\theta_i}{\theta_l} \right) + (1-\theta_i) \ln \left(\frac{1-\theta_i}{1-\theta_l} \right)$.

Lemma 2 shows that the loss of any consistent strategy scales at least as $\omega(\ln T)$. An intuitive explanation of this loss is that we need to spend at least $O(\ln T)$ time slots on sampling each of the channels with smaller θ_i , in order to get a reasonably accurate estimate of θ , and hence use it to determine the channel having the largest θ_i to sense. We say that a strategy Γ is order optimal if $L(\theta; \Gamma) \sim O(\ln T)$.

Before proceeding to the proposed low complexity order-optimal strategy, we first analyze the loss order of some heuristic strategies which may appear to be reasonable.

The first simple rule is the random strategy Γ_r where, at each time slot, the cognitive user randomly chooses a channel from the available N channels. The fraction of time the cognitive user spends on each channel is therefore $1/N$, leading

to the loss $L(\theta; \Gamma_r) = \frac{B \sum_{i=1}^N (\theta_i^* - \theta_i)}{N} T \sim O(T)$.

The second one is the myopic rule Γ_g in which the cognitive user keeps updating $f^j(\theta)$, and chooses the channel with the largest value of $\hat{\theta}_i = \int \theta_i f^j(\theta) d\theta$ at each stage. Since there are no convergence guarantees for the myopic rule, that is $\hat{\theta}$ may never converge to θ due to the lack of sufficiently many samples for each channel [6], the loss of this myopic strategy is $O(T)$.

The third protocol we consider is *staying with the winner and switching from the loser* rule Γ_{SW} where the cognitive user randomly chooses a channel in the first time slot. In the succeeding time-slots 1) if the accessed channel was found to be free, it will choose the same channel to sense; 2) otherwise, it will choose one of the remaining channels based on a certain switching rule.

Lemma 3 *No matter what the switching rule is, $L(\theta; \Gamma_{SW}) \sim O(T)$.*

There are several strategies that have loss of order $O(\ln T)$. We adopt the following linear complexity strategy from [7].

Rule 1 (Order optimal single index strategy)

The cognitive user maintains two vectors \mathbf{X} and \mathbf{Y} , where each X_i records the number of time slots in which the cognitive user has sensed channel i to be free, and each Y_i records the number of time slots in which the cognitive user has chosen channel i to sense. The strategy works as follows.

1. *Initialization: at the beginning of each block, each channel is sensed once.*
2. *After the initialization period, the cognitive user obtains an estimate $\hat{\theta}$ at the beginning of time slot j , given by $\hat{\theta}_i(j) = X_i(j)/Y_i(j)$, and assigns an index $\Lambda_i(j) = \hat{\theta}_i(j) + \sqrt{2 \ln j / Y_i(j)}$ to the i^{th} channel. The cognitive user chooses the channel with the largest value of $\Lambda_i(j)$ to sense at time slot j . After each sensing, the cognitive user updates \mathbf{X} and \mathbf{Y} .*

The intuition behind this strategy is that as long as Y_i grows as fast as $O(\ln T)$, Λ_i converges to the true value of θ_i in probability, and the cognitive user will choose the channel with the largest θ_i eventually. The loss of $O(\ln T)$ comes from the time spent in sampling the inferior channels in order to learn the value of θ . This price, however, is inevitable as established in the lower bound of Lemma 2.

Finally, we observe that the difference between the myopic rule and the order optimal single index rule is the additional term $\sqrt{2 \ln j / Y_i(j)}$ added to the current estimate $\hat{\theta}_i$. Roughly speaking, this additional term guarantees enough sampling time for each channel, since if we sample channel i too sparsely, $Y_i(j)$ will be small, which will increase the probability that Λ_i is the largest index. When $Y_i(j)$ scales as $\ln T$, $\hat{\theta}_i$ will be the dominant term in the index Λ_i , and hence the channel with the largest θ_i will be chosen much more frequently.

4. CONCLUSIONS

This work has developed a unified framework for the design and analysis of cognitive medium access based on the classical bandit problem. Our formulation highlights the tradeoff between exploration and exploitation in cognitive channel selection. A linear complexity cognitive medium access algorithm, which is asymptotically optimal as the number of time slots increases, has also been proposed.

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