CLASSIFICATION OF GMSK SIGNALS WITH DIFFERENT BANDWIDTHS

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ABSTRACT

This paper studies a Bayesian classifier which recognizes Gaussian minimum shift keying (GMSK) modulated signals with different bandwiths. We focus on identifying two different GMSK signals with $BT = 0.25$ and $BT = 0.5$ standardized by the consultative committee for space data system (CCSDS) for future space missions. The main idea of the proposed classifier is to compute the posterior probability of the observation sequence given each possible model by a modified Baum-Welch (BW) algorithm. The received GMSK signals are then classified according to the maximum a posteriori (MAP) rule.

***Index Terms*—** modulation classification, Baum-Welch (BW) algorithm, GMSK modulation, Bayes classifier.

1. INTRODUCTION

Digital modulation classification consists of recognizing the type of a modulated signal corrupted by noise and other impairments. It is required in many communication applications including cooperative and non-cooperative scenarios (see for instance [1, 2] and references therein). Most studies have concentrated on identifying various types of linear modulations by using likelihood or feature-based classifiers. Likelihood-based classifiers are based on hypothesis testing theory and required to determine the probability density function (pdf) of the received signal conditioned on each class. This determination is often a difficult problem which may require intensive computational cost methods. Conversely, appropriate features can be extracted from the received signal and the modulation classification problem is reformulated in this new feature space. The features which have shown interesting properties for classifying linear modulations include the instantaneous amplitude, phase and frequency, statistical moments, higher-order statistics or wavelet coefficients of the received signal (see [2] for details).

Surprisingly, the classification of non-linear modulations has received less attention in the literature though these modulations play a great deal in modern communications. Polydoros studied different methods for classifying non-linear modulations with different modulation indexes [3,4]. A classifier based on an approximate likelihood function for a multiple M-ary frequency-shift keying (MFSK) signal (transmitted through a Rayleigh fading channel) was also studied in [5]. However, classification problems involving GMSK modulations have not been considered in the literature (to the best of our knowledge), despite the popularity of GMSK signals. This paper studies a Bayesian classifier which recognizes GMSK signals with different bandwidths $BT = 0.25$ and $BT = 0.5$ as recommended by the CCSDS [6]. The proposed algorithm assumes that these two non-linear modulations have been pre-identified from other linear modulation candidates. This preprocessing step might be achieved by feature-based classifiers that discriminate constant and nonconstant envelope signals. For instance, the maximum of the squared Fourier transform of the normalized signal amplitudes has been used for this purpose in [7]. Note that the classifier performance will be studied especially at small SNRs as required by GMSK modulation applications.

The classification of linear modulation signals propagating via unknown intersymbol interference (ISI) channels has been recently studied in [8]. The first step of the proposed algorithm estimated the channel coefficients (which are related to the signal means) and noise variance by using the Baum-Welch (BW) algorithm. The received communication signal was then identified according to the MAP rule. In this work, we modify the algorithm proposed in [8] to handle non-linear modulations corrupted by additive white Gaussian noise (AWGN). As explained before, this paper is devoted to the classification of GMSK signals. The choice of GMSK modulations can be motivated by many interesting properties including spectrum efficiency, capacity of supporting several receivers, and high immunity against interference (see [9] and references therein). The CCSDS standard uses two GMSK signals with $BT = 0.25, L = 4$ and $BT = 0.5, L = 2$. As it is widely known, the GMSK modulation is a non-linear continuous phase modulation (CPM) with memory. After constructing the state trellis associated to a GMSK signal, the BW algorithm can be applied to estimate the posterior probability of the received modulated signal as done in [8] for linear modulations in presence of residual channel interferences.

This paper is organized as follows. Section 2 gives some useful information regarding GMSK signals. Section 3 presents the signal model used for modulation classification. The received signal is modeled as a probabilistic function of an hidden state represented by a first order hidden Markov model (HMM). Section 4 recalls the main steps of the BW algorithm which determines the posterior probability of the observation sequence given the model and estimates the unknown model parameters. Section 5 studies the performance of the MAP rule based on the posterior probabilities computed by the BW algorithm. Simulation results and conclusions are reported in Sections 6 and 7.

2. GMSK SIGNALS

GMSK signals are partial CPM signals (with modulation index $h = 0.5$ and Gaussian frequency shaping) defined as [10]:

$$x(t) = A\cos\left(2\pi f_c t + \Phi(t, \mathbf{a})\right), \qquad t \in \mathbb{R},$$

where f_c is the carrier frequency and $\Phi(t, \mathbf{a})$ is the so-called *excess phase*. The transmitted data sequence of M-ary symbols selected from the alphabet $\pm 1, \pm 3, \ldots, \pm(M-1)$ denoted as $\mathbf{a} = \{a_k\}$ is embedded in the *excess phase*

$$\Phi(t, \mathbf{a}) = 2\pi h \sum_{k=-\infty}^{\infty} a_k q(t - kT), \tag{1}$$

where $q(t) = \int_{-\infty}^{t} g(\tau)d\tau$ and T is the symbol duration. The frequency shape pulse $g(t)$ has a smooth phase shape over a finite time interval $0 \leq t \leq LT$ (where L is the pulse length) and is approximately zero outside this interval. For a GMSK signal, $g(t)$ is defined as

$$g(t) = \frac{1}{2T}\left[Q\left(2\pi B \frac{t - \frac{T}{2}}{\sqrt{\ln 2}}\right) - Q\left(2\pi B \frac{t + \frac{T}{2}}{\sqrt{\ln 2}}\right)\right],$$

where B is the 3dB bandwidth of the lowpass Gaussian filter (with $0 \leq BT \leq 1$) and $Q(t) = \int_{t}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau^2}{2}\right) d\tau$. The excess phase during interval $[kT, (k+1)T]$ can be written as

$$\Phi(t, \mathbf{a}) = \theta_k(t, \mathbf{a}) + \phi_k,$$

where $\theta_k(t, \mathbf{a})$ is the *instant phase*

$$\theta_k(t, \mathbf{a}) = 2\pi h \sum_{i=k-L+1}^{k} a_i q(t - iT),$$

and ϕ_k is the accumulated phase (memory) of all symbols up to time $k - L$ (sometimes called *cumulant phase*)

$$\phi_k = h\pi \sum_{i=-\infty}^{k-L} a_i \pmod{2\pi}.$$

The cumulant phase represents the constant part of the total excess phase in $[kT, (k+1)T]$, and is equal to the sum of the maximum phase changes contributed to each symbol, accumulated along the time axis up to the $(k-L)^{th}$ symbol interval. It can be recursively computed as

$$\phi_{k+1} = \phi_k + h\pi a_{k-L+1}.$$

The instant phase $\theta_k(t, \mathbf{a})$ is determined by the data symbol a_k and the previous $L - 1$ symbols. If h is rational, i.e. $h = 2q/p$, the number of distinct values of ϕ_k is p. The state of a CPM signal at $t = kT$ is classically defined as the vector

$$s_k = (\phi_k, a_{k-1}, a_{k-2}, \ldots, a_{k-L+1}).$$

Each state corresponds to a specific value of the excess phase.

3. SIGNAL MODEL

The baseband GMSK signal can be written as $u(t) = \exp[j\Phi(t, \mathbf{a})]$, where the phase $\Phi(t, \mathbf{a})$ has been defined in (1). The baseband signal is modulated by a local oscillator $\exp(j\omega_c t)$. The signal is corrupted by additive white Gaussian noise $w(t)$, with spectral density $\mathrm{N}_0/2$. At the receiver side, the received signal is multiplied by the synchronous carrier $\exp(-j\omega_c t)$, followed by low pass filters to generate the real and imaginary parts of the complex envelope of the received signal. After downconversion, we obtain the received baseband signal

$$y(t) = u(t) \otimes f(t) + z(t), \quad t \in \mathbb{R},$$

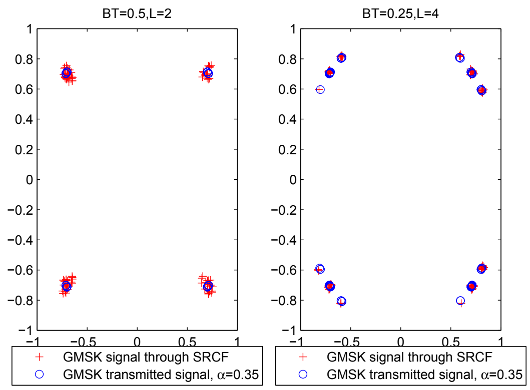

Fig. 1. GMSK constellations (one sample per symbol).

where $f(t)$ is the impulse response of the lowpass filter, $z(t) = w(t) \otimes f(t)$ is a normalized complex-valued additive Gaussian noise process with variance σ_z^2 and " $\otimes$ " denotes convolution. The baseband complex envelope of the received modulated signal sampled at one sample per symbol ($t = kT$) at the output of the lowpass filters can be written as:

$$y(k) = u(k) \otimes f(k) + z(k), \quad k = 1, ..., N_s,$$

where N_s is the number of symbols in the observation interval. Two GMSK signal "constellations" obtained at the output of a square root raised cosine filter (roll-off factor $\alpha = 0.35$ and cutoff frequency adapted to symbol duration) in the absence of noise are shown in Fig. 1. The two constellations are clearly similar even if they are obtained from two distinct GMSK modulations.

The received signal $y(k)$ can be modeled as a probabilistic function of an hidden state at time k which is represented by a first order HMM. This model will be used efficiently for classifying two nonlinear GMSK modulations with different bandwidths (denoted as λ_1, λ_2). The main HMM characteristics are summarized below:

1. The state of the HMM at time instant k is $\mathbf{s}_k$ which belongs to an alphabet denoted as $\{s(1), s(2), ..., s(N)\}$ of size $N = 4M^{L-1}$, where $s(j)$ is the jth possible value of $\mathbf{s}_k$. As an example, for binary symbols and GMSK modulation with $BT = 0.5, L = 2$, hence $N = 8$ different states. For binary symbols and GMSK modulation with $BT = 0.25, L = 4$, yielding $N = 32$ different states.

2. The state transition probability distribution is

$$d_{ij} = P[s_{k+1} = s(j)|s_k = s(i)],$$

which equals $1/M$ when all symbols are equally likely.

3. The initial state distribution vector $\pi = (\pi_1, ..., \pi_N)^T$ is defined by $\pi_i = P[s_1 = s(i)] = 1/N, \ i = 1, ..., N$.

4. The pdf of the observation $y(k)$ conditioned on state i, denoted as $p_i(y(k)) \triangleq p(y(k)|s(i))$ can be written

$$p_i(y(k)) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{|y(k) - m_i|^2}{2\sigma_z^2}\right),$$

for $i = 1, ..., N$, where m_i is the ith value of $e^{j\Phi(kT, \mathbf{a})}$. We denote as $\boldsymbol{m} = [m_1, ..., m_N]^T$ the vector containing all possible "constellations" points.

4. BW PARAMETER ESTIMATION

Given the above HMM, the BW algorithm can be used to determine the posterior probability of the observation sequence $\boldsymbol{y}$ given the model $\lambda \in \{\lambda_1, \lambda_2\}$ and estimate the unknown model parameters $\boldsymbol{m}$ and σ_z^2. The BW algorithm is based on a forward-backward procedure which estimates iteratively the unknown model parameters maximizing the posterior probability of the unknown parameters. After convergence, the BW algorithm provides MAP estimates of $\boldsymbol{m}$ and σ_z^2 such that:

$$(\widehat{\boldsymbol{m}}, \widehat{\sigma}_z^2) = \arg\max_{\boldsymbol{m}, \sigma_z^2} P(\boldsymbol{m}, \sigma_z^2 | \boldsymbol{y}, \lambda).$$

The algorithm needs a forward operation to compute $P(\boldsymbol{y}|\boldsymbol{m}, \sigma_z^2, \lambda)$ whereas a forward/backward procedure is necessary to estimate the unknown parameters $\boldsymbol{m}$ and σ_z^2. This section describes the principles of the standard BW algorithm detailed for instance in [11]. An LMS-type update BW algorithm is also presented.

4.1. The Standard BW Algorithm

The standard BW algorithm estimates $P(\boldsymbol{y}|\boldsymbol{m}, \sigma_z^2, \lambda)$ by using the following three step procedure iteratively:

1. Compute the normalized forward variable $\alpha_i(k)$.
   Initialization:

$$\alpha_i(1) = \pi_i p_i(y(1)), \quad 1 \leq i \leq N,$$
$$c(1) = \left(\sum_{i=1}^{N} \alpha_i(1) \right)^{-1}.$$

   Induction: for $k = 1, ..., N_s - 1$, $j = 1, ..., N$

$$\alpha_j(k+1) = c(k) p_j(y(k+1)) \sum_{i=1}^{N} \alpha_i(k) d_{ij},$$
$$c(k+1) = \left(\sum_{i=1}^{N} \alpha_i(k+1) \right)^{-1}.$$

2. Compute the normalized backward variable $\beta_i(k)$.
   Initialization: $\beta_i(N_s) = c(N_s)$, $1 \leq i \leq N$,
   Induction: for $k = N_s - 1, ..., 1$, $i = 1, ..., N$,

$$\beta_i(k) = c(k) \sum_{j=1}^{N} d_{ij} p_j(y(k+1)) \beta_j(k+1),$$

3. Estimate the model parameters

$$\widehat{m}_i = \frac{\sum_{k=1}^{N_s} \gamma_i(k) y(k)}{\sum_{k=1}^{N_s} \gamma_i(k)},$$
$$\widehat{\sigma}_z^2 = \frac{1}{N_s} \sum_{n=1}^{N_s} \sum_{i=1}^{N} \gamma_i(n) |m_i - y(n)|^2,$$

   where $\gamma_i(k) = \alpha_i(k) \beta_i(k)$.

In a batch mode implementation, steps 1 to 3 are carried out iteratively with updated values of $p_j(y(k))$ until convergence. Thus, the estimated probability of the observation sequence given the model is computed as follows

$$\widehat{P}(\boldsymbol{y}|\boldsymbol{m}, \sigma_z^2, \lambda) = \frac{\sum_{i=1}^{N} \alpha_i(N_s)}{\sum_{i=1}^{N_s} c(i)}. \tag{2}$$

Different modifications have been applied to the standard BW algorithm to improve its performance or reduce computation complexity. One of these modifications is presented in Section 4.2.

4.2. The LMS-type Update Algorithm

The standard BW algorithm suffers from the "curse of dimensionality" because the computation complexity and memory requirement are proportional to the square of the number of the states. Furthermore the convergence rate is rather slow. Thus, it is worth seeking improvements in terms of memory and computation speed. In this paper, we have implemented the LMS-type update algorithm initially presented in [12]:

$$m_i(k) = m_i(k-1) + \mu_m \gamma_i(k) e_i(k),$$
$$\sigma_z^2(k) = (1 - \mu_s) \sigma_z^2(k-1) + \mu_s \left(\sum_{i=1}^{N} \gamma_i(k) |e_i(k)|^2 \right),$$

where $e_i(k) = y(k) - m_i(k-1)$ for $i = 1, ..., N$. The initialization and time-induction calculation for the forward variable can be computed as in the standard BW algorithm. The calculation of backward variable can be obtained by using the fixed-lag or sawtooth-lag schemes [13]. In this paper, we have used the fixed-lag scheme as explained in [8].

5. CLASSIFICATION RULE

The MAP classification rule used to recognize GMSK signals is defined as follows:

Assign $\boldsymbol{y}$ to λ_i if $\widehat{P}(\boldsymbol{y}|\lambda_i) P(\lambda_i) \geq \widehat{P}(\boldsymbol{y}|\lambda_j) P(\lambda_j), \forall j = 1, ..., c,$

where c is the number of possible modulations (or the number of classes) and $\widehat{P}(\boldsymbol{y}|\lambda_i) \triangleq \widehat{P}(\boldsymbol{y}|\boldsymbol{m}, \sigma_z^2, \lambda_i)$ is obtained from (2). Note that the whole sequence of length N_s is required to estimate $\widehat{P}(\boldsymbol{y}|\lambda_i)$ even if the online LMS-type update algorithm has been used for the computation of $m_i(k)$ and $\sigma_z^2(k)$. Note also that the observation length N_s required to properly identify the modulation constellations should be greater than the maximum number of HMM states in the class dictionary so that every possible state can be reached by the algorithm. This paper assumes that the different modulation formats are equally likely e.g., $P(\lambda_i) = 1/c, i = 1, ..., c$.

6. SIMULATION RESULTS

Many simulations have been carried out to evaluate the performance of the proposed classifier. All constellations have been normalized to unit energy and generated with the bit duration $T = 1$ and the sampling rate $F_e = 10$. The signal to noise ratio per bit is defined as $\mathrm{E_b/N_0}$, where $\mathrm{E_b}$ is the energy per bit at the input of the receiver. The classification performance is the average probability of correct classification defined as

$$P_{\text{cc}} = \frac{1}{c} \sum_{i=1}^{c} P \left[\text{assigning } \boldsymbol{y} \text{ to } \lambda_i | \boldsymbol{y} \in \lambda_i \right].$$

Figure 2 displays the classification performance as a function of $\mathrm{E_b/N_0}$ for the two GMSK modulations (five different values of the number of observations N_s are considered). This figure allows one to appreciate good classification performance especially for small $\mathrm{E_b/N_0}$. Figure 3 shows the classification performance versus $\mathrm{E_b/N_0}$

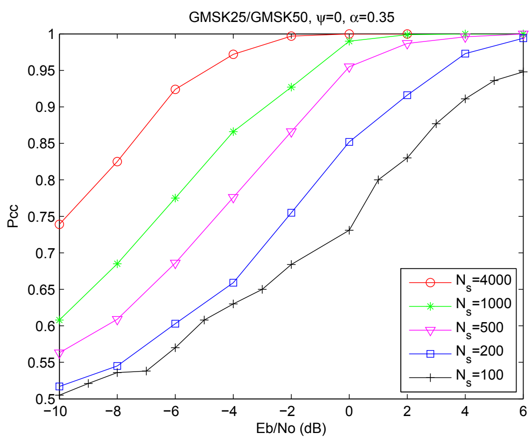

Fig. 2. Classification performance versus $\mathrm{E_b/N_0}$ for different numbers of observation symbols N_s.

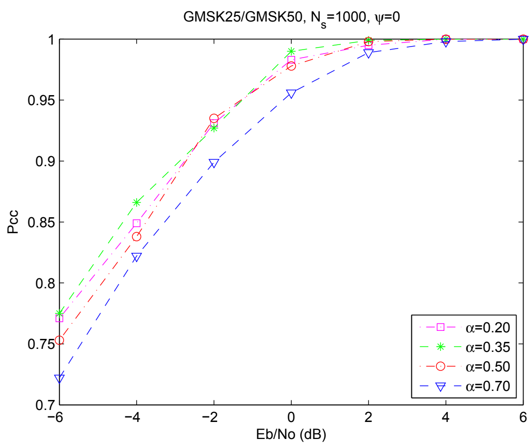

Fig. 3. Classification performance versus $\mathrm{E_b/N_0}$ for different roll-off factors α.

for different values of roll-off factor α. Clearly, the roll-off factor has an impact on the performance and it should be adjusted as a function of signal bandwidth in practical scenarios. The last simulations study the effect of a phase offset obtained by rotating the constellation with an angle ψ (this phase offset is due to synchronization errors at the receiver). Figure 4 shows that the classification performance seems to be robust to moderate synchronization errors especially for $\mathrm{E_b/N_0} \geq 0\mathrm{dB}$.

7. CONCLUSIONS

This paper addressed the problem of classifying GMSK signals with different values of BT transmitted through AWGN channels. The received communication signal was classified according to an MAP rule. This rule required to estimate the posterior distribution of the received communication signal conditionally to each modulation belonging to a known dictionary. This estimation was conducted by using the Baum-Welch algorithm for HMM which has shown interesting properties for speech recognition and blind channel characterization. The performance of the proposed classifier was assessed by

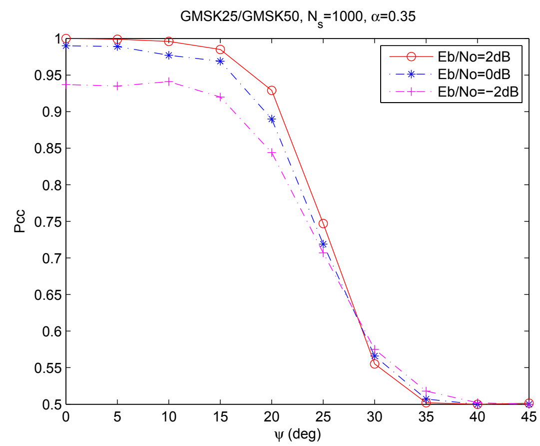

Fig. 4. Classification performance versus phase offset.

means of several simulation results. Perspectives include the joint classification of linear and nonlinear modulations using the Baum-Welch algorithm.

8. ACKNOWLEDGMENTS

The authors would like to thank Josep Vidal from Universitat Politcnica de Catalunya (UPC) for fruitful discussions regarding the Baum-Welch algorithm.

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