ACTIVE NOISE CONTROL OF NOISY PERIODIC SIGNALS USING SIGNAL SEPARATION

Govind Kannan, Ali A. Milani, student members, Issa Panahi, member IEEE

Dept of Electrical Engineering, The University of Texas at Dallas

ABSTRACT

Periodic signals (since they can be easily predicted) can be canceled much more effectively when compared to non-periodic/stochastic signals. A large class of acoustic noise sources have an underlying periodic process that generates a periodic noise component, and thus the acoustic noise can in general be modeled as the sum of a periodic signal and a random signal (usually a background noise). In this paper we present the idea that separating the acoustic noise into periodic and random noise components and doing separate active noise control(ANC) for each tends to increase the overall Noise Attenuation Level (NAL). Formulae for the exact improvement in noise attenuation levels are derived. A novel signal separation and noise cancelation scheme based on adaptive filtering is developed and its effectiveness is shown for several periodic signal in white noise cases.

Index Terms— Active noise control, Acoustic signal processing, ANC

1. INTRODUCTION

Active noise control is a technique of canceling acoustic noise by generating an appropriate antinoise using loud-speakers, and directing it towards the region where noise cancelation is required. Formulation of ANC from a signal-processing perspective enables us to recognize (as we show later) that it is a combination of three sub-problems: 1) System identification, 2) Prediction, 3) Equalization.

Usually an adaptive control algorithm handles these tasks. For periodic noise the problem of prediction is trivial. System identification and equalization can also be done accurately since they amount to estimation of magnitude and phase of a transfer function only at the sinusoidal components of the periodic signals. Hence, very good noise attenuation levels can be reached. For stochastic signals which are usually broadband, NAL is usually low. For a mixture of periodic and stochastic noise the achievable NAL is in between (depending on the SNR between periodic and random noise components). In this paper we develop the notion of separating the periodic and random noise components and generating corresponding antinoise components. We show that such a 'parallel' form of ANC tends to increase the achievable NAL . We also outline and simulate a simple adaptive filtering scheme that can realize the idea.The ANC method we use here is known as the feedforward method in ANC literature [1] and is shown in Fig.1. The reference microphone captures a version of the acoustic noise, $x(n)$ termed as the reference signal input. The error microphone is placed at the point where the cancelation is desired. $d(n)$ is the signal at the point where cancelation is desired. The transfer function that transforms $x(n)$ to $d(n)$ is termed as $P(z)$, the primary path. The goal of the ANC setup is to generate an antinoise $y'(n)$ through loudspeakers, directed towards error microphone, such that the error microphone's measurement signal $e(n)$ is driven to zero. The antinoise $y(n)$ that the ANC system generates gets modified into $y'(n)$ when it reaches the error microphone due to the effect of D/A, power amplifier, loud-speaker response, and the acoustic path between loud-speaker and the error microphone. All these effects are lumped into a transfer function $S(z)$ called the secondary path. The distortion due to $S(z)$ is handled by filtering $x(n)$ with an identical filter $\hat{S}(z) \approx S(z)$ before feeding it into LMS algorithm. This approach is termed as the FilteredX-LMS or FXLMS [1], [2]. It is clear from Fig.1 that the optimum control filter $W_{opt}(z)$, is given by:

$$W_{opt}(z) = \frac{P(z)}{S(z)} \quad (1)$$

Thus the task of the adaptive filter is to *identify* $P(z)$ and *equalize* $S(z)$. When $S(z)$ has zeros outside the unit circle, i.e when it is non-minimum phase (the typical case), a causal and stable $W_{opt}(z)$ cannot model $\frac{P(z)}{S(z)}$, thus limiting performance. However a non-causal and stable $W_{opt}(z)$ does exist which implies that future samples of $x(n)$ are needed which in turn implies the need for *prediction*. Thus for predictable processes, the impairment due to non-minimum phase can be addressed easily.

Since periodic signals are perfectly predictable [3] compared to stochastic signals, much higher NAL's can be reached. When both signals (periodic and random) are present, it follows that by separating them and using two different control

Supported by a subcontract from the Epidemiology Division, Department of Internal Medicine, and UT Southwestern Medical Center at Dallas under grant no. DAMD17-01-1-0741 from the U.S. Army Medical Research and Material Command. The content of this paper does not necessarily reflect the position or the policy of the U.S. government, and no official endorsement should be inferred. contacts:govind@student.utdallas.edu,ali.a.milani@student.utdallas.edu, issa.panahi@utdallas.edu.

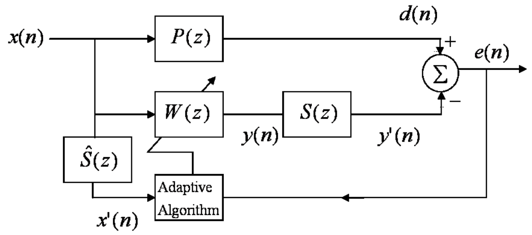

Fig. 1. The FXLMS algorithm. $W(z)$ generates an appropriate anti-noise so as to drive $e(n)$ to zero

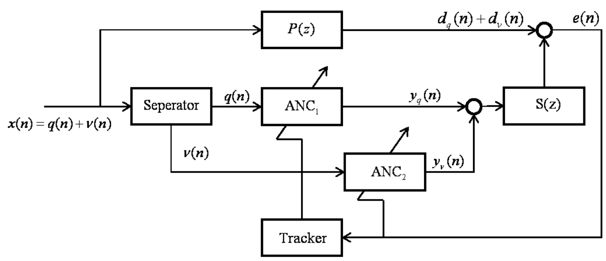

Fig. 2. Active noise control employing separation of periodic and random parts of the reference signal.

filters (one causal and another noncausal), the NAL should improve. We explore the idea of using two separate control filters connected in a parallel fashion. We quantify the improvement in NAL in terms of system parameters. We also derive a formula for the maximum possible NAL achievable by our approach . Our preliminary simulation results have verified our approach and conform with the upper bounds derived.

2. PROBLEM FORMULATION

The various signals, blocks and terms that will be used in the paper are listed as follows

$\xi_{(.)}$ gives the signal energy i.e $\xi_{(.)} = E((.)^2)$. For example for a signal $w(n)$, the energy is given by $\xi_w = E(w^2(n))$.

$f_{(.)}$ The subscript for any quantity f denotes the component of f, corresponding to the subscripted variable.

$x(n)$ The version of acoustic noise recorded by a reference microphone placed somewhere in the noisy area. This measurement is known as the reference signal in ANC literature.

$q(n)$ The periodic noise component which can be written as a sum of sinusoids i.e $q(n) = \sum_{k=1}^{L} A_k cos(\omega_k n + \phi_k)$.

$\nu(n)$ The stochastic/random noise component. It is assumed to be zero mean. In this paper $x(n) = q(n) + \nu(n)$.

η The SNR of periodic component to the random component. i.e $\eta = \frac{\xi_q}{\xi_\nu}$.

$e(n)$ The measurement from error microphone. The cancelation is achieved around the error microphone. The error signal corresponding to a particular component will have a subscript to denote that. For example ,the component of error signal corresponding to $q(n)$ will be denoted as $e_q(n)$ and its power will be ξ_{e_q}.

$P(z)$ The transfer function between reference and error microphones, known as the *primary path*. The corresponding impulse response is $p(n)$.

$S(z)$ represents the transfer function of electrical path (microphones, digital circuitry, canceling loudspeaker) plus the acoustic path between the loud speaker and the error microphone known as the *secondary path*.

$\hat{S}(z)$ An estimated model of $S(z)$, which is placed in the reference signal path. In all our discussions $\hat{S}(z)$ is considered as an accurate model i.e $\hat{S}(z)$=$S(z)$.

$d(n)$ The noise signal after passing through $P(z)$ is called $d(n)$ and represents the signal to be canceled.

$W(z)$ is the main adaptive filter which generates the appropriate anti-noise based on the error and reference input.

$y(n)$ The control signal generated by the adaptive filter $W(z)$. $y(n)$ gets shaped into $y'(n)$ when it reaches the error microphone due to the effect of the secondary path $S(z)$.

Γ_w Noise Attenuation Level (NAL) of an ANC system which has the reference input $w(n)$.It is given by $\Gamma_w = \frac{\xi_{d_w}}{\xi_{e_w}}$ It is the performance metric we use in this paper.

The separation based ANC is formulated as follows:1.) Given an acoustic noise process $x(n) = q(n) + \nu(n)$ devise a method of separating $x(n)$ into $q(n)$ and $\nu(n)$ 2.) Design an ANC system that has two parallel anti-noise generating adaptive filters $W_q(z)$ and $W_\nu(z)$. The overall system should monitor the error microphone output $e(n)$ to adjust its weights.

3. QUANTIFYING IMPROVEMENT

In this section we quantify the improvement in performance due to the proposed method in terms of η , Γ_ν and $P(e^{j\omega})$. Γ_x is by definition,

$$\Gamma_x = \frac{\xi_d}{\xi_{e_q} + \xi_{e_\nu}} \tag{2}$$

Due to linearity:

$$\begin{aligned} d(n) &= p(n) * (x(n)) = p(n) * (q(n) + \nu(n)) \\ &= d_q(n) + d_\nu(n) \end{aligned}$$

Since $q(n)$ (deterministic) and $\nu(n)$ are independent:

$$\Rightarrow \xi_d = \xi_{d_q} + \xi_{d_\nu} \tag{3}$$

$$\Rightarrow \Gamma_x = \frac{\xi_{d_q} + \xi_{d_\nu}}{\xi_{e_q} + \xi_{e_\nu}} \tag{4}$$

The performance gain due to the proposed separation based method can be understood from (4). By using a dedicated canceling filter, the periodic part can be almost perfectly canceled i.e $\xi_{e_q} \approx 0$. This is not the case with conventional ANC since a single filter has to generate anti-noises for both the periodic and random part. Hence the denominator of (4) is always higher for the conventional method. If we assume that the periodic part can be perfectly canceled, (4) becomes

$$\Gamma_x \approx \frac{\xi_{d_q} + \xi_{d_\nu}}{\xi_{e_\nu}} \tag{5}$$

This assumption in fact makes Γ_x of (5) more of an upper bound since it is the maximum NAL possible under the assumptions of perfect separation and perfect cancelation of the periodic part $q(n)$. Denoting the performance of an ANC system to a white noise input by $\Gamma_\nu = \frac{\xi_{d_\nu}}{\xi_{e_\nu}}$ we can write (5) as

$$\Gamma_x = \Gamma_\nu(1 + \frac{\xi_{d_q}}{\xi_{d_\nu}}) \tag{6}$$

$$\Gamma_x = \Gamma_\nu(1 + \frac{\sum_{k=1}^{L} \frac{A_k^2}{2} |P(e^{j\omega_k})|^2}{\xi_\nu \sum_{n=1}^{\infty} |p(n)|^2}) \tag{7}$$

In case of a single tone ($L = 1$), (7) becomes

$$\Gamma_x = \Gamma_\nu(1 + \eta \frac{|P(e^{j\omega_k})|^2}{\sum_{n=1}^{\infty} |p(n)|^2}) \tag{8}$$

Let

$$\alpha = \frac{|P(e^{j\omega_k})|^2}{\sum_{n=1}^{\infty} |p(n)|^2} \tag{9}$$

Then

$$\Gamma_x = \Gamma_\nu(1 + \eta\alpha) \tag{10}$$

The improvement in NAL of a mixture of periodic and random noise is greater than that of the performance of a purely random noise by a factor of $(1+\eta\alpha)$. If we assume $\eta\,\alpha \gg 1$, then $\Gamma_x \simeq \Gamma_\nu \eta\alpha$. α is low when there are deep notches in $P(e^{j\omega_k})$ at the harmonic components , in which case the relative position of reference microphone should be changed.

4. SIGNAL SEPARATION BASED ANC-METHOD

The method is elaborated in Fig.2. It is done in three parts 1.) Estimation of frequency of tones 2.) Signal Separation 3.) Generating anti-noises corresponding to the periodic and random parts.

4.0.1. Tonal Frequency Estimation

This block estimates the frequencies present in the noisy periodic signal. The spectrum of a periodic signal is basically a line-spectrum. For high SNR and reasonably widely spaced tones, FFT is adequate to determine the frequencies. For lower values of η subspace methods like MUSIC/ ESPIRIT [4] or statistical methods like AR modeling can be used. The estimated frequencies are then used to generate a reference periodic signal $r(n) = q(n) = \sum_{k=1}^{L} cos(\omega_k n)$. $r(n)$ is different from $q(n)$ only in the amplitudes A_k's and phases ϕ_k's the values of which the next block determines.

4.0.2. Signal Separation

This block essentially estimates $q(n)$ from $r(n)$ by trying to cancel $q(n)$ from $x(n) = q(n) + \nu(n)$ by estimating $q(n)$ from $r(n)$. It can be easily seen that after convergence, the error signal $\epsilon(n) \approx \nu(n)$ and canceling signal $y(n) \approx -q(n) \Rightarrow q(n) \approx -y(n)$. Thus separation is achieved. The first few seconds of the reference signal $x(n)$ is used to estimate the tones and achieve separation. The separated signals will be denoted by $\hat{q}(n)$ and $\hat{\nu}(n)$ to indicate that they are estimates. For high η, the estimates are highly accurate.

4.0.3. Anti noise Generation

This block generates the anti-noise in two steps via two parallel ANC systems , ANC$_1$ and ANC$_2$. ANC$_1$ uses $\hat{q}(n)$ as the reference and generate an antinoise $y_q(n)$ which attempts to cancel $d_q(n)$. The error signal $e_q(n)$ is given by

$$e_q(n) = d(n) - y'_q(n) = d_\nu(n) + (d_q(n) - y'_q(n)) \tag{11}$$

The adaptive filter $W_q(z)$ and $S(z)$ linearly operate on the periodic signal $\hat{q}(n)$ to produce $y'_q(n)$ which will lie in the same subspace as that of $\hat{q}(n)$. Since $d_\nu(n)$ is uncorrelated with $d_q(n)$, convergence in this case basically means that $(d_q(n) - y'_q(n))$ is driven to zero. Thus after convergence

$$e_q(n) \approx d_\nu(n). \tag{12}$$

After convergence is attained in ANC$_1$ the adaptation is stopped and W_q is fixed to the steady-state value. The system ANC$_1$ continues to generate antinoise which cancels $d_q(n)$. The error microphone's output is $d_\nu(n)$. The next step is to switch on ANC$_2$ whose reference input is the separated random part $\hat{\nu}(n)$ and error input is $d_\nu(n)$ which it tries to minimize by generating the antinoise $y_\nu(n)$. After convergence the error is $e_\nu(n)$ and should not contain any component of the periodic noise as long as the acoustic system is stationary.To handle non-stationarity, the 'tracker' block shown in Fig.2 monitors the error $e(n)$ for harmonic components. If harmonics are sensed (above a threshold value), it implies that the acoustic system has changed, and $W_q(z)$ has to be adapted again. The two step adaptation scheme is repeated.

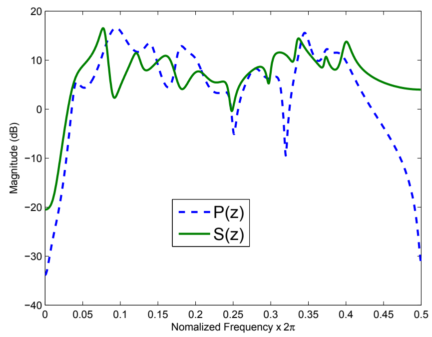

Fig. 3. Frequency responses of the primary and secondary paths used in the simulation

5. SIGNAL SEPARATION BASED ANC-RESULTS

Simulation of the proposed scheme (using FXLMS for adaptation) was implemented by artificially generating $x(n) = q(n) + \nu(n)$ for different values of the SNR η and different kinds of $q(n)$. Single and multi-tone signals of various frequencies were synthesized and $\nu(n)$ of varying power was added. The $P(z)$, $S(z)$ pair used in all the simulations were obtained from [1] and represent real life acoustic transfer functions (Fig.3). The performance of the proposed method is compared against the existing conventional single filter FXLMS method [1]. The experiments and results are shown in Table 1. The last column shows the best achievable Γ_x as given by (5). It can be seen NAL's much closer to Γ_x are attained using our proposed when compared with the existing method. For low SNR's (like 10dB) the performance improvement will be less noticeable (as indicated by (10)) when compared to higher SNR's. It should also be noticed that the upper-bound NAL (Γ_x) cannot be achieved in reality since in its derivation it is assumed that the periodic part is perfectly separated and canceled (i.e $\xi_{e_q} \approx 0$), while in reality it is not.

6. CONCLUSION

In this paper we have focussed on the active noise control of acoustic signals which have a rich periodic/harmonic structure along with a random noise. Our key idea stems from the observation that the periodic component and the random component might need different noise canceling filters for the best performance. We have proposed a *parallel* form of ANC, in which the reference input is separated into periodic and random components. A novel adaptive filtering based implementation of signal separation and anti-noise generation is developed and simulated . A simple but an insightful formula is derived to quantify performance. The method is shown to work very well with single and multi-tone signals in white noise. The method is currently being expanded to effectively cancel fMRI acoustic noise, owing to its periodic nature [5], [6].

Table 1. Performance comparison of the proposed separation based parallel ANC and conventional (existing) ANC for different tones, SNR combination. Γ_x(dB) is the maximum possible NAL.

#	Number of Tones L	η(dB)	Proposed method (dB)	Existing method (dB)	Γ_x(dB)
1	1	10	25.57	24.61	26.38
2	1	20	33.69	20.16	36.33
3	1	30	44.56	25.97	46.24
4	1	40	53.03	35.60	56.24
5	2	10	31.83	33.21	34.20
6	2	20	42.88	32.20	44.09
7	2	30	52.34	35.91	54.05
8	2	40	58.00	44.64	64.00
9	3	10	31.96	31.86	33.00
10	3	20	41.99	30.69	43.00
11	3	30	50.03	35.34	53.80
12	3	40	55.86	44.43	64.00
13	4	10	31.54	31.43	33.00
14	4	20	40.36	30.5	42.67
15	4	30	48.32	35.49	53.64
16	4	40	58.40	44.35	63.00

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