EVALUATION OF TORQUE ESTIMATION USING GRAY-BOX AND PHYSICAL CRANKSHAFT MODELING

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ABSTRACT

In-cylinder pressure and combustion torque provide feedback information that can be utilized in advanced control and diagnosis strategies for combustion engines. One challenge in torque estimation is the compensation of the torsional effects of the crankshaft. This paper presents a novel torque estimation method using a gray-box model (motivated from a physical model) of the crankshaft in combination with one in-cylinder pressure and one engine speed measurement. The MISO system inversion is described and the approach is benchmarked against an existing method. The results using a four-cylinder spark ignition engine encourage the further investigation of the approach.

Index Terms— Torque, Identification, Road vehicle propulsion

1. INTRODUCTION

Increasing awareness of global climate change has an impact on the development of future propulsion systems. Since the combustion engine will remain the dominating technology in the automotive industry for many years to come, a lot of research is being undertaken to improve its efficiency and emissions. One technique being investigated is the use of new homogeneous combustion processes which use feedback information from the combustion chamber to allow a stable mode of operation. In-cylinder pressure sensors provide such a feedback information for necessary engine control and diagnosis. However, an engine fully-equipped with such sensors is not cost efficient and therefore other methods are investigated to reconstruct feedback information from the combustion chamber.
The most commonly used method in the literature is the evaluation of the engine speed signal ([1], [2], [3]). Alternatively, Larsson examined pressure estimation considering a torque sensor mounted at the crankshaft [4]. Structure-born sound, commonly used for knock detection in spark-ignition engines, was used by Villarino [5] for pressure reconstruction. Hamedović *et al.*'s method ([6],[7]) for pressure reconstruction uses combined processing of one in-cylinder pressure signal, together with the engine speed signal. Their approach doesn't compensate for torsional deflections of the crankshaft which occur at higher engine speeds. In order to take these into account, this work considers a dynamical torsional crankshaft model with processing one pressure in the so called key-cylinder, as well as engine speed. The investigation in [8] has shown that the use of a physical crankshaft model with two engine speed signals can yield encouraging results. Based on the physical crankshaft model, this article describes a gray-box modeling approach using subspace identification with only one engine speed measurement (hereafter referred to as Subspace method). Furthermore, a new torque estimation algorithm is derived. The estimation results are compared for combustion feature estimation with the physical model approach using an Unscented Kalman Filter described in [8] (hereafter referred as UKF approach).
The article is structured as follows: Section 2 describes the physical equations for torsional crankshaft modeling and the resulting gray-box model approach using subspace identification. Section 3 illustrates the inversion of the MISO system using the in-cylinder pressure for compression torque estimation and a separation algorithm for torque reconstruction. In Section 4 the performance of this approach is compared with the UKF approach for combustion feature estimation using measurement data from a spark-ignition engine.

2. CRANKSHAFT MODELLING

The firing of each cylinder drives the crankshaft at whose ends the engine speed can be measured. The relationship between the indicated torque and engine speed is therefore described by the dynamics of the crankshaft. Since in multi-cylinder engines the crankshaft is exposed to torsional deflections which influence the engine speed signal, a torsional crankshaft model is chosen to describe this relationship. According to [9] the torque balance equation of the torsional crankshaft model can be described as follows:

$$\mathbf{\Theta}\,\underline{\ddot{\varphi}}+\mathbf{D}\,\underline{\dot{\varphi}}+\mathbf{K}\,\underline{\varphi}=\underline{\tau}_{ind}\,(\underline{\varphi})+\underline{\tau}_{mass}\,(\underline{\varphi})+\underline{\tau}_{load}\,(\underline{\varphi})+\underline{\tau}_{fric}\,(\underline{\varphi}). \tag{1}$$

where the matrices $\mathbf{\Theta}$, $\mathbf{D}$, $\mathbf{K}$ are symmetric and stand for the rotating moment of inertia, damping and stiffness behavior of the crankshaft, respectively. The crank angle vector $\underline{\varphi}$ describes the torsion of the crankshaft. The indicated torque $\tau_{ind,l}$ of cylinder l results from the in-cylinder pressure p_l as follows:

$$\tau_{ind,l}\,(\varphi)=(p_l(\varphi)-p_0)\,h(\varphi-(l-1)\,\frac{4\pi}{z}) \tag{2}$$

with $h(\varphi)\;=\;A\,r\,\left(\sin\varphi+\frac{\lambda\,\sin\varphi\,\cos\varphi-\mu\,\cos\varphi}{\sqrt{1-\lambda^2\,\sin^2\varphi+2\,\lambda\,\mu\,\sin\varphi-\mu^2}}\right)$, where p_0 is the ambient pressure, z the number of cylinders, l the cylinder index according to the firing order, A the piston area, r the crank radius, λ the connecting rod ratio, and μ the axial offset ratio. In the following $z=4$ is considered.
The oscillating movement of the pistons and rods leads to the following mass torque:

$$\tau_{mass,l}\,(\varphi)=-\,\theta_l(\varphi)\,\frac{d\dot{\varphi}}{d\varphi}\,\dot{\varphi}-\frac{1}{2}\,\frac{d\theta_l(\varphi)}{d\varphi}\,\dot{\varphi}^2, \tag{3}$$

where $\theta_l(\varphi)$ stands for the moment of inertia of the oscillating parts of cylinder l. The friction and load torque are assumed to be constant for one operating cycle and can be estimated from the mean value of the indicated torque of the key-cylinder 1:

$$\underline{\tau}_{fric} + \underline{\tau}_{load} = \overline{\tau}_{ind,1} \tag{4}$$

The torsional crankshaft model is illustrated in Figure 1.
As already described in [8] the physical crankshaft model can be linearized by neglecting the torsional differences for the calculation of the mass torque. The resulting linear time invariant system can then be described by the following state-space description regarding engine speed measurements at the free end $\dot{\varphi}_{fe}$ and at the flywheel $\dot{\varphi}_{flwh}$:

$$\begin{aligned} \dot{\underline{x}} &= \mathbf{A}\,\underline{x} + \mathbf{B}\,\underline{u} \\ \underline{y} &= \mathbf{C}\,\underline{x} + \mathbf{D}\,\underline{u} \end{aligned} \tag{5}$$

with $\underline{u} = [\tau_{ind,1} + \tau_{mass,1}, \ldots, \tau_{ind,4} + \tau_{mass,4}, \tau_{load}]^T$ and $\underline{y} = [\dot{\varphi}_{fe}, \dot{\varphi}_{flwh}]^T$. Considering the fact that the parameters of the physical crankshaft model are not known exactly, they need to be identified. The structure of the system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ is well defined by the spring, damping and inertia coefficients. However, a more flexible structure for system identification might be preferable finding a good model. Furthermore, the system structure of the physical crankshaft model according to (1) leads to a high system order, since two system states are introduced per degree of freedom. The system order of the crankshaft model in Figure 1 is 16, for example.

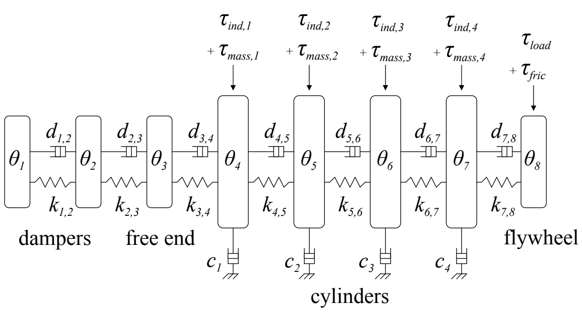

Fig. 1. Crankshaft model for $z = 4$ cylinders

With this in mind, a gray-box model approach is considered here. This approach results in a reduced model order by incorporating higher flexibility in the system matrices.
The system matrices of (5) are identified by subspace identification. The model order can be estimated through singular value decomposition using the input and output measurements. Alternatively, it can be chosen manually in order to include previous knowledge of the system. Afterwards the states are estimated using the orthogonal matrices of the singular value decomposition. The system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ can then be determined by linear regression. For more detailed information on the `n4sid` subspace identification algorithm the interested reader is referred to [10].
Since the load torque can also be estimated directly from the key-cylinder pressure measurement, the gray-box model is used for modeling torque fluctuations. Therefore, the load torque is removed from the input torque vector which can be done for constant operating points. Considering furthermore a sensor configuration using only the engine speed signal at the crankshaft leads to a MISO system with 4 inputs and 1 output.
Instead of using the engine speed signal directly for system identification, this work suggests the use of the engine speed acceleration which showed more successful results. For differentiation a real differentiator with a limited bandwidth was used. The corresponding filter characteristic can be seen in Figure 2.

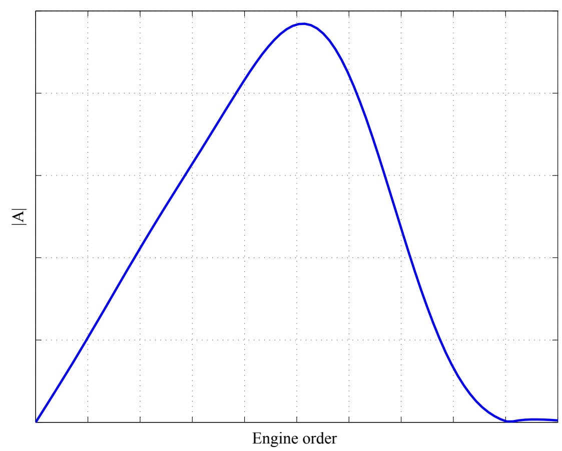

Fig. 2. Filter characteristic of the differentiator

The engine order is defined as crankshaft frequency. It is a convenient frequency definition in engine applications, since all measurements are sampled with respect to crank angle degrees. The constant engine order cut-of frequency allows the same frequency information of the indicated torques at different engine speeds.

3. INVERSION OF THE GRAY-BOX MODEL USING THE SEPARATION ALGORITHM

In order to estimate cylinder-wise torque, the MISO system described in the previous section needs to be inverted. Instead of introducing an input torque model according to [8], this approach suggests a separation of the input sources according to [11] which allows a direct inversion afterwards. Whereas Andersson *et al.* use a torque sensor and a pressure model for inversion, this approach allows the integration of the key-cylinder pressure in combination with engine speed.
In the first step of the input separation, the indicated torque of each cylinder is divided into a compression $\tau_{cp}(\varphi)$ and a combustion component $\tau_{comb}(\varphi)$:

$$\begin{aligned} \tau_{cp}(\varphi) &= (p_{cp} - p_0)\, h(\varphi) \\ \tau_{comb}(\varphi) &= p_{comb}\, h(\varphi) \end{aligned} \tag{6}$$

Therefore, the compression pressure is estimated from the key-cylinder pressure measurement using an adiabatic model:

$$p_{cp} V(\varphi)^{\kappa} = C \tag{7}$$

The adiabatic exponent κ and the constant C are estimated with an LS algorithm assuming compression pressure and key-cylinder pressure measurement to be equal in the crank angle region from -180 to 0 degrees before injection [7]:

$$
\begin{aligned}
& ln\ p_{cp}(\varphi) = ln\ C - \kappa\ lnV(\varphi) \\
& \underline{\gamma} = (\mathbf{\Psi}'\mathbf{\Psi})\mathbf{\Psi}'\underline{y}\,, \qquad \mathbf{\Psi} = \begin{pmatrix} 1 & -lnV(\varphi_1) \\ \vdots & \vdots \\ 1- & lnV(\varphi_n) \end{pmatrix}
\end{aligned} \tag{8}
$$

with $\underline{\gamma} = [ln\ C,\ \kappa]^T$ and $\underline{y} = [ln\ p(\varphi_1), \ldots, p(\varphi_n)]^T$.
The estimated compression pressure can be assumed to be identical for all cylinders and the corresponding compression torque component can be approximated:

$$
\tau_{cp} = \tau_{cp}(\varphi - (l-1)\ \pi) \approx \tau_{cp,l}\big|_{l=2,3,4} \tag{9}
$$

The compression torque τ_{cp} allows the inversion of the MISO system of the gray-box model according to Figure 3.

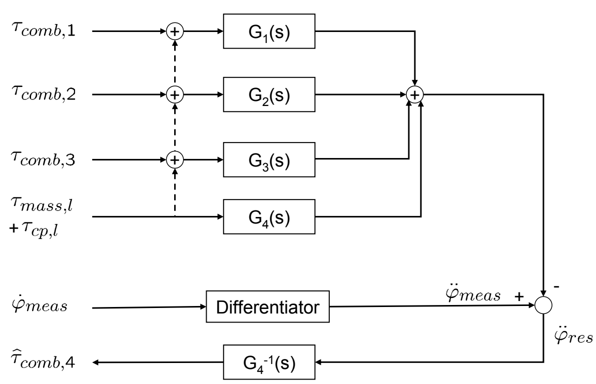

Fig. 3. Inversion strategy of the linear crankshaft system. In this case the combustion torque $\hat{\tau}_{comb,4}$ of cylinder 4 is estimated using the known inputs of the cylinders 1-4. The dashed line connects the mass torque $\tau_{mass,l}$ and compression torque $\tau_{comp,l}$ of cylinder l which is summed to the corresponding combustion torque.

Besides the compression torques $\tau_{cp,l}$, the mass torques of each cylinder $\tau_{mass,l}$ can also be calculated from the crank-slider geometries and the engine speed measurement according to (3) and are therefore known. The unknown combustion torques are now gained by an iterative procedure where in each step the combustion torque of one cylinder is estimated considering the combustion torques of the other cylinders as known. For initialization, the combustion torque of the key-cylinder can be used for the approximation of the unknown combustion torques.
After filtering the engine speed measurement with the differentiator in Figure 2, the residual engine acceleration can be found by the convolution of the known input torques with the impulse responses of the corresponding cylinders:

$$
\begin{aligned}
\ddot{\varphi}_{res} = & \sum_{i=1, i\neq l}^{4} g_i * \Big[\tau_{comb,i} + \tau_{cp,i} + \tau_{mass,i}\Big] \\
& + g_l * \Big[\tau_{cp,l} + \tau_{mass,l}\Big]
\end{aligned} \tag{10}
$$

The combustion torque of cylinder l can then be estimated by SISO inversion as follows:

$$
\hat{\tau}_{comb,l} = \mathcal{F}^{-1}\Big\{G_l^{-1}(j\omega)\mathcal{F}\{\ddot{\varphi}_{res}\}\Big\} \tag{11}
$$

where G_l is the system response of cylinder l. For the estimation of the next combustion torque in firing order, the combustion torque of cylinder 4 is used as a known input and the described procedure from above is repeated.

4. EXPERIMENTAL RESULTS

In this section the results of the torque estimation using the Subspace method and the UKF approach according to [8] are compared. Both methods are investigated with measurements from a 4-cylinder spark-ignition engine. In-cylinder pressure measurements with 1 crank angle degree (CAD) resolution, as well as engine speed measurements made with an optical sensor at the free end (1 CAD resolution) and an anisotropic magnetoresistive sensor at the flywheel (3 CAD resolution) were available. The gray-box model consists of 5 engine speed models. The parameters for the physical model (UKF approach) were obtained from the manufacturer. For comparison of both algorithms, measurements from 32 operating points in different engine speeds and load cases were made. The range of available engine speed measurements was 1500-3500 rpm and the range of load was 3-9 bar. In total, 19200 combustions were used for corresponding torque estimation. In Figure 4 the estimated and reference torque trace are illustrated for one operating cycle. Only the combustion torques of cylinder 2-4 are estimated, since cylinder 1 is the key-cylinder and its combustion torque is therefore known from available pressure measurements.

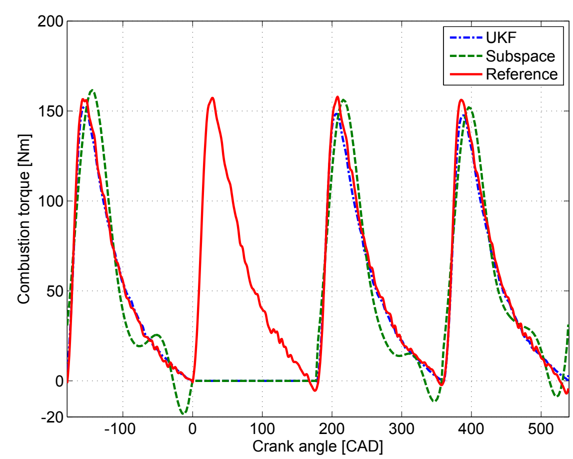

Fig. 4. Exemplary combustion torque trace at 1500 rpm and 3 bar load.

For comparison of both algorithms, the confidence intervals of two combustion features extracted from the estimated torque were considered. Figures 5 and 6 show the performance of both methods for PMI- and combustion phase estimation. The 95 % confidence interval of the combustion feature PMI is between +/- 0.8 bar for the UKF and +/- 0.96 bar for the Subspace method. For the combustion phase feature the UKF achieved a 95 % confidence with the bounds +/- 4.23 CAD and the Subspace method with the limits of +/- 4.7 CAD error. Both methods show feasible results for combustion feature estimation. The slight performance drop of the Subspace method can be explained by the usage of only one engine speed measurement.

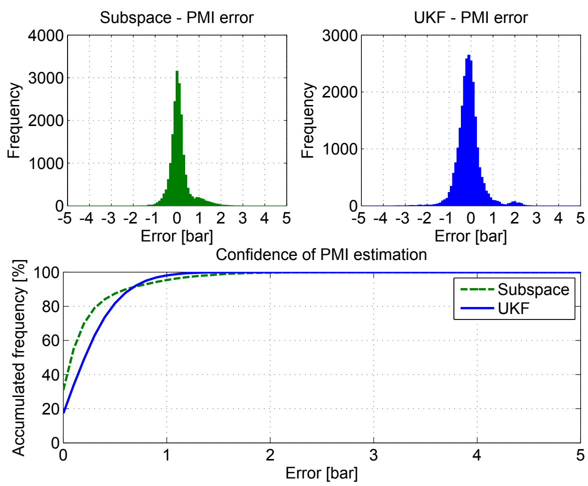

Fig. 5. Estimation of combustion feature PMI for 32 operating points in the range of 1500 to 3500 rpm and 3 to 9 bar. The UKF uses a physical model and engine speed measurements at both ends of the crankshaft. The subspace method using a gray-box model of the crankshaft with only one engine speed measurement at the flywheel.

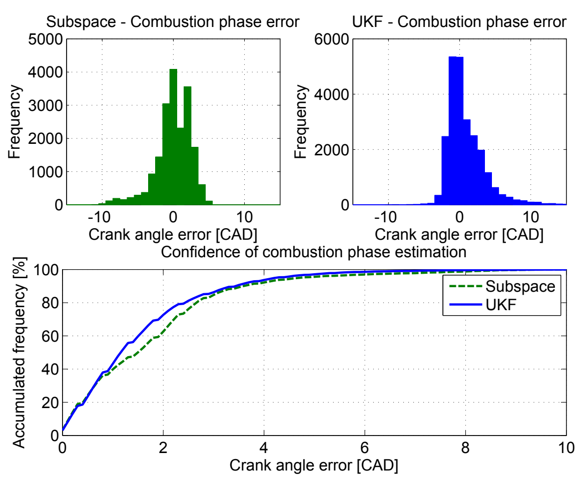

Fig. 6. Estimation of combustion feature combustion phase for 32 operating points in the range of 1500 to 3500 rpm and 3 to 9 bar. The UKF uses a physical model and engine speed measurements at both ends of the crankshaft. The subspace method uses a gray-box model of the crankshaft with only one engine speed measurement at the flywheel.

5. CONCLUSIONS

This article presented a new torque estimation approach for a combined processing of in-cylinder pressure and engine speed. A gray-box model of the crankshaft was identified using subspace identification. The inversion of the MISO system using the key-cylinder pressure and an engine speed signal at the flywheel was explained. The results were presented at a 4-cylinder spark-ignition engine and compared with an existing torque estimation approach. They showed good performance and encourage further investigation of the approach.

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