EXTRACTION OF POLYPHASE RADAR MODULATION PARAMETERS USING A WIGNER-VILLE DISTRIBUTION – RADON TRANSFORM

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Abstract - Often used in low probability of intercept continuous waveform (CW) emitters, polyphase modulations can have extremely long code lengths (large processing gain), good sidelobe performance and robust Doppler tolerance. This paper presents an efficient algorithm to autonomously extract the polyphase radar modulation parameters from an intercepted waveform using a Wigner-Ville Radon transform. Results show that our method results in a small relative error in the extracted parameters for signal-to-noise ratios as low as –6 dB.

Index Terms**—** Wigner-Ville distribution, Modulation parameters, Radon transform.

1. INTRODUCTION

Polyphase modulation of a continuous wave (CW) carrier frequency is becoming increasingly important in low probability of intercept wideband emitter design [1]. Non-cooperative intercept receivers looking for these emitters must detect the modulation across a broad spectrum in the presence of noise and multi-path. The intercept receiver can increase its processing gain by implementing time-frequency (T-F) signal processing such as the pseudo Wigner-Ville distribution (PWVD). The T-F output images can provide details about the polyphase CW modulation parameters that are unavailable using power spectral density techniques. The need for human interpretation of the T-F results however limits the extraction of the waveform parameters to non-real time electronic intelligence receivers.

Autonomous parameter extraction of the LPI emitter modulations can eliminate the need for a human operator and enable near real-time coherent handling of the threat emitters being intercepted. Parameter extraction followed by correlation with existing emitters in a database (identification) can then aid in signal tracking and coherent response management. This paper investigates an algorithm to efficiently extract the polyphase radar modulation parameters (bandwidth *B*, cycles of the carrier per subcode *cpp*, code length N_c, code period *T* and carrier frequency f_c) using a Wigner-Ville distribution – Radon transform. We show that the Radon transform is particularly useful for this time-frequency signal processing task since the majority of polyphase modulations are developed by approximating a linear frequency modulation waveform.

We evaluate the sensitivity of the algorithm using the five polyphase modulations Frank, P1, P2, P3, and P4 for signal-to-noise ratios (SNRs) of 0 dB and -6 dB. To illustrate the algorithm, a Frank code is used with N_c subcodes, a carrier frequency of $f_c = 1495$ Hz and an analog-to-digital converter (ADC) sampling frequency of $f_s =$ 7 kHz with SNR = 0 dB. The Frank code is a polyphase code with each sub-code phase defined as

$$\phi_{i,j} = \frac{2\pi}{M}(i-1)(j-1) \tag{1}$$

where $i = 1, 2, \ldots M$ with $N_c = M^2 = 36$ total sub-codes. The number of carrier frequency cycles within a subcode is $cpp = 1$, giving a transmitted bandwidth $B = f_c / cpp = 1495$ Hz and a code period of $T =$ 24.1 ms [1].

2. PWVD–RADON TRANSFORM ALGORITHM

A block diagram of the autonomous PWVD – Radon transform algorithm is shown in Figure 1. The first step is to compute the Wigner-Ville distribution. The carrier frequency f_c is then extracted by finding the location of the maximum intensity level within the PWVD image. In order to extract the code length T and bandwidth B, the Radon transform is computed from the T-F PWVD image. The Radon transform is the projection of the image intensity along a radial line oriented at a specific angle. It transforms the 2-D image with line-trends into a domain of the possible line parameters ρ and θ, where ρ is the smallest distance from the origin and θ is its angle with the x-axis. In this form, a line is defined as $\rho = x\cos\theta + y\sin\theta$ [2]. Using this, the Radon transform of a 2-D image $f(x, y)$ is

$$R(\rho,\theta) = \int_{-\infty}^{+\infty} f(\rho\cos\theta - s\sin\theta, \rho\sin\theta + s\cos\theta)ds \tag{2}$$

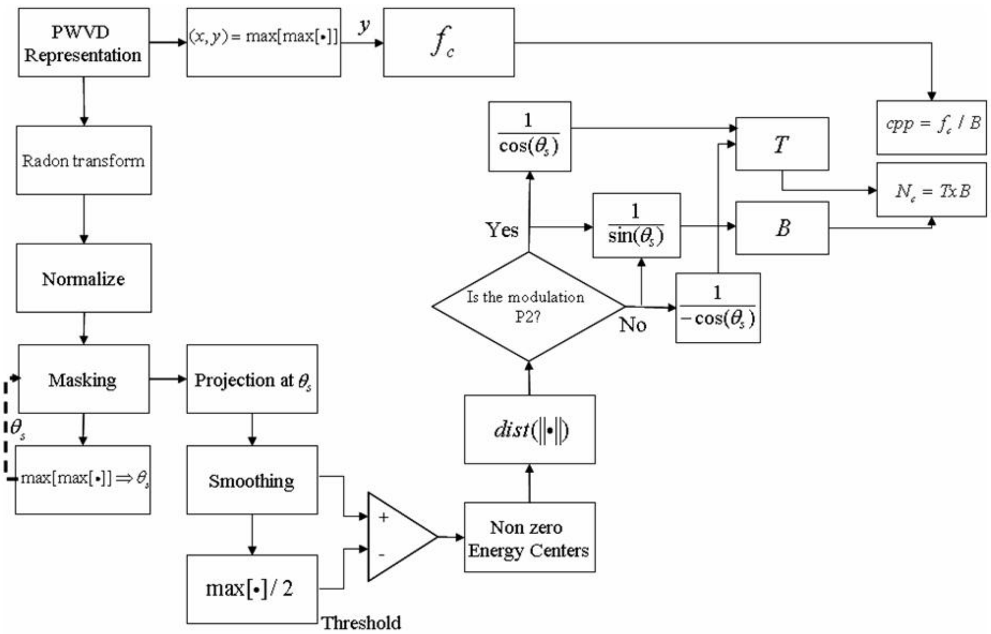


Figure 1. Parameter extraction block diagram.

where the *s*-axis lies along the line perpendicular to ρ as shown in Figure 2. Here *s* can be calculated as

$$s = y\cos\theta - x\sin\theta \tag{3}$$

Note ρ and *s* can be calculated from *x, y* and θ [3].

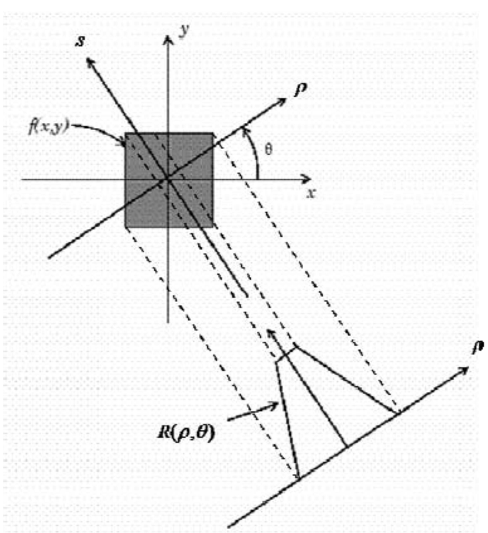


Figure 2. Geometry of the Radon Transform.

In this work the projection of the images are computed as line integrals from multiple sources along parallel paths in a given direction. The beams are spaced 1 pixel unit apart. Figure 3 shows the Gray-scale image from the PWVD illustrating the parameters to be extracted i.e., signal bandwidth B and polyphase code period T. The algorithm measures B and T by implementing the Radon transform to find θ_s and *d*. Here *d* is the perpendicular distance in pixels between two consecutive linear energy lines at the modulation angle θ_s [4].

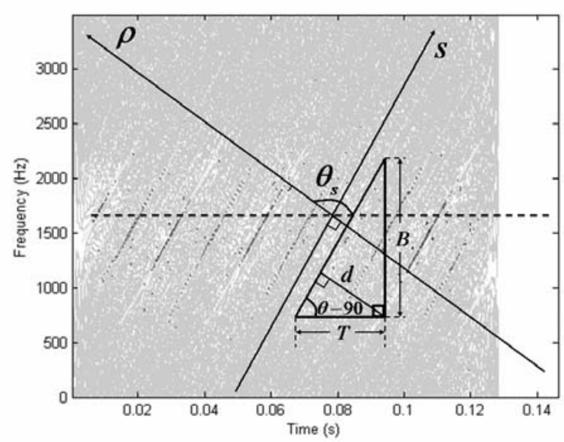


Figure 3. Radon Transform Geometry on PWVD image.

Once θ_s and d for the modulation are determined, B and T can be calculated using geometrical relations [4]. The Radon transform is implemented so that the parallel-beam projections of the image are taken between $[0^{\circ}, 179^{\circ}]$. Once the transform is completed it is normalized. In some cases the maximum intensity on the transform may occur around $\theta = 90^{\circ}$ which corresponds to the marginal frequency distribution (MFD) and around $\theta = 0^{\circ}$ which corresponds to the time marginal. In order to avoid the detection of the angle corresponding to the MFD and marginal time distribution, we assume that the slope of linear energy lines are not between $[10^{\circ}, -10^{\circ}]$ or between $[85^{\circ}, 95^{\circ}]$. The projections on angles between $\theta = [80^{\circ}, 100^{\circ}], [0^{\circ}, 5^{\circ}], [175^{\circ}, 179^{\circ}]$ are masked, and set to zero. After masking, the location of the maximum intensity level of the transform is found. The corresponding projection angle at this location gives θ_s. Once θ_s is found the projection at angle θ_s is cropped from the masked Radon transform and a projection vector is obtained. Figure 4 illustrates the cropping of the projection vector $\mathbf{A}$ at angle θ_s from the masked Radon transform of the Frank code.

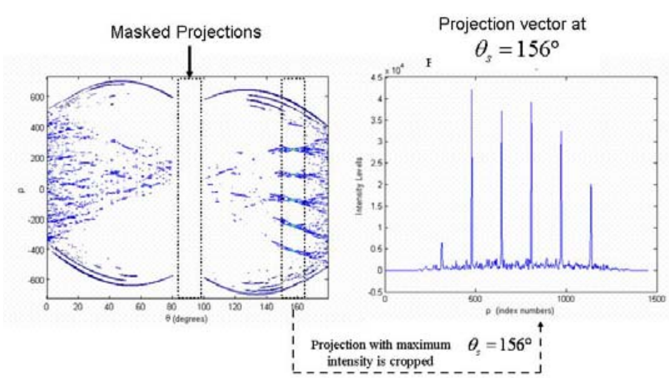


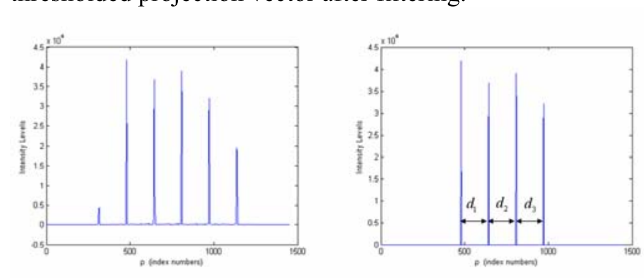
Figure 4. Radon transform and projection cropping on the angle θ_s.

From Figure 4 the number of modulation energy lines contained in the PWVD image (number of code periods intercepted) can easily be detected from both the Radon transform and the projection vector at angle θ_s. The ripples between each modulation energy component correspond to the additive noise and the *cross term* integration at angle θ_s. The projection vector is then smoothed with a Wiener filter

$$b(n) = \mu + \frac{\max\left(\sigma^2 - \nu^2, 0\right)}{\sigma^2}(\mathbf{A}(n) - \mu) \tag{4}$$

where n is an index into the local neighborhood of size η, μ is the estimated local mean, σ^2 is the estimated local variance and ν^2 is the estimated noise variance obtained by using the average of all the estimated local variances. A local neighborhood of $\eta = 10$ is used in the adaptive filter.

Following smoothing, the projection vector is thresholded with a threshold equal to one half of the maximum value of the projection vector. Figure 5(a) shows the filtered projection vector and Figure 5(b) shows the thresholded projection vector after filtering.



(a) (b)

Figure 5. (a) Filtered projection vector and (b) thresholded projection vector after filtering.

After thresholding several distances can be found between the nonzero values in the projection vector which correspond to the consecutive modulation energy components. The final distance d (pixels) can be determined by finding the mean value of these distances. In Figure 3, once d is found the modulation code period T can now be found [4]:

$$T = -\frac{1}{f_s}\left(\frac{d}{\cos(\theta_s)}\right) \tag{5}$$

and the bandwidth B can be found:

$$B = \Delta f\left(\frac{d}{\cos(\theta_s)}\right) / \tan(\theta_s) \tag{6}$$

where Δf is the frequency resolution of the PWVD image [4]. Note that (5) is not applied to P2 coded signals since the modulation has an opposite T-F slope. For P2 code modulation, the following relationship applies:

$$T = \frac{1}{f_s}\left(\frac{d}{\cos\theta_s}\right) \tag{7}$$

Once f_c, T and B are obtained the code length N_c can be found using $N_c = T \times B$ and the number of carrier frequency cycles per sub-code *cpp* can be obtained using $cpp = f_c / B$.

4. DATABASE DESCRIPTION

The parameter extraction algorithm is tested with 6 signals for the Frank, P1, P2, P3 and P4 as shown in Table

1. The parameters used to generate the polyphase modulations are $f_s = 7000$ Hz, $f_c = 1495$ Hz (signals 1 to 3), $f_c = 2495$ Hz (signals 4 to 6), $N_c = 9$, 16, 25, 36, $cpp = 1$, 2, 3, 4, 5, 6 and SNRs of 0 dB and -6 dB. Note that the modulation bandwidths range from 299 Hz to 1495 Hz [4].

	Signal #	Carrier Frequency	Number of Subcodes	Carrier Cycles per Subcode	Code Period	Bandwidth
Frank	1	1495	9	5	0.0301	299.00
	2	2195	16	4	0.0292	548.75
	3	2195	16	5	0.0364	439.00
	4	1495	25	2	0.0334	747.50
	5	2195	25	3	0.0342	731.67
	6	1495	36	1	0.0241	1495.00
P1	1	1495	9	4	0.0241	373.75
	2	1495	9	5	0.0301	299.00
	3	2195	16	4	0.0292	548.75
	4	2195	16	5	0.0364	439.00
	5	2195	16	6	0.0437	365.83
	6	1495	25	2	0.0334	747.50
P2	1	1495	16	2	0.0214	747.50
	2	1495	16	3	0.0321	498.33
	3	2195	16	4	0.0292	548.75
	4	2195	16	5	0.0364	439.00
	5	1495	36	1	0.0241	1495.00
	6	2195	36	3	0.0492	731.67
P3	1	1495	9	4	0.0241	373.75
	2	1495	9	5	0.0301	299.00
	3	2195	16	4	0.0292	548.75
	4	2195	16	6	0.0437	365.83
	5	1495	25	2	0.0334	747.50
	6	2195	25	3	0.0342	731.67
P4	1	1495	9	4	0.0241	373.75
	2	1495	9	5	0.0301	299.00
	3	2195	16	4	0.0292	548.75
	4	2195	16	6	0.0437	365.83
	5	1495	25	2	0.0334	747.50
	6	2195	25	3	0.0342	731.67

Table 1. Polyphase Modulation Parameters.

5. RESULTS AND CONCLUSION

If a^* is a measurement value of a quantity whose exact value is a, then the absolute value of the relative error ε_r is defined by [1]

$$\varepsilon_r = \left|\frac{a^* - a}{a}\right| = \left|\frac{\text{Error}}{\text{True value}}\right| \tag{9}$$

The absolute value of the relative error is plotted for each of the extracted parameters in Figure 6 for the modulations under test. The carrier frequency error is very small for 0 dB but for -6 dB higher errors occur for small values of N_c. If the frequency resolution of the PWVD is increased (integration of more samples from the ADC), the error in estimating f_c decreases. The error in the estimation of N_c is related to T and B since $N_c = T \times B$. The overall errors are reasonably small for 0 dB. For SNR = -6 dB, note that the largest errors occur for $N_c = 9{,}16$. That is, the simulation shows the important result that for smaller values of SNR, the error in the extracted parameters are smaller for larger values of N_c. This is due to the larger processing gain obtained with larger numbers of sub-codes. Note that another advantage to our approach is that the extraction algorithm is not affected from the cross terms present within the PWVD images. The reason is that integration of the cross term projections is very small compared to the modulation projections.

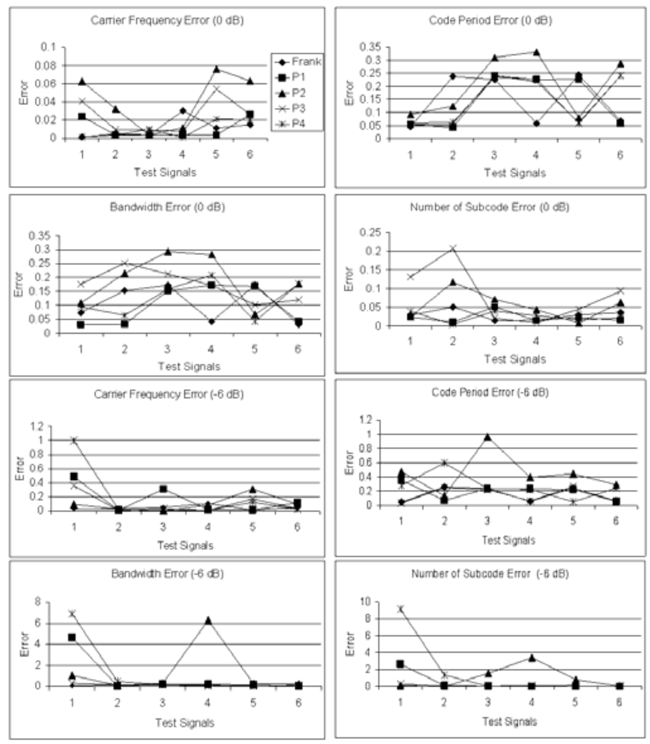


Figure 6. Relative Error.

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