RADAR TARGET DOA ESTIMATION: MOVING WINDOW VS AML ESTIMATOR

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ABSTRACT

In this paper we compare two radar target direction-of-arrival (DOA) estimation algorithms, the classical moving window (MW) and the asymptotic maximum likelihood (AML) estimators. The first technique for azimuth DOA estimation exploits multiple detections in the same time-on-target and the second one exploits the fact that the radar antenna mechanical scanning impresses an amplitude modulation on the signals backscattered by the target. Performances of the estimators are numerically investigated through Monte Carlo simulation in terms of root-mean-square-error (RMSE), probability of detection for a fixed probability of false alarm, and probability of "splitting". The obtained results show that the asymptotic maximum likelihood estimator generally outperforms the classical moving window estimator.

Index Terms— Radar signal processing, direction of arrival estimation

1. INTRODUCTION

The estimation of target direction of arrival is not new in radar literature, in both tracking and searching radars. One of the most common techniques is the monopulse [1] which in principle can work with just a single pulse using for azimuth estimation two tightly matched receiving channels: the sum and the difference channels.

In this work we compare two multi-pulse algorithms that work with only one channel, the moving window [1], often implemented in commercial radars, and the more modern asymptotically maximum likelihood estimator [2] with the aim of measuring vantages and advantages of both techniques. The first technique for azimuth DOA estimation exploits multiple detections in the same *time-on-target*, and the second one exploits the knowledge of the antenna main beam pattern and the fact that the antenna mechanical scanning impresses an amplitude modulation on the signals backscattered by the targets.

The paper is organized as follows. After this short introduction, in Section 2 the data model and the problem statement are presented and explained. In Section 3 and 4 the MW and the AML estimators are summarized. Section 5 shows some results and draws some conclusions.

2. DATA MODEL AND PROBLEM STATEMENT

Consider a radar antenna which rotates mechanically with constant angular velocity ω_R *rad/s* and denote by $h(\theta)$ the one-way antenna beam pattern, by G_0 the maximum gain, and by θ_B the -3 dB azimuth beam width, i.e. the angle such that $h^2(\pm\theta_B/2) = G_0/2$. The number N of pulses collected during the time-on-target (*ToT*) by the radar within the -3 dB points is given by $N = \theta_B/(\omega_R T)$, where T=1/*PRF* is the radar pulse repetition time (*PRT*) and *PRF* is the pulse repetition frequency. The number N of pulses can be changed by changing the angular velocity ω_R of the rotating antenna or the *PRF* value.

Assume that M point-like targets, with direction of arrivals $\{\theta_{TGi}\}_{i=1}^{M}$ and Doppler frequencies $\{f_{Di}\}_{i=1}^{M}$, are present in the same range-azimuth resolution cell under test. The data vector $\mathbf{z}$ is composed by the collection of the N echoes received during the *ToT*. The nth element of $\mathbf{z}$ is given by

$$z(n) = \sum_{i=1}^{M} b_i G(n,\theta_{TGi})\, e^{j2\pi f_{Di} n} + d(n)\,,\ n = 0,1,\ldots N-1\,, \quad (1)$$

where b_i is the unknown deterministic complex amplitude of the ith target signal, $G(n,\theta_{TGi}) = h^2(\theta_{TGi} - n\omega_R T)$ is the two-way antenna gain for the nth pulse from the ith DOA, $\theta_{TGi} \in [0,\theta_B)$,[1] and $f_{Di} \in (-0.5, 0.5)$ is the Doppler frequency of the ith target normalized to the *PRF*. The term $d(n)$ models the disturbance that, in general can be composed of thermal noise and clutter. In this work we suppose that $d(n)$ is only white Gaussian noise.

In vector notation, the data model for M targets is given by

[1] The assumption that $\theta_{TGi} \in [0,\theta_B)$ is equivalent to assume that no contaminating targets are present in adjacent azimuth cells.

$$\mathbf{z} = \mathbf{A}(\theta)\mathbf{b} + \mathbf{d}\,, \tag{2}$$

where $\mathbf{z} = [z(0) \ \cdots \ z(N-1)]^T$ is the $N \times 1$ complex data vector, $\mathbf{b} = [b_1\, b_2 \cdots b_M]^T$ is the $M \times 1$ vector of the unknown complex amplitudes, $[\cdot]^T$ denotes the transpose operation, $\mathbf{A}(\theta) = [\mathbf{a}(\theta_{TG1}, f_{D1}) \ \ \mathbf{a}(\theta_{TG2}, f_{D2}) \ \ \cdots \ \ \mathbf{a}(\theta_{TGM}, f_{DM})]$ is the $N \times M$ steering matrix, $\theta = [\theta_{TG1} \cdots \theta_{TGM}\ f_{D1} \cdots f_{DM}]^T$ is the $2M \times 1$ vector of the unknown DOAs and Doppler frequencies, $\mathbf{a}(\theta_{TGi}, f_{Di})$ is an $N \times 1$ vector that can be factored as $\mathbf{a}(\theta_{TGi}, f_{Di}) = \mathbf{g}(\theta_{TGi}) \odot \mathbf{p}(f_{Di})$, whose elements are given by, for $n = 0, 1, \ldots, N-1$:

$$[\mathbf{a}(\theta_{TGi}, f_{Di})]_n = [\mathbf{g}(\theta_{TGi})]_n [\mathbf{p}(f_{Di})]_n = G(n, \theta_{TGi})\, e^{j2\pi f_{Di} n} \tag{3}$$

where $\odot$ represents the Hadamard product or element-wise multiplication, $[\mathbf{g}(\theta_{TGi})]_n = G(n, \theta_{TGi})$ and $[\mathbf{p}(f_{Di})]_n = e^{j2\pi f_{Di} n}$. Note that $\mathbf{g}(\theta_{TGi})$ depends only on θ_{TGi}, whereas $\mathbf{p}(f_{Di})$ is only function of f_{Di}.

The $N \times 1$ disturbance vector $\mathbf{d}$ is modeled as a complex zero-mean white Gaussian vector. In shorthand notation, we write $\mathbf{d} \sim CN(\mathbf{0}, \sigma_n^2 \mathbf{I})$, where σ_n^2 is the variance of each noise component and $\mathbf{I}$ is the $N \times N$ identity matrix.

3. THE MOVING WINDOW

The scheme of the moving window is shown in Figure 1 [1]. The echoes collected by the radar during the scanning in the *ToT* are saved for each range cell into a shift register (SR) (first line in the scheme). The amplitude of each echo is compared with a first threshold η, that is

$$|z(n)| \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{4}$$

In this way the string of amplitudes is converted in a string of bits "0" and "1", "1" when the threshold is overcome, "0" conversely (second line in the scheme). Last *N* bits of this string are added and compared with a second threshold *K*. A target is declared present if this second integer threshold is overcome (*K* out of *N* detection rule).

The threshold η, related to the probability of false alarm P_{FA}^s on the single pulse, and *K* are chosen such that the overall probability of false alarm P_{FA} after the pulse integration assumes some desired value P_{FA}^*. Different combinations of η and *K* can provide the same P_{FA}^*. For maximizing the detection probability, often the second threshold is chosen such that $K = N/2$ [1]

It is easy to verify that, if the probability of false alarm on the single pulse is $P_{FA}^s = P_0$, the P_{FA} after integration is

$$P_{FA} = \sum_{j=K}^{N} \binom{N}{j} P_0^{\,j} (1-P_0)^{N-j}\,. \tag{5}$$

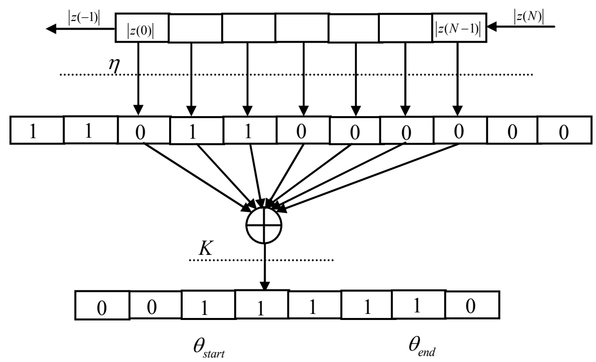

Fig.1 - Moving window scheme

All the detections after the second threshold are saved in a second string[2] (last line in the scheme) that is used by the radar processor for the target DOA estimation.

Let's define θ_{start} as the angular position corresponding to the first detection (first bit 1 in the second string) and θ_{end} as the angular position of the last detection. The target DOA is estimated as [1]

$$\hat{\theta}_{TMW} = \frac{\theta_{start} + \theta_{end}}{2} \tag{6}$$

In picking θ_{start} and θ_{end} in the string, the processor checks for the continuity of the 1s. If more than two consecutive zeroes are present, the estimation is not done and a "split" is declared. The split corresponds to a possible presence of more than one target in the same range-azimuth cell.

4. THE AML ESTIMATOR

A detailed theoretical derivation and analysis of the asymptotic maximum likelihood (AML) estimator of target DOA and Doppler frequencies[3] for the data model at hand (eq. 2) has been presented in [2] and [4]. Here we summarize some results. The AML estimator for M targets can be found by solving the following maximization

[2] Bit 1 for detection, bit 0 for the alternative hypothesis.

[3] The moving window estimates only the DOA, does not estimate also the Doppler shift of the target.

problem with respect to the vector of unknowns $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_M, f_1, f_2, \cdots, f_M)^T$:

$$\hat{\boldsymbol{\theta}}_{AML} = \underset{(\theta_1, \theta_2, \cdots, \theta_M, f_1, f_2, \cdots, f_M)}{\arg\max} \sum_{i=1}^{M} \Gamma_N(\theta_i, f_i) \quad (7)$$

where in our case, $\Gamma_N(\theta_i, f_i) = \left|\mathbf{z}^H \mathbf{a}(\theta_i, f_i)\right|^2 \Big/ \left\|\mathbf{a}(\theta_i, f_i)\right\|^2$. Observe that $\Gamma_N(\theta, f)$ is the functional that must be maximized when the radar knows that only one target is present in the same range-azimuth cell.

In the scenario that we analyze, we simulate only one target but we suppose that the radar does not know the value of M. It knows only that the maximum number of targets is M=2, then in eq. (7) the vectors of unknowns is $\boldsymbol{\theta} = (\theta_1, \theta_2, f_1, f_2)^T$. For estimating the number of targets actually present in the cell under test (0, 1 or 2) the radar performs a successive hypothesis test (SHT) as in [3]. The successive hypotheses test is a procedure for model order selection which tests a set of mutually exclusive hypotheses H_m and alternatives K_m. At step m the SHT procedure tests the hypothesis H_{m-1}, "*There are* $m-1$ *targets*" against the alternative K_{m-1}, "*There are* m *targets*", by comparing a certain test statistic $S_m(\mathbf{z})$ with a certain threshold λ_m, In this work, as test statistic $S_m(\mathbf{z})$, we adopt the generalized likelihood ratio test (GLRT) [3]. Then, the test at step m is given by

$$S_m(\mathbf{z}) = 2\ln L_{G,m}(\mathbf{z}) = 2\ln\left\{\frac{p_{\mathbf{z}|K_{m-1}}(\mathbf{z}|K_{m-1}; \hat{\boldsymbol{\theta}}_m)}{p_{\mathbf{z}|H_{m-1}}(\mathbf{z}|H_{m-1}; \hat{\boldsymbol{\theta}}_{m-1})}\right\} \underset{H_{m-1}}{\overset{K_{m-1}}{\gtrless}} \lambda_m \quad (8)$$

where $L_{G,m}(\mathbf{z})$ denotes the generalized likelihood ratio (GLR) for hypothesis H_{m-1} and alternative K_{m-1}, $p_{\mathbf{z}|H_m}(\mathbf{z}|H_m; \boldsymbol{\theta}_m,)$ is the data probability density function (pdf) under hypothesis H_m and $\hat{\boldsymbol{\theta}}_m$ is the maximum likelihood (ML) estimate of $\boldsymbol{\theta}_m$ under hypothesis H_m (note that $K_{m-1} \equiv H_m \equiv \{m$ targets are present$\}$) as stated in (8). Therefore, hypotheses $\{H_0, H_1, \cdots, H_{M_{\max}-1}\}$ are tested in sequence, going to the next one only if previous hypotheses have been rejected. The procedure stops the first time the statistic does not exceed the threshold or when the number m of hypothesized targets reaches the maximum value $M_{\max}$ (at step $M_{\max}$). If the procedure stops at step m we estimate $\hat{M} = m-1$, otherwise $\hat{M} = M_{\max}$. In our simulation $M_{\max} = 2$. The thresholds $\{\lambda_m\}_{m=1}^{2}$ are selected in order to provide the desired P_{FA} at the end of the entire detection procedure. In radar detection applications it is essential to control the probability of false alarm. The great advantage of using an SHT approach relies on the possibility of upper bounding P_{FA}. In fact, if all the thresholds $\{\lambda_m\}$ are selected to provide the same local probability of false alarm α at each step, i.e. $P_{FA}(m) = \Pr\{S_m(\mathbf{z}) > \lambda_m | H_{m-1}\} = \alpha$, then the probability of false alarm of the entire procedure, $P_{FA} = \Pr\{\hat{M} > M\}$, is upper bounded by α, i.e. $P_{FA} \le \alpha$. This property makes the SHT procedure extremely appealing, but at the same time, difficult to implement. In fact, to select λ_m, it is necessary to know the pdf of $S_m(\mathbf{z})$ conditioned on the hypothesis H_{m-1}. Unfortunately, this pdf is not available or very difficult to compute in closed form. Therefore, we resort to large sample-size analysis, both to derive the statistics of the tests $\{S_m(\mathbf{z})\}$ that can be calculated in real-time with affordable computational complexity, and to properly set the thresholds $\{\lambda_m\}$. Details on this derivation are in [3].

5. NUMERICAL RESULTS AND CONCLUSIONS

In the simulation the pattern of the antenna array has been chosen as in [5]

$$T_{2N_e} = \begin{cases} \cos\left(2N_e \cos^{-1}(u)\right) & -1 \le u \le 1 \\ \cosh\left(2N_e \cosh^{-1}(u)\right) & 1 \le |u| \end{cases} \quad (9)$$

where $2N_e$+1 is the number of array elements that depends on the beamwidth of the antenna, $u = u_0 \cos(\psi/2)$, $\psi = 2\pi d \sin(\theta)/\lambda$, d is the distance between two elements of the array, λ is the radar wavelength. The value of u_0 depends on the side lobe level of the antenna pattern, that is $u_0 = \cosh\left(\frac{1}{2N_e}\ln\left[\eta + \sqrt{\eta^2+1}\right]\right)$, where $\eta = 10^{SLL/20}$, and SLL is the side lobe level in dB.

Due to the two-way antenna pattern in eq. (1) we have

$$\begin{aligned} G(n, \theta_{TG}) &= T_{2N_e}^2(n, \theta_{TG}) \\ &= \cos^2\left(2N_e \cos^{-1}\left(u_o \cos\left(\pi \frac{d}{\lambda}\sin\left(\theta_{TG} - n\omega_R T\right)\right)\right)\right) \end{aligned} \quad (10)$$

When only one target is present the observed signal is

$$z(n) = bG(n, \theta_{TG})e^{j2\pi f_D n} + d(n), \; n = 0, 1, \ldots N-1, \quad (11)$$

The parameters of the analyzed scenario are summarized in Table 1. The signal-to-noise ratio is defined as $SNR = b^2/\sigma_n^2$.

Target Doppler shift	f_D =0.3
Number of pulses per cell	N=28
Second threshold of MW	$K=14$
Signal-to-noise ratio	SNR=30 dB
Antenna beamwidth	$\theta_B = 3^{\circ}$
Sidelobe level	SLL=30 dB
Probability of false alarm	$P_{FA} = 10^{-3}$
DOA of target	$\theta_{TG} = 0.5^{\circ}$

Table 1 – Parameters of analyzed scenario.

In the figures 2-4 the DOA of the target has been set to $\theta_{TG} = 0.5^{\circ}$, but we obtained very similar results for other angular positions in the antenna mainbeam.

In figure 2 we report the RMSE of the DOA for both estimators as a function of the signal-to-disturbance ratio. It is evident that the AML estimator always outperforms the MW, particularly for high *SNR*. It is important to observe that the estimation is performed by both algorithms only when at least one target is detected. The probability of detection as a function of *SNR* is shown in Figure 3. In that sense the performances of MW and AML algorithms are very similar.

Figure 4 shows the probability of splitting. For the MW a split corresponds to the presence of at least two consecutive zeroes in the string of detections, for the AML it corresponds to a declaration by the successive hypothesis test that the targets present in the cell under test are two instead of only one. It is evident from Figure 4 that the AML algorithm is more robust to the target splitting than the MW.

Based on our results we can conclude that the AML estimator always outperforms the MW at the cost of more complex signal processing devices and a higher number of operations.

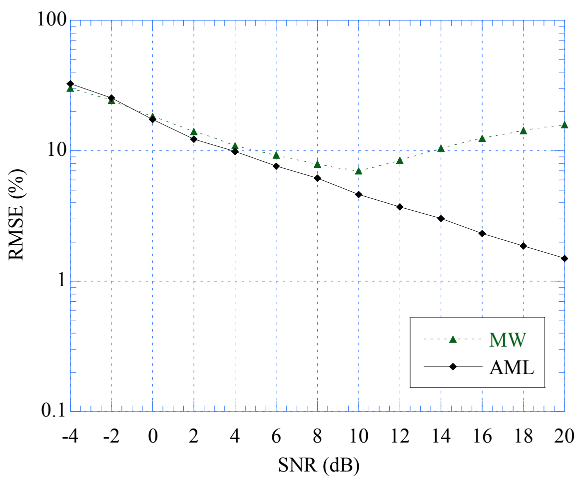

Figure 2 – Root-Mean-Square Error of the MW and AML estimators.

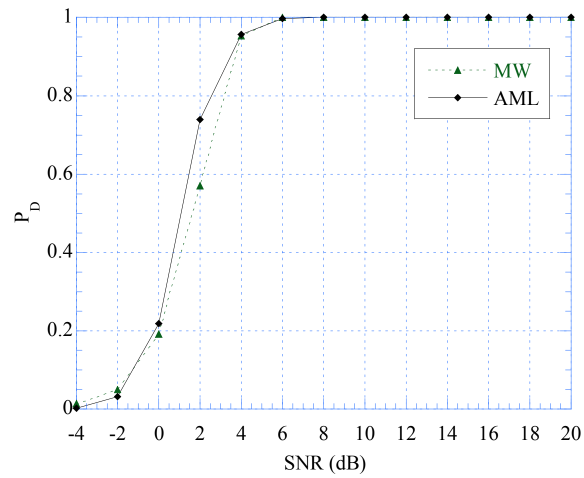

Figure 3 – Probability of detection of the MW and AML techniques, P_{FA}=10^{-3}.

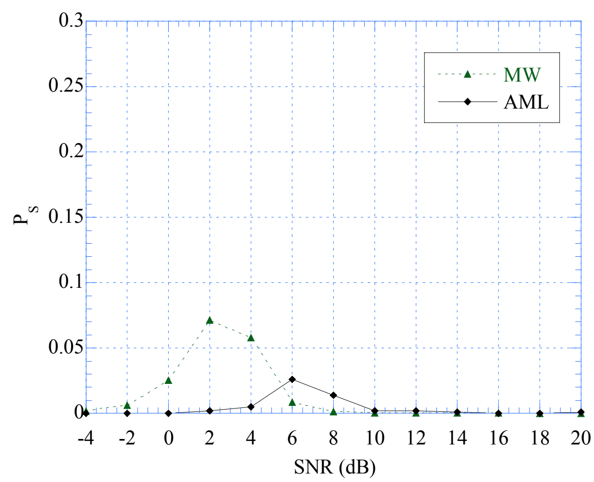

Figure 4 – Probability of splitting of the MW and AML techniques, P_{FA}=10^{-3}.

6. REFERENCES

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