APPLICATION OF DOPPLER RESILIENT COMPLEMENTARY WAVEFORMS TO TARGET TRACKING

*Sofia Suvorova**, *Bill Moran**, *Stephen Howard*†, *Robert Calderbank*‡

(*) Melbourne University, (†) Defence Science and Technology Organization, (‡) Princeton University

ABSTRACT

The use of complementary codes as a means of reducing radar range sidelobes is well-known, but lack of resilience to Doppler is often cited as a reason not to deploy them. This work describes techniques for providing Doppler resilience with an emphasis on tailoring Doppler performance to the specific aim of target tracking. The Doppler performance can be varied by suitably changing the order of transmission of multiple sets of complementary waveforms. We have developed a method that improves Doppler performance significantly by arranging the transmission of multiple copies of complementary waveforms according to the first order Reed-Müller codes. Here we demonstrate significant tracking gains in the context of accelerating targets by the use of adaptively chosen waveform sequences of this kind, compared to both a fixed sequence of similar waveforms, and an LFM waveform.

Index Terms— Doppler tolerance, complementary waveforms, radar, tracking

1. INTRODUCTION

An important problem in radar performance, especially in the context of multi-target detection and tracking, and in clutter-limited situations, is the presence of sidelobes in the auto-correlation of the illuminating waveform. Pulse compression methods in radar require the use of waveforms with low sidelobes, but the mathematics of the ambiguity function prevent this. For single waveforms, sidelobes must exist. From a radar performance viewpoint, this means that targets of interest can be hidden in the sidelobes associated with a nearby strong reflector. Complementary waveforms provide the possibility of zero range sidelobes, at least in theory. When these are transmitted separately their summed auto-correlations, at zero Doppler, form a true "thumbtack" response with no sidelobes. Unfortunately this does not remain true off the zero Doppler axis. The presence of delay (range) sidelobes off the zero-Doppler axis is a significant problem in the deployment of complementary waveform techniques. We remark that suitable separation of the received waveforms is crucial to the complementary waveform technique. In our application the waveforms are transmitted on time separated pulses. A discussion of frequency separation is given in [**?**]. We show that, by careful design, of the order of the train of complementary pulses it is possible to achieve superior range sidelobe performance in the presence of moving targets.

In section **??** we briefly introduce Golay sequences and their performance in terms of radar range sidelobes. This topic is taken up in more detail in, for example [**?**]. We also define first order Reed-Müller codes and give the formulae for the construction of such codes. Section **??** discusses the problems associated with the use of complementary sequences in radar applications. In particular, the problem of range sidelobes introduced by the doppler shift of the target, when the complementary waveforms are transmitted in a time separated way. The section shows the way to mitigate these sidelobes, by transmitting the pulses in a particular pulse train, given by a binary sequence with special properties. We give an algorithm for choosing the optimum Reed-Müller sequence in [**?**]. Section **??** presents an example of scheduling of the Reed-Müller sequence in an application to a single target tracking. In this example we demonstrate significant improvement in performance compared to the non-scheduled case.

2. GOLAY COMPLEMENTARY SEQUENCES

A complementary pair of sequences satisfies the property that the sum of the out-of-phase auto-correlation coefficients is zero. Let $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1})$ be a sequence of length N (usually a power of 2) such that $x_i \in \{+1, \ -1\}$. Define the auto-correlation function of x by

$$X(k) = \sum_{i=0}^{N-k-1} x_i x_{i+k}, \qquad 0 \le k \le N-1. \tag{1}$$

Let $\mathbf{y}$ be another such sequence, and Y the corresponding auto-correlation function. The pair $(\mathbf{x}, \mathbf{y})$ is called a *Golay complementary pair* if

$$X + Y = 2N\delta(k), \tag{2}$$

where δ denotes the Kronecker delta function: $\delta(k) = 1$ if $k = 0$ and zero otherwise. Each member of the Golay complementary pair is called a *Golay complementary sequence*. Implementation in a radar context involves transmission separately on, say, time separated pulses, and the addition of the match-filtered returns to provide essentially zero range sidelobes.

We will also need to recall Walsh matrices. The Walsh matrices H_n of order n; that is, of size $2^n \times 2^n$, are constructed by induction as follows:

$$H_1 = \begin{pmatrix}1\end{pmatrix},$$

$$H_2 = \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix},$$

and

$$H_k = \begin{pmatrix}H_{k-1} & H_{k-1}\\ H_{k-1} & -H_{k-1}\end{pmatrix} = H_2 \otimes H_{k-1},$$

for $2 \leq k \in N$, where $\otimes$ denotes the Kronecker product. The rows of the Walsh matrix H_n can be viewed as a length 2^n linear error-correcting first order Reed-Müller code of rank n. The Walsh matrix can also be obtained by defining the element in the kth row and mth column of H as

$$h(k,m) = (-1)^{\sum_{i=0}^{n-1} k_i m_i} = \prod_{i=0}^{n-1} (-1)^{k_i m_i}$$

where $\{k_0, k_1, \ldots, k_{n-1}\}$ and $\{m_0, m_1, \ldots, m_{n-1}\}$ are the binary representations of k and m, respectively.

3. DOPPLER TOLERANCE

Let $\mathbf{x}_0$ and $\mathbf{x}_1$ be a complementary pair of waveforms with auto-correlation functions X_0 or X_1, respectively. The two are pulses modulated onto a carrier. They are then transmitted with a time separation, that is, pulse repetition interval (PRI), of ΔT, in such a way that the phase information from the carrier is retained. It is enough for our current purposes to consider the case of a single scatterer at delay τ and Doppler frequency ϕ "at baseband" (without inclusion of the carrier term $\exp 2\pi i f_c t$). Here $\phi = \frac{2v}{\lambda}$, where v is the target velocity and λ is the wavelength. The returns from the first and the second pulses are then

$$\mathbf{x}_{n-1}(t-\tau) e^{2\pi\phi(t+(n-1)\Delta T)} \tag{3}$$

with an appropriate multiplicative factor to account for the decay due to range and the reflectivity of the scatterer. We can assume that ϕ is small enough so that $e^{2\pi\phi t}$ is almost constant for the pulse duration t, which is typically a lot smaller than ΔT. At this point the return signal is filtered against the transmit signal. From a mathematical point of view this results in the following match-filtered response:

$$\int_{\mathbb{R}} \mathbf{x}_{n-1}(t'-\tau) e^{2\pi i\phi(t'+(n-1)\Delta T)} \overline{\mathbf{x}_{n-1}(t'-t)} dt', \tag{4}$$

where $\bar{\cdot}$ denotes the combination of complex conjugation and time reversal. Ignoring the $e^{2\pi\phi t}$ term, we obtain an approximation to the match-filtered return,

$$e^{2\pi i\phi(n-1)\Delta T} X_{n-1}(t-\tau), \tag{5}$$

where X_{n-1} is the auto-correlation function of $\mathbf{x}_{n-1}$. The sum of the match-filtered returns for the two pulses takes the form

$$F(\theta) = X_0 + \exp(i\theta) X_1 \tag{6}$$

where $\theta = 2\pi\phi\Delta T$ is the Doppler offset.

The inter-pulse time ΔT is assumed long enough that most of the return from a given transmit pulse arrives back within the subsequent listening period.

At zero Doppler we have the desired sum $X_0 + X_1$, which by the Golay property is a delta function, or to be precise, a triangle function of 2 chips width, but this degrades quickly as $|\theta|$ increases.

To mitigate the effects of the Doppler offset we construct a length 2^M pulse train, represented by a string of ± 1's, where -1's denotes a pulse with auto-correlation function X_0 and 1's a pulse with auto-correlation function X_1. In [**?**], the initial segments of length 2^n of the Prouhet-Thue-Morse pulse train are used to provide a method for choosing the order of the transmission of the waveforms in a time-separated way. That paper showed that, by the correct choice of order, it was possible to significantly reduce the effects of Doppler on the sidelobes of the summed auto-correlation function. In [**?**] we have generalize this result to an arbitrary pulse train and perform the analysis for the first order Reed-Müller code pulse trains.

Consider a binary sequence of -1's and 1's $\mathbf{p} = \{p_n\}, n = 1, \ldots, 2^M$ and the $\mathbf{p}$ pulse train. The sums of auto-correlation functions in this case can be calculated as

$$F_{\mathbf{p}}(\theta) = X_0 \sum_{p_n=-1} \exp(i\theta n) + X_1 \sum_{p_n=1} \exp(i\theta n). \tag{7}$$

For a non-zero θ, after a simple algebraic manipulation, we have

$$\begin{aligned} F_{\mathbf{p}}(\theta) = {} & \frac{1}{2}(X_0 + X_1) \sum_{n=0}^{2^M-1} \exp(i\theta n) \\ & + \frac{1}{2}(X_1 - X_0) \sum_{n=0}^{2^M-1} p_n \exp(i\theta n). \end{aligned} \tag{8}$$

The first term of Eq. (**??**) represents the main lobe and for a Golay pair of waveforms is equal to

$$N\delta(t) e^{(2^{M-1}-1)i\theta} \frac{\sin(2^{M-1}\theta)}{\sin(\theta/2)}. \tag{9}$$

The second term of Eq. (**??**) represents the range sidelobes of the ambiguity function for non-zero Doppler. Observe that the size of the second term depends on the spectrum of the sequence $\mathbf{p}$. It is also clear from Parseval's theorem, that the spectrum has finite positive energy and cannot be zero everywhere. Instead, the shape of the spectrum can, to some degree, be tailored to be small at a particular θ, resulting in range sidelobe reduction for a particular Doppler.

In [?] we showed how to select Reed-Müller pulse train to minimize the ambiguity sidelobes for a given Doppler frequency θ.

4. TARGET TRACKING USING RM$(1, M)$ PULSE TRAINS

Here we present simulation results of a target tracking application of the ideas described above, where the pulse trains are scheduled on the basis of the prior state estimate of the target. Our simulations are based on an X-band radar simulation with 10cm wavelength, and 100 μsec PRI. Each pulse consists of 64 chips, each of length 10 nanoseconds. We are able to demonstrate significant gains in tracking performance by the use of scheduling.

We state the tracking problem as follows. Our aim is to track an accelerating target, evolving at discrete epochs according to dynamic equation

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \omega, \tag{10}$$

where $x = \begin{pmatrix} r & \dot{r} & \ddot{r} \end{pmatrix}^T$, r is range, $\dot{r}$ is range rate, $\ddot{r}$ is acceleration and $\mathbf{F} = \begin{pmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix}$, T is the time interval between epochs and ω is zero mean Gaussian noise with covariance matrix $\mathbf{Q}$. The target is illuminated by a radar, which transmits an appropriate pulse train of a pair of complementary waveforms. We assume, for simplicity, that there is a constant variance (uniform) Gaussian clutter in each range and Doppler bin. The measurements from the target are given with probability p_d by

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \nu, \tag{11}$$

where $H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and ν is independent zero mean Gaussian noise with covariance matrix $\mathbf{R}$, given by the Hessian of the sum of the ambiguity functions of a complementary pair of waveforms at the main peak [?]. In this example we use a Kalman filter based tracking algorithm. Define by d_k a detection indicator taking the value 1 if the target is detected with probability p_d at time k and 0 otherwise. The prediction is performed in the usual way for most tracking filters, using the target dynamic equation, in our case Eq. (**??**).

$$\begin{aligned} \mathbf{x}(k|k-1) &= \mathbf{F}\mathbf{x}(k-1|k-1) \\ \mathbf{P}(k|k-1) &= \mathbf{F}\mathbf{P}(k-1|k-1)\mathbf{F}^T + \mathbf{Q}, \end{aligned} \tag{12}$$

where $\mathbf{x}(k|k-1)$, $\mathbf{P}(k|k-1)$ and $\mathbf{x}(k-1|k-1)$, $\mathbf{P}(k-1|k-1)$ are prior and posterior state estimate means and covariances. The next step is to schedule the pulse train. In this example we choose the RM$(1, M)$ pulse train which minimizes the sidelobes of the sum of matched filtered returns. This, in effect, maximizes signal-to-clutter ratio (SCR) and therefore p_d in the context of uniform clutter. Assuming that the detection was obtained by a likelihood ratio test realized via, for example, the cell averaging CFAR method, we can write the well known formula for p_d,

$$p_d = 1 - \Phi(\Phi^{-1}(1 - p_{\text{fa}}) - \text{SCR}), \tag{13}$$

where Φ is the Gaussian cumulative distribution function [?]. We next assume that probability of a false alarm p_{fa} is fixed, which is consistent with the uniform model of clutter. From Eq. **??** it is clear that for fixed p_{fa}, maximizing SCR is equivalent to maximizing p_d. To maximize SCR over RM$(1, M)$ codes we use the algorithm of [?].

Next the measurements are collected: these are received from the target with probability p_d. No detection results in no measurement and the detection indicator d_k is set to 0, a detection results in 1 measurement and the detection indicator d_k is set to 1. The tracker state update incorporating these measurements is

$$\begin{aligned} \mathbf{x}(k|k) &= \mathbf{x}(k|k-1) + d_k\mathbf{P}\mathbf{H}^T\mathbf{S}^{-1}(\mathbf{y}_k - \mathbf{H}\mathbf{x}(k|k-1) \\ \mathbf{P}(k|k) &= \mathbf{P}(k|k-1) - d_k\mathbf{P}(k|k-1)\mathbf{H}^T\mathbf{S}^{-1}\mathbf{H}\mathbf{P}(k|k-1), \end{aligned} \tag{14}$$

where $\mathbf{S} = \mathbf{H}\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{R}$. When the target is not detected, i.e $d_k = 0$, the state update is equal to the predicted state. When the target is detected $d_k = 1$, the state update follows the standard Kalman update [?, ?].

Below we presents the result of Monte-Carlo simulations for tracking an accelerating target using three different methods of illumination:

- a pulse train consisting of repetitions of a linear frequency modulated pulse (linear chirp) with 20 MHz bandwidth,
- a PTM pulse train, identical for all epochs and
- epoch-to-epoch scheduled RM$(1, M)$ pulse train based on the current knowledge of the target state, using the algorithm of [?]. The optimum choice of RM$(1, M)$ sequence maximizes the probability of target detection.

The target trajectory is given in Fig. **??** over a period of 50 epochs, each 1 sec long. The measurements were collected by a cell averaging CFAR detection method with the following parameters: 25 dB additive threshold, and 25 range cells before and after the cell of interest. Fig. **??** shows average signal-to-clutter ratio (SCR) at each epoch for the three methods. Although almost independent of the speed of the target, the SCR for an LFM waveform is significantly lower than the SCR for a complementary pair. Fig. **??** shows the number of detections at each run. Notice, that in the scheduled case, p_d is close to 1, but is only around 0.73 for PTM and even lower (0.38) for LFM. Figs. **??** and **??** show position and velocity RMSE for the three methods. Clearly in these experiments, scheduling of measurements gains considerable improvements in accuracy of tracking over an unscheduled methods.

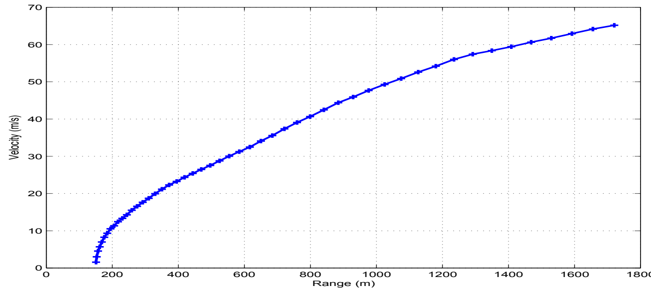

Fig. 1. Target trajectory

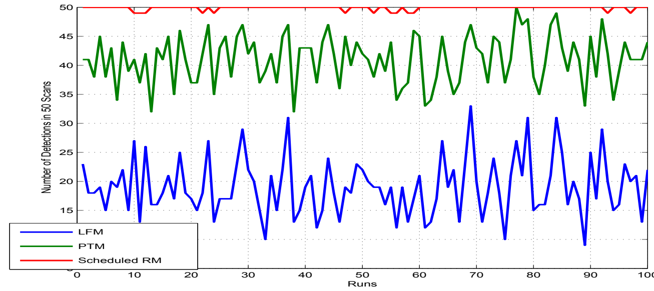

Fig. 3. Number of detections at each simulation run

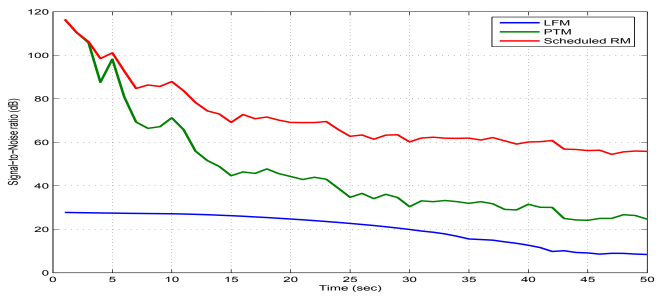

Fig. 2. Average signal-to-noise ratio at each epoch

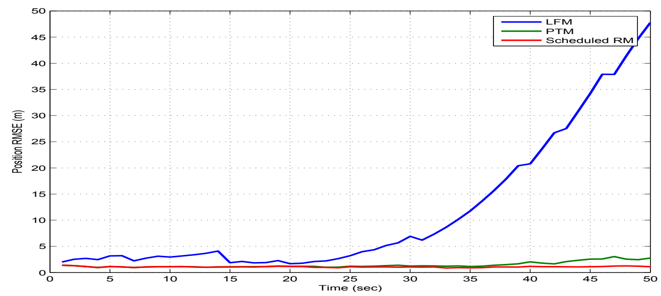

Fig. 4. Target position root mean square error at each epoch

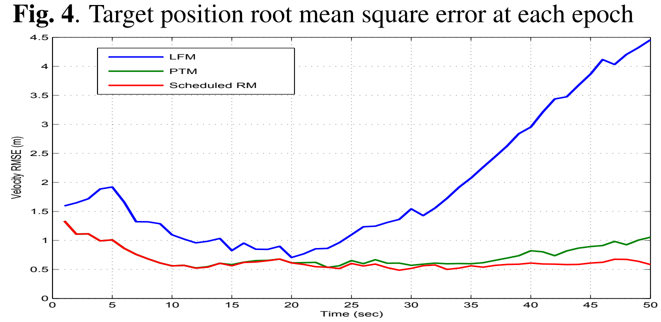

Fig. 5. Target velocity root mean equare error at each epoch

5. CONCLUSION

We have demonstrated a method for scheduling sequences of complementary waveforms to provide Doppler resilience around a particular Doppler bin. This permits the implementation of an adaptive scheme for optimal waveform transmission for detection of a low SNR accelerating target in a clutter-limited environment. This scheme provides significant improvement in target detection and track error over both an unscheduled scheme for Doppler resilience of complementary waveforms, and a conventional LFM illumination scheme.

6. REFERENCES

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