DETECTION CAPABILITIES OF RANDOMLY-DEPLOYED UNDERWATER SENSORS

Jill K. Nelson

George Mason University
Fairfax, VA 22030

Errol G. Rowe, G. Clifford Carter

Naval Undersea Warfare Center
Newport, RI 02841

ABSTRACT

A problem of interest in ocean engineering is the defense of a particular region of the ocean, often referred to as a sea base. Among the primary challenges in defending the sea base is effective and efficient detection of intruders that travel through the space. A recent focus on undersea distributed networked systems (UDNS) has raised the question of how effectively small, randomly distributed sensors can detect such intruders relative to larger sensing platforms. We develop an analytical framework for characterizing the detection performance of a set of sensors deployed within a sea base. Focusing on the detection of a target traveling a straight line, we employ our framework to compare the performance of various sensing approaches as a function of the number of sensors, sensing radius, sensor location, and sensor cost. Our results reveal that, when sensor cost scales with sensing radius, the detection vs. cost trade-off can be improved by employing a larger number of less powerful sensors.

Index Terms— Underwater detection, sensor networks, distributed detection

1. INTRODUCTION

A significant challenge in ocean engineering is protecting the sea lines of communications by defending a strategic region of the ocean, which we refer to here as a sea base. The first line of defense is the sensors and associated signal processing that perform detection, classification, localization, and tracking (DCLT). Traditionally, DCLT tasks have been performed by large, expensive sensor platforms (costing on the order of one billion 2008 U.S. dollars) with long detection ranges and sophisticated signal processing capabilities. Recent advances in the detection capabilities of networks composed of small, low-cost wireless sensors [1, 2] have raised the possibility of similar approaches in an underwater environment. These approaches, called undersea distributed networked systems (UDNS), are becoming a research topic of interest in the underwater community [3].

Much of the research pursued thus far has focused on the detection capabilities of sparsely distributed sensors whose sensing regions are assumed not to overlap [4]. In [5], for example, Traweek and Wettergren characterize the sensing range that optimizes the detection vs. cost trade-off in a sparse sensor field. In contrast to previous work, we consider how to choose the number and sophistication of sensors to maximize detection performance when sensor placement is random and the region of interest is densely populated. Such a model is particularly relevant when small, low-cost sensors are used, since deployment may be as simple as dropping sensors from a moving platform. In [6], Rowe showed how to estimate the coverage provided by models of this type. Once deployed in the ocean environment, floating sensors will be relocated by waves, tides, and other ocean forces. In our work, we explore detection performance as a function of both the number of sensors deployed and their individual detection radii. We are interested in exploring the performance of such systems in the limit as the number of sensors grows and their respective sophistication/cost decreases.

2. SYSTEM MODEL

We consider sensors that have a disc-shaped detection region with sensing radius R. A sensor is assumed to detect a moving target if the target's path intersects the sensing region of that sensor. We model the region of interest (or sea base) as a square with sides of length $2U$. We employ a cartesian coordinate system in which the sea base is centered at $(x, y) = (0, 0)$. In our analysis, we study the likelihood that various sensors or sets of sensors detect a target that travels in a straight line through the sea base. Without loss of generality, we assume that the target enters the sea base through the lower edge of the square. The target enters at point $(u, -U)$ and with angle θ, where $u \in [-U, U]$, and $\theta \in [0, \pi]$. An example trajectory through the sea base is shown in Figure 1. We model both the point u at which the target enters the sea base and the angle θ with which it enters as uniformly distributed random variables with $u \in [-U, U]$ and $\theta \in [0, \pi]$.

3. LIKELIHOOD OF DETECTION USING A SINGLE RANDOMLY-PLACED SENSOR

Consider the likelihood that a single, randomly-placed sensor with sensing radius R will detect a target moving in a straight line through the sea base. Using a geometric approach, we

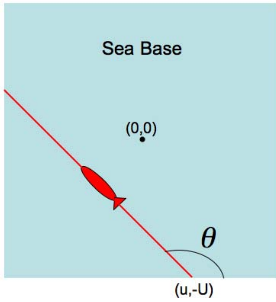

Fig. 1. An example target trajectory through the sea base is shown. The target enters at point $(u, -U)$ on the lower edge of the square and with angle θ relative to the horizontal.

compute the likelihood that the sensor will detect a moving target that enters the sea base at a certain point u and with angle θ. To obtain a general expression for detection likelihood, we then average over both u and θ.

Let the sensor be centered at (x_c, y_c), where $x_c \in [-U, U]$ and $y_c \in [-U, U]$. For a target entering the seabase at point $(u, -U)$ and with angle θ, the equation of the line traveled by the target is given by $y = x\tan\theta - U - u\tan\theta$. The probability of detection (P_d) across all sensor locations can be written as

$$P_{d|\theta,u} = \int_{x_c}\int_{y_c} P_{d|(x_c,y_c),\theta,u}\ f_{x_c,y_c}(x_c, y_c)\ dx_c dy_c.$$

The probability of detection for a sensor centered at (x_c, y_c) is given by

$$P_{d|(x_c,y_c),\theta,u} = \begin{cases} 1, & d_{min}(x_c, y_c, u, \theta) \leq R \\ 0, & d_{min}(x_c, y_c, u, \theta) > R \end{cases},$$

where $d_{min}(x_c, y_c, u, \theta)$ denotes the minimum distance between the sensor center and the closest point on the line traveled by the target. Assuming the sensor location is uniformly distributed across the square, we can compute P_d for a given u and θ as

$$P_{d|\theta,u} = \frac{1}{4U^2}\int_A dx_c dy_c,$$
$$A = \{(x_c, y_c) : d_{min} \leq R, x \in [-U, U], y \in [-U, U]\}.$$

The closest point on the target path to the sensor center is the solution to $\frac{d}{dx}\left(x - x_c\right)^2 + \left(y(x) - y_c\right)^2 = 0$, which is given by

$$x = \frac{x_c + y_c\tan\theta + U\tan\theta + u\tan^2\theta}{1 + \tan^2\theta}, \quad \text{and}$$
$$y = \frac{x_c\tan\theta + y_c\tan^2\theta - U - u\tan\theta}{1 + \tan^2\theta}.$$

The minimum distance between the sensor center and a point on the line traveled by the target is then computed as

$$d_{min} = \sqrt{\left[\frac{my_c - mb - m^2x_c}{1 + m^2}\right]^2 + \left[\frac{mx_c + b - y_c}{1 + m^2}\right]^2},$$

where $m = \tan\theta$ and $b = -U - u\tan\theta$ are the slope and y-intercept, respectively, of the target path. The region of the sea base for which $d_{min} \leq R$ can be expressed in terms of x_c and y_c as

$$mx_c + b - R\sqrt{1 + m^2} \leq y_c \leq mx_c + b + R\sqrt{1 + m^2}.$$

Graphically, the region of interest lies between the target path shifted upward by $R\sqrt{1 + m^2}$ and the target path shifted downward by $R\sqrt{1 + m^2}$. The perpendicular distance between the target path and either of the shifted paths is R. Hence, the result is intuitively pleasing, as it concludes that a sensor whose center is within distance R of the target path will detect the target.

The region of interest is simply the area within the sea base for which $d_{min} \leq R$. The form taken by this region depends upon the point and angle at which the target enters the sea base, as well as the sensing radius. While it is possible to determine the ranges of u, θ, and R that uniquely identify each class and to compute the area of the region of interest in each case, such an approach is tedious and cumbersome. Instead, we employ a simplified approach that, as we will show, yields results that are very similar to empirical results for the range of R in which we are interested. Rather than computing the exact area of the detection region, we compute the area of the rectangle whose longer central axis is given by the target path within the square and whose width is given by $2R$, the perpendicular distance between the shifted paths.

For ease of presentation, let $U = 1$. To compute the length of the target path within the sea base (and hence the approximate area of the detection region), we consider three cases that correspond to the side of the square (right, left, or top) through which the target exits. For each case, the applicable ranges of u and θ, as well as the resulting area of the rectangle, are given below:

1. $0 \leq \theta \leq \tan^{-1}\left(\frac{2}{1-u}\right)$, $A_1 = \frac{2R(1-u)}{\cos\theta}$
2. $\tan^{-1}\left(\frac{2}{1-u}\right) \leq \theta \leq \tan^{-1}\left(\frac{2}{-1-u}\right)$, $A_2 = \frac{4R}{\sin\theta}$
3. $\tan^{-1}\left(\frac{2}{-1-u}\right) \leq \theta \leq \pi$, $A_3 = \frac{-2R(u+1)}{\cos\theta}$

The probability of detection across all u and θ is then given by

$$\begin{aligned} P_d &\approx \int_u\int_\theta P_{d|\theta,u} f(\theta)f(u)d\theta du \\ &= \frac{R}{2\pi}\int_{-1}^{1}\int_{0}^{\tan^{-1}(2/(1-u))}\frac{1-u}{\cos\theta}d\theta du \\ &\quad + \frac{R}{\pi}\int_{-1}^{1}\int_{\tan^{-1}(2/(1-u))}^{\pi/2}\frac{1}{\sin\theta}d\theta du \\ &= \frac{R}{\pi}\left(1 - \sqrt{2} + \cosh^{-1}(3) - \ln\left(\frac{\sqrt{2}}{2+\sqrt{2}}\right)\right) \\ &\approx 0.7098R, \end{aligned}$$

where the second equality employs even symmetry of the integrand about $\theta = \pi/2$. (In general, the approximation is given by $P_d \approx 0.7098R/U$.) The rectangle-based approach yields an approximation for the probability of detection that is linear in the sensing radius R.

To evaluate the accuracy of the analytically-derived approximation to P_d, we simulate the likelihood of detection for a single sensor randomly placed in the sea base for values of R between 0 and 1. Figure 2 shows the approximate and simulated values of P_d as a function of R. The linear approximation generated via analytical means provides results nearly identical to those generated empirically for values of R below approximately 0.75. In practical target-detection scenarios, the size of the region to be monitored is considerably larger than the sensing range of an individual sensor, even when sophisticated platform-based sensors are employed. Hence, we argue that the values of R of interest are those that are significantly less than U, and thus the linear approximation to probability of intersection can be employed with minimal introduction of error.

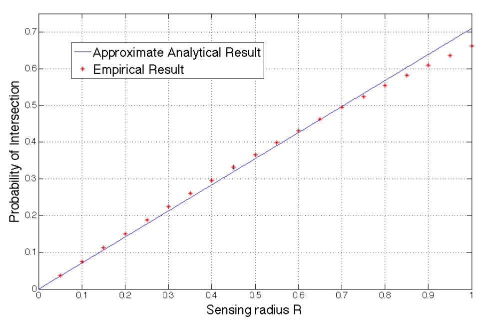

Fig. 2. Probability that a straight-line target is detected by a single sensor with radius R and center randomly located within the sea base, $U = 1$.

4. LIKELIHOOD OF DETECTION USING M RANDOMLY-PLACED SENSORS

We now extend the results derived in Section 3 to compute the likelihood that a target is detected by at least one of M randomly-placed sensors with radius R. We place no restrictions on the locations of the M sensors, thus modeling as closely as possible the scenario of low-cost sensors being randomly deployed (perhaps via mass drop) in the region of interest. In particular, note that the sensing regions of the arbitrarily-placed sensors may intersect, and as the number of sensors increases, such intersection will become more likely.

Because the sensors are randomly placed within the sea base, the probability that any particular sensor detects a target is independent of the probability of detection for all other sensors. Hence, the number of sensors detecting a target follows a binomial distribution, and the likelihood that at least one of the M sensors detects the target can be approximated as

$$P_d^{(M)} = 1 - (1 - P_d)^M \approx 1 - (1 - .7098R)^M.$$

Figure 3 compares the analytical approximation of the likelihood of detection using M sensors to simulation results for the same scenario. Sensing radii in the range $0 < R < 1$ are considered for sensor sets of size $M = 2$, 10, 50, and 100. The comparison shows that, across various numbers of sensors, the analytical approximation to $P_d^{(M)}$ nearly identically matches simulation results for detection likelihoods up to 0.6. (Of course, the radius R required to achieve $P_d^{(M)} = 0.6$ varies with M.) For likelihood of detection above 0.6, the analytical approximation overestimates the true (simulated) likelihood of detection, but it provides an upper bound that is within 5% of the true value for the scenarios considered. Since probability of detection is a function of many variables that are not fully controllable (number of functioning sensors, sensitivity of the detection device, etc.), such an approximation provides sufficiently accurate information in most applications of interest.

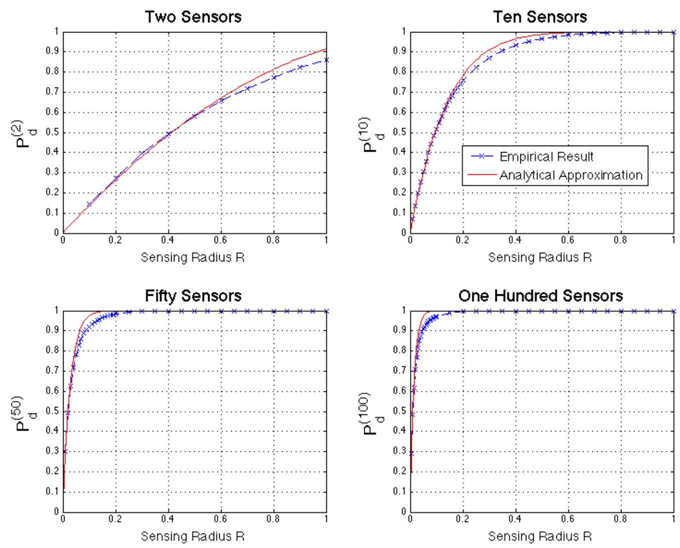

Fig. 3. Analytical and simulated likelihood of detection when $M = 2$, 10, 50, and 100 sensors with sensing radius R are randomly deployed in the sea base.

5. DETECTION PERFORMANCE VS. COST

Figure 4 shows the performance of various sizes of sensor sets as a function of total sensing area. For example, P_d for a sensor with $R = R_1$ is compared to $P_d^{(M)}$ for M sensors with $R = R_1/\sqrt{M}$. One can quickly glean that, despite the possibility for overlap, detection performance improves as the number of sensors increases while total sensing area is held constant.

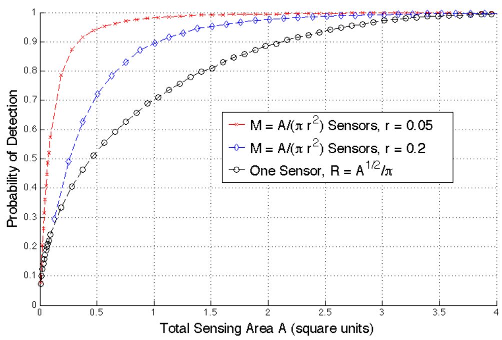

Fig. 4. Likelihood of detection as a function of total sensing area for sets of sensors with sensing radius $R = .05$, $R = 0.2$, and $R = \sqrt{A}/\pi$, where A denotes total sensing area.

Perhaps more applicable to the design of real-world systems is the question of whether or not larger sets of less powerful sensors yield better detection when cost is held constant. To address this question, we consider a sensor cost model that includes a fixed cost component, as well as a variable cost that scales linearly with sensing radius R, e.g. $C(R) = f_c + \alpha R$ for some constant α. Figure 5 shows the detection performance of sensor sets as a function of total cost for $f_c = 0.1$ and $f_c = 0.01$ and $R = 0.05, 0.2,$ and 0.4. In all cases, $\alpha = 1$. It is clear from the figure that the degree to which larger sets of less powerful sensors can improve upon the performance of smaller sets of high-powered sensors is highly dependent upon the relative size of fixed vs. variable cost. Using these results, we can determine an upper bound on fixed-cost in order to make randomly-distributed sensors an economically viable detection technique. Stated differently, if small sensors are relatively expensive, the design trade space tips in favor of fewer sophisticated sensors.

6. CONCLUSIONS

We have developed a framework for analytically approximating the likelihood of detecting a straight-line target traveling through a sea base as a function of the number and size of randomly-distributed sensors employed. Via comparison to simulation results, we have shown that the analytical approximation yields nearly error-free results for detection probabilities below 0.6 and an upper bound within a 5% margin for larger values of $P_d^{(M)}$. In addition, we have shown that the relative cost effectiveness of large sets of small sensors depends strongly on the fixed cost associated with such sensors. Future work will address the tracking capabilities of randomly-deployed sensors.

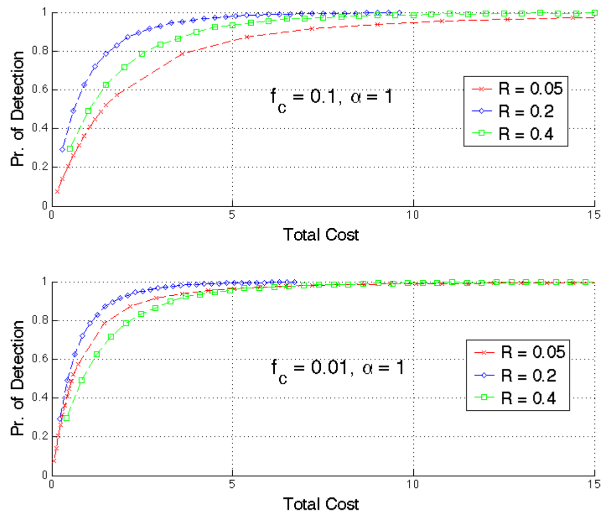

Fig. 5. Likelihood of detection as a function of total cost for sets of sensors with sensing radius $R = .05$, $R = 0.2$, and $R = 0.4$ and for fixed costs of $f_c = 0.1$ and $f_c = 0.01$.

7. REFERENCES

[1] D. Li, K.D. Wong, Y.H. Hu, and A.M. Sayeed, "Detection, classification, and tracking of targets," *IEEE Signal Processing Magazine*, vol. 19, no. 2, pp. 17–29, March 2002.

[2] J.-F. Chamberland and V.V. Veeravalli, "Wireless sensors in distributed detection applications," *IEEE SIgnal Processing Magazine*, vol. 24, no. 3, pp. 16–25, 2007.

[3] National Research Council, *Distributed Remote Sensing for Naval Undersea Warfare*, The National Academies Press, Washington, D.C., 2007.

[4] H. Cox, "Cumulative detection probabilities for a randomly moving source in a sparse field of sensors," in *Proceedings of the Twenty-Third Asilomar Conference on Signals, Systems, and Computers*, October 1989, vol. 1, pp. 384–389.

[5] C.M. Traweek and T.A. Wettergren, "Efficient sensor characteristic selection for cost-effective distributed sensor networks," *IEEE Journal of Oceanic Engineering*, vol. 31, no. 2, pp. 480–486, April 2006.

[6] E. Rowe, "Coverage statistics of distributed sensor fields with heterogeneous range sensitivity," Tech. Rep. 11785, Naval Undersea Warfare Center, Newport, RI, December 2006, available from http://stinet.dtic.mil as report number ADA464030.