CONSTRAINED OPTIMIZATION ALGORITHM FOR DIGITAL CAMERA-BASED SPECTROMETER

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ABSTRACT

Spectrometers that provide spectral decomposition at different locations of a scene are required in many applications. A simple device based on a multi-channel digital camera system has recently been developed to provide pixel level spectrum estimation. The current spectrum estimation algorithm for the device is based on PCA analysis. This paper develops a new estimation approach in an optimization framework that is based on a model of the optical path. The new method provides better spectrum estimates. Theoretical development and experimental examples are provided.

***Index Terms*—** Spectrometry, Principal Component Analysis.

1. INTRODUCTION

Spot-wise measurement of spectra as a function of the visible wavelengths is required in many industrial and graphic applications. An example is that of color reproduction in images for artwork preservation. A six-channel digital camera system consisting of a R,G,B color filter array (CFA) sensor and two absorption (colored) filters has recently been developed for this application, As shown in Figure.1, the RGB filter array output is recorded twice, once with Filter A in the optical path and once with Filter B. Thus for each pixel in the image of the scene, a six dimensional vector is recorded. Compared to traditional high resolution imaging spectrometers, this system is simpler, lower in cost, and easier to use. Methods have been developed to estimate scene spectra from the measured digital camera signals in this system [1],[2],[3]. Most of these reflectance reconstruction techniques are based on what is known as learning-based indirection reconstruction, in which a calibration target is employed to build a transform matrix to map the recorded six- dimensional vector to a spectrum profile. The spectrum itself is typically sampled at 31 wavelengths and thus the matrix maps a 6-dimensional space to a 31-dimensional space. As we explain in the following sections, this approach has problems. We therefore propose a new computation model and a reconstruction algorithm which combines physical constraints and optimization resulting in a transform matrix that provides better estimates of the spectra.

2. PHYSICAL MODEL

Suppose the spectrum of the illuminating light source is represented by a (31x31) diagonal illumination spectral power distribution matrix $P = diag\left(p_1, p_2, \cdots, p_{31}\right)$ and the reflectance of the object at these different wavelengths is represented by a vector $\mathbf{r} = [r_1, r_{2,} \cdots, r_{31}]'$. Then the spectrum of the reflected light from the object is given by

$$L = \mathrm{P}\,\mathbf{r} \tag{1}$$

Let $F_{fA} = diag\left(f_{A1}, f_{A2}, \cdots, f_{A31}\right)$ and $F_{fB} = diag\left(f_{B1}, f_{B2}, \cdots, f_{B31}\right)$ be the diagonal 31x31 spectra transmittance matrixes of filters A and B in Figure.1. That is, $F_{fA}L$ and $F_{fB}L$ are the output spectra of filters A and B respectively. We form the 62x31 matrix F_f as

$$F_f = \begin{bmatrix} F_{fA} \\ F_{fB} \end{bmatrix} \tag{2}$$

The output spectra of filters A and B are each transformed to representative RGB values by multiplying them by the transpose of a 31x3 spectral sensitivity matrix F_C. Let $\left[d_1, d_2, d_3\right]'$ and $\left[d_4, d_5, d_6\right]'$ be the corresponding RGB vectors. The camera spectral sensitivity is characterized as a 6 x62 matrix F_{CS}

$$F_{CS} = \begin{bmatrix} F_C' & 0 \\ 0 & F_C' \end{bmatrix} \tag{3}$$

Denoting $\mathbf{d} = \left[d_1, d_2, d_3, d_4, d_5, d_6\right]'$, we have,

$$\mathbf{d} = F\mathbf{r} \tag{4}$$

where F is the 3x31 system filter matrix given by

$$F = (F_{CS}F_f)P \tag{5}$$

The key fact to note from (4) is that the transformation from the 31-dimensional reflectance to the 6-dimensional measurement vector is linear. This model works well with normal white light illumination.

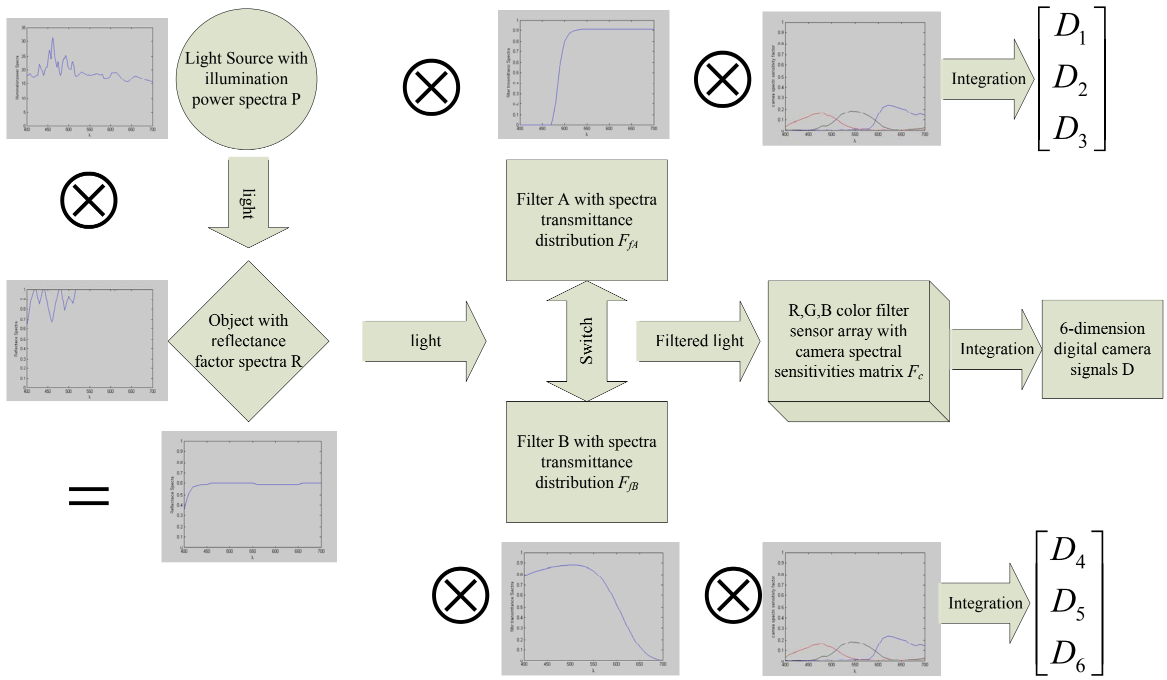

Figure.1 System diagram

2.2. Inverse problem

The problem of interest here is that of mapping an observed **d** to a reflectance vector **r**. This is a 6-dimensional to 31-dimensional mapping and hence there is no unique solution. Use of principal component analysis (PCA) has been proposed as follows. Suppose the source spectra form a set of N (31-dimensional) vectors, which are recorded in columns of a matrix R_T, and the corresponding 6-dimensional vectors from the camera are recorded in columns of a matrix D_T, where the subscript T stands for training set targets. Currently a GretagMacbeth ColorChecker DC, which contains 232 color patches, is our calibration source leading to N=232. To reduce the dimensionality, PCA is employed and the original training set reflectance spectra are reconstructed as,

$$\hat{R}_E = EA_T \tag{6}$$

where $\hat{R}_E$ is the (31xN) estimated reflectance spectra matrix , E is the (31x6) matrix whose columns are the first six eigenvectors of R_T in order of the magnitude of their eigenvalues resulting from the PCA of the R_T matrix, and $A_T = E'R_T$ is a 6xN matrix whose Nth column is the projection of Nth column of R_T onto the columns of E.

To establish connections between the measured digital signals and the projections of spectra onto training set eigenvectors, a 6x6 transformation matrix M_S is generated from the training set data to satisfy

$$A_T = M_s D_T \tag{7}$$

Then M_S could be solved as shown in Eq.(8) ,

$$M_S = A_T D_T^{\dagger} \tag{8}$$

The operation $\dagger$ is the Moore-Penrose pseudo-inverse. Once M_s is computed from the training set by a generalized inverse matrix calculation, it could be applied to other measured camera signals D_M to estimate the corresponding eigenvector scalars A_M, where subscript M stands for measured target. Concomitantly, from equation (6) and (7), the estimated spectral reflectance $\hat{R}_M$ would be:

$$\hat{R}_M = EM_s D_M \tag{9}$$

3. NEW ALGORITHM INCORPORATING OPTICAL PATH MODEL

The pseudo-inverse approach introduced in the previous section does not incorporate the camera model. It is based on the premise that, since the forward transformation from object to digitized signal is linear, the inverse of this transformation is also linear and hence the pseudo-inverse solution. On the face of it this appears *ad hoc*. However, an optimization interpretation can be attached to the procedure as follows.

The mean squared-error (MSE) of the training set can be expressed as:

$$
\begin{aligned}
MSE &= (R_T - EM_S D_T)'(R_T - EM_S D_T) \\
&= R_T' R_T - R_T' EM_S D_T - D_T' M_S' E' R_T - D_T' M_S' E' EM_S D_T \\
&= R_T' R_T - R_T' EM_S D_T - D_T' M_S' E' R_T - D_T' M_S' M_S D_T
\end{aligned}
\tag{10}
$$

where we have used the fact that $E'E = I$ since E is the eigenvector matrix. To solve the optimized M_S for RMSE, we differentiate the above with respect to M_S and set the result to the null matrix. It must be noted that the form of the MSE is different from the form $\|\mathbf{y} - A\mathbf{x}\|^2$ that is usually encountered in optimization problems. Proceeding with the optimization,

$$
\begin{aligned}
&\frac{\partial MSE}{\partial M_S} = 0 \Rightarrow \\
&\frac{\partial MSE}{\partial M_S} = \frac{\partial (R_T' R_T - R_T' EM_S D_T - D_T' M_S' E' R_T + D_T' M_S' M_S D_T)}{\partial M_S} \\
&= -R_T' ED_T - D_T' E' R_T + D_T' M_S' D_T + D_T' M_S D_T \\
&= D_T'(M_S D_T - E' R_T) + (D_T' M_S - R_T' E) D_T \\
&= D_T'(M_S D_T - E' R_T) + (D_T'(M_S D_T - E' R_T))' = 0 \\
&\Rightarrow M_S D_T - E' R_T = 0 \\
&\Rightarrow M_S D_T = E' R_T \\
&\Rightarrow M_S = E' R_T D_T^{\dagger}
\end{aligned}
\tag{11}
$$

The last result in Equation (11) is identical to that in Equation (8), indicating that the pseudo-inverse solution is the optimum solution to minimizing the MSE in (10).

However, there are problems with the approach. One problem is that a six-dimensional space is being mapped to a six-dimensional subspace of a 31-dimesnional space. Consequently, there is a good chance that the spectrum estimated from the camera measurement is substantially different from the true spectrum.

Another problem is that the system parameters are completely ignored. In fact, for the obtained solution to be tenable, some of the transmittance values for the system should be negative, which is a violation of the physics of the set up. Given M_S, the matrix F of Equation (4) can be obtained (using Eqs. 1),(2),(3)) as follows:

$$
\begin{aligned}
&A_T = M_S D_T \Rightarrow \\
&E' R_T = M_S F R_T \\
&\Rightarrow E' = M_S F \\
&\Rightarrow M_S = E' F^{\dagger} = E'((F_{CS} F_f) P)^{\dagger} \\
&\Rightarrow F = M_S{}^{\dagger} E'
\end{aligned}
\tag{12}
$$

As shown in Figure 2, various values of the F matrix are negative for M_S obtained with current approach. We now propose a new approach for solving M_S that takes the required positivity of elements of F into account.

Eq. (12) shows that if camera spectral sensitivities matrix F_c, the spectral transmittances matrix F_f of filters, and the diagonal illumination spectral power distribution matrix P, could all be precisely measured, even in the absence of digital image signals of calibration target, initial M_S could be generated with only the knowledge of the reflectance spectra of calibration target. It also shows that the general filter matrix F could be estimated from M_S and E. While the general filter matrix F could be regarded as the whole system filter for the target spectral reflectance factor, the physical constraint for F is that all the components of F should be non-negative.

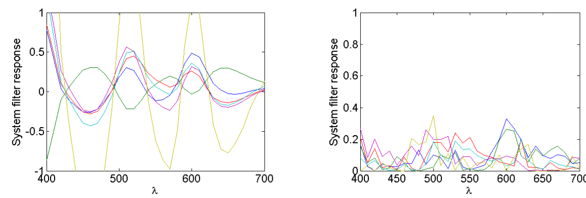

Figure.2 Left: The system filters response of pseudo-inverse approach. Right: The system filters response of the non-linear constrained optimization approach.

We formulate a constrained optimization problem as

$$
\begin{aligned}
M_S^* &= \min \arg_{M_S} (\|R_T - \hat{R}_T\|^2) \\
&= \min \arg_{M_S} (\|R_T - EM_S D_T\|^2) \\
&\text{subject to} (M_S)^{\dagger} E' \geq 0
\end{aligned}
\tag{13}
$$

For initial value of M_S, we can choose the result from Eq. (8). The non-linear optimization can use sequential quadratic programming (SQP) method.

4. EXPERIMENTAL RESULTS

We first compare the values of elements of the system filter matrix F obtained with the paper's method to the previous pseudo-inverse approach. With the M_S^* obtained from Eq.(13), the corresponding F was calculated using Eq.(12) and its rows were plotted (plot on right in Figure 2). Compared to the pseudo-inverse approach, the system filters responses after the non-linear constrained optimization are now all positive.

The experimental results have also shown that the new approach with physical constraints provides improved estimates of the source spectra. Figure 3 compares spectra estimated with the pseudo-inverse method and the new method with ground-truth spectra measured using a high resolution spectrometer. The estimated spectra calculated from the new approach is closer to the true spectra than the ones obtained from the old approach.

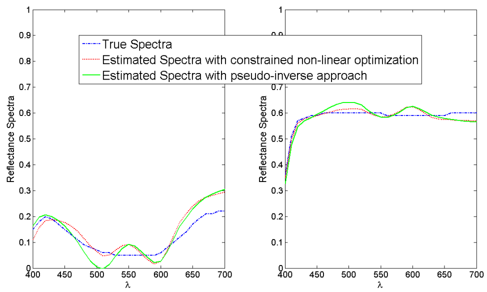

Figure.3 Graph of estimated and true Spectra

As another evaluation of performance of this method relative to the pseudo-inverse approach, color reproduction metrics such as color difference [4] and metamerism index [5] were also calculated. First, color coordinates are calculated by integrating the spectra of illumination and object reflectance with human color matching functions over visible wavelength, therefore, even a very small change in the spectra might lead to a big difference in color perception, which could be gauged in CIEDE2000 color difference equation. Different object spectra may appear to trigger the same color perception under one illumination condition. However if the illumination changes, these spectra may show large color difference. This phenomena is called metamerism, and can be measured with the metamerism index equation. Using GretagMacbeth ColorChecker DC with 232 effective color patches as training set, the two spectral reflectance factor estimation approaches, without constraint, and with constraint, were carried out on the 24 color patches of the verification GretagMacbeth ColorChecker SG target. The results are shown as below in Table. I

Table.I Performance Comparison for Color Checker SG spectral reflectance factor estimation. ΔE00 is the color difference. MSI is the Metamerism Index with lighting source changing from the D65 lighting source to A lighting source

COLOR CHECKER SG		MEAN	STD	MAX	MIN
Pseudo-inverse approach	ΔE00	1.97	1.28	6.19	0.36
	MSI	0.62	0.42	1.81	0.08
Non-linear optimization with constraint.	ΔE00	1.78	1.07	5.78	0.16
	MSI	0.53	0.52	2.04	0.04

The color difference and metamerism performance results in Table.I proved that the new approach provides better performance in color reproduction. t-tests have also been carried out on all the difference between these two methods. All the differences have failed the null hypothesis that the two methods yield statistically identical results at the 5% significance level, which indicates the performance of the new approach is significantly improved over the old approach.

5. CONCLUSIONS

A new algorithm for spectral reflectance factor estimation was developed for a camera-based spectrometer. As shown in the paper, the motivation for the development came from the fact that the current algorithm does not constrain the solution to the physics of the optical path and, in fact, violates it. The optimal solution developed in the paper was shown not only to conform to the positivity condition of the system transmittance but also to provide spectrum reconstructions that are closer to the ground truth spectra. The concept has the potential to be extended to various application areas such as multispectral and hyperspectral data analysis.

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6. REFERENCES

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