FREQUENCY OFFSET ESTIMATION FOR MULTI-STATIC RADAR SYSTEM

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ABSTRACT

Bi-static and multi-static radar requires synchronization between transmitter(s) and receiver(s) due to their separated locations. Especially, for the target doppler estimation, the carrier frequency offset occurred by the discord between the transmit and the receive ends should be estimated before the processing. In this paper, the system model for multi-static radar is provided as a similar form of *multi-input multi-output* (MIMO) communication system. However, unlike the MIMO case the frequency offset for each receiver is not identical to each other which makes the modeling and problem complicated. Based on the system model, the joint maximum likelihood (ML) solution and the suboptimal estimation is presented for frequency offset compensation. In this approximated approach, each receiver frequency offset is decoupled to others and the computational complexity becomes extremely low while it provides reasonable error performance for small frequency offsets and high signal to noise ratio (SNR).

***Index Terms*—** frequency offset, ML estimation, true time delay, orthogonal sequence

1. INTRODUCTION

Bi-static and multi-static radars of which the transmitter(s) and receivers(s) are positioned apart have been studied over the recent years. Currently, the deployment of unmanned air vehicles (UAV's) is making multi-static radar systems an interesting field of study. Some advantages make the bi-/multi-static radar be attractive compared to the mono-static system. Since the receivers are operated in a passive mode, they are undetectable at a field of battle. Besides, the bi-/multi-static systems have anti-stealth capability resulting from the different aspect angle of the transmitter and the receiver at the target point of view. However, having the transmitter and the receiver at different locations, synchronization between them is required to achieve appropriate performances [1]-[2]. Especially, the carrier frequency synchronization should be guaranteed to retrieve the doppler of the target signal. In this system, the frequency offset is induced by the instability of oscillator or limited oscillator precision, resulting in the inaccurate doppler frequency estimation. Therefore, it is required to measure and compensate the frequency offset at the receiver.

In digital communication systems, the frequency offset problem have been dealt as an important issue by many researchers. In many algorithms, training sequences (TSs) are used to solve this problem easily and to achieve higher performance [3]-[6]. Unfortunately, in the radar system, a few works have been done for this issue.

This work was supported by Brain Korea 21 Project, the school of information technology, KAIST in 2007 and by the SamsungThales.

The synchronization of the bi-/multi-static radar using reference signal disciplined to Global Positioning System (GPS) has been investigated [7]. However, the frequency estimation without any data support except the received signal has not been considered. This issue is motivated from the frequency offset estimation in wireless communication system [5]-[6].

This paper addresses two issues raised by the carrier frequency estimation for the multi-static radar system. One is the novel system model which explains the received data using true time delay [8] for the coherent processing in multi-static system. This model has similar structure to the MIMO systems. Hence, note that the MIMO techniques, especially for frequency offset estimation, are applicable to the model herein for multi-static radar systems. The other issue is the frequency offset estimation. In this paper, based on the proposed signal model, joint ML estimation of the channel response and the frequency offsets is presented. Moreover, the suboptimal scheme which requires low complexity and makes it possible to estimate each receiver's offset, independently is introduced.

The remainder of this paper is organized as follows. In section 2, the appropriate signal model is provided by introducing the true time delay. Section 3 is occupied with ML based carrier frequency offset estimator and the suboptimal scheme containing some approximations. Numerical examples illustrating the performance of the introduced estimators are presented in Section 4, and some conclusions and suggestions for future directions are offered in Section 5.

2. SYSTEM MODEL

In multi-static radar systems, N_t transmitters and N_r receivers which are located apart from one another are considered. Since a multi-static radar can be treated as nothing but the extended version of a bi-static system, at first, it is essential to look into a bi-static radar system.

The illuminated signal from the transmitter is represented as

$$x(t) = u(t)e^{j2\pi f_0 t + j\phi}, \tag{1}$$

where $u(t)$ is the complex envelope of a baseband signal including signal amplitude, f_0 is the carrier frequency and ϕ, uniformly distributed on [0 2π), is the random phase. At the receiver site, the target echo including doppler effect is

$$y(t) = A(t)u(t - \tau^{TX} - \tau^{RX})e^{j2\pi(f_0+f_d)(t-\tau^{TX}-\tau^{RX})+j\phi}. \tag{2}$$

where $A(t)$ is the echo amplitude, τ^{TX} and τ^{RX} are traverse times from a transmitter to a target and from a target to a receiver, respectively. The overall delay from a transmitter to a receiver via a target can be represented as $\tau = \tau^{TX} + \tau^{RX}$, f_d is the doppler frequency induced by transmitter, receiver, and the target velocities and λ_0 is the wavelength corresponding to the carrier frequency f_0.

After down-conversion with the discorded frequency $\tilde{f}_0 = f_0 - \Delta f_0$ which has the frequency offset Δf_0 at the receiver end, the signal becomes

$$y(t) = A(t)u(t-\tau)e^{j2\pi(\Delta f_0 + f_d)(t-\tau)+j\phi}. \qquad (3)$$

Based on bi-static radar signal model, extension to the multi-static case is easy to achieve. The m-th receiver signal which is transmitted from n-th transmitter is denoted as

$$y_{mn}(t) = A_{mn}(t)u_n(t-\tau_{mn})e^{j2\pi(\Delta f_0 + f_{d,mn})(t-\tau_{mn})+j\phi}$$
$$n = 1, 2, \cdots, N_t \qquad m = 1, 2, \cdots, N_r. \qquad (4)$$

where the subscript used herein denotes the related site for parameters, m is the receiver and n is the transmitter part, respectively.

In the multi-static system, all the data received from each receiver are combined coherently at the center location to detect target and estimate parameters. For coherent data processing at a time, it is necessary to introduce the true time delay concept [8].

Assume that all transmitters and receivers are focusing at the look-point (X_t, Y_t, Z_t). When we let the n-th transmitter position be (X_n, Y_n, Z_n), the distance of the look point to the n-th illuminator is denoted as $D_{nt} = \sqrt{(X_n - X_t)^2 + (Y_n - Y_t)^2 + (Z_n - Z_t)^2}$. The true time delay between these two points is presented as

$$\Delta T_{nt} = \frac{\mathtt{max}(D_{nt}) - D_{nt}}{c}, \qquad (5)$$

where c is the speed of light and $\mathtt{max}(D_{nt})$ means the maximum distance among the distances from N_t transmitters to the look point. To make all transmit signals reach to the look point at a same time, the transmitting time of each transmitter is modified using the true time delay. As a result, at the look point, illuminated signals from all transmitters are regarded as if those are from multiple antennas of a single transmitter.

Similarly, $D_{tm} = \sqrt{(X_m - X_t)^2 + (Y_m - Y_t)^2 + (Z_m - Z_t)^2}$ is the distance of the look point to the m-th receiver at (X_m, Y_m, Z_m) and $\Delta T_{tm} = (\mathtt{max}(D_{tm}) - D_{tm})/c$ is the true time delay.

When the m-th receiver signal is sampled at a time $t = kT_s - \Delta T_m$ where k is the time index and T_s is the sampling interval, the received data can be processed at a same time. This sampling process makes the signals arrived at each receiver at each different time, be the responses of the target started from the look point at a same time. After modification of transmit and sampling time, there is no need to consider the effect of distributed positions among transmitters or among receivers. It can be treated as a bi-static radar which has multiple antennas at both transmitter and the receiver. This modification is based on the assumption of the knowledge of the locations at receivers, transmitters and the look point.

The transmitted signal at time l takes the form as (6) for notation simplicity.

$$\mathbf{x}_l = [x_l(1)x_l(2)\cdots x_l(N_t)]^T, \qquad (6)$$

where $x_l(n), l = 0, 1, \cdots, L-1$ is the complex baseband signal of the n-th transmitter, L is the length of the code in one pulse and $(\cdot)^T$ is a vector transposition.

The random amplitude $A_{mn}(t)$ in (4) is obtained directly from the radar equation. The only random parameter in radar equation is a radar cross section. When we assume the target is modeled as swerling I or swerling III (this assumption will be removed later), the random amplitude can be treated as a deterministic constant in a coherent processing interval (CPI). From this assumption, the received signal is represented by matrix form using $N_r \times N_t$ channel matrix $\mathbf{H}$ with entries h_{mn} which is the amplitude of the sampled target signal from the n-th transmitter to the m-th receiver.

When transmitters are frequency synchronized perfectly with each other but not with receivers, frequency offset of the receiver is different each other. The received signal with frequency offset Δf_m at the m-th receiver at time l is given by

$$y_l(m) = e^{j2\pi(\Delta f_m)lT_s} \sum_{n=1}^{N_t} h_{mn} x_l(n) e^{j2\pi(\Delta f_{d,mn})lT_s} + v_l(m) \qquad (7)$$

where $v_l(m)$ is the zero-mean complex Gaussian noise with variance $\sigma_v^2 = E|v_l(m)|^2$.

For simplicity in frequency offset estimation, the doppler frequency effect induced by transmitter, receiver, and target motions is assumed to be irrelevant to the subject. Under this condition, the overall received signals at time l can be written more compactly using (8).

$$\mathbf{y}_l = \mathbf{F}^l \mathbf{H} \mathbf{x}_l + \mathbf{v}_l, \qquad 0 \le l \le L-1. \qquad (8)$$

where $\mathbf{F}$ is a diagonal matrix

$$\mathbf{F} = \mathtt{diag}(e^{j2\pi\Delta f_1 T_s}, \cdots, e^{j2\pi\Delta f_{N_r} T_s}). \qquad (9)$$

It has a similar form to the MIMO wireless digital communication system model [3]-[6]. However, in multi-static radar system, the frequency offset for each receiver is different each other whereas the MIMO system has same carrier frequency offset for all receiver antennas. Hence, the modeling is complicated and not concise compared to the MIMO system.

The overall received signal which is processed coherently, can be represented as a matrix form by stacking all snapshot vectors as a tall column

$$\mathbf{y} = \mathbf{F}_\Sigma \mathbf{H}_\Sigma \mathbf{x} + \mathbf{v}, \qquad (10)$$

where $\mathbf{F}_\Sigma$ and $\mathbf{H}_\Sigma$ are block diagonal matrices

$$\begin{aligned} \mathbf{F}_\Sigma &= \mathtt{diag}(\mathbf{I}, \mathbf{F}, \cdots, \mathbf{F}^{L-1}), \\ \mathbf{H}_\Sigma &= \mathtt{diag}(\mathbf{H}, \mathbf{H}, \cdots, \mathbf{H}), \end{aligned} \qquad (11)$$

and $\mathbf{y}, \mathbf{v}$ are the $N_r L \times 1$ stacked snapshot vectors, $\mathbf{x}$ is the $N_t L \times 1$ also stacked transmitted code (training sequence).

In this paper, each transmitter uses its own code which is orthogonal to each other. This code is transmitted through just one pulse for frequency estimation, hence also swerling II and IV modeled target amplitude can be treated as constant through all process. The sampling interval T_s is determined by the pulse duration over the length of the code. The length of the code is equivalent to the number of snapshots.

Transmitted codes are known at the receiver, but $\mathbf{H}$ is not known and should be estimated jointly with the frequency offsets $\Delta f_1, \Delta f_2, \cdots, \Delta f_{N_r}$.

3. FREQUENCY-OFFSET ESTIMATION

3.1. ML based estimation

In radar systems, the received signal contains interference plus noise signal instead of additive white noise only signal. In general, the interference plus noise signal $\mathbf{v}$ is assumed to be zero-mean Gaussian distributed with the covariance matrix $\mathbf{R}$. In this paper , to make problem simple, the covariance matrix $\mathbf{R}$ is assumed to be

$\sigma_v^2\mathbf{I}$, where $\mathbf{I}$ is a $N_rL \times N_rL$ identity matrix. Thus, for a given $\mathbf{H}$ and $\Delta f_1, \Delta f_2, \cdots, \Delta f_{N_r}$, the received vector $\mathbf{y}$ is Gaussian with the mean $\mathbf{F}_\Sigma\mathbf{H}_\Sigma\mathbf{x}$ and covariance matrix $\sigma_v^2\mathbf{I}$.
The likelihood function for the parameter $(\mathbf{H}, \Delta f_1, \Delta f_2, \cdots, \Delta f_{N_r})$ is given by

$$\begin{aligned}\Lambda(\mathbf{y}|\ \tilde{\mathbf{H}},\ \Delta\tilde{f}_1, \Delta\tilde{f}_2, \cdots, \Delta\tilde{f}_{N_r})\\ = [\mathbf{y} - \tilde{\mathbf{F}}_\Sigma\tilde{\mathbf{H}}_\Sigma\mathbf{x}]^H[\mathbf{y} - \tilde{\mathbf{F}}_\Sigma\tilde{\mathbf{H}}_\Sigma\mathbf{x}],\end{aligned} \qquad (12)$$

where $\{\tilde{\mathbf{H}}, \Delta\tilde{f}_1, \Delta\tilde{f}_2, \cdots, \Delta\tilde{f}_{N_r}\}$ is the set of candidate values of $\mathbf{H}$ and $\Delta f_1, \Delta f_2, \cdots, \Delta f_{N_r}$ and $(\cdot)^H$ denotes the complex conjugate transposition. The candidate set which makes the likelihood function of (12) to be a minimum is the joint ML estimate of $\mathbf{H}$ and $\Delta f_1, \Delta f_2, \cdots, \Delta f_{N_r}$.

For arbitrary frequency offsets, the estimate $\hat{\mathbf{H}}$ which minimize $\Lambda(\mathbf{y}|\tilde{\mathbf{H}}, \Delta\tilde{f}_1, \Delta\tilde{f}_2, \cdots, \Delta\tilde{f}_{N_r})$ is given by

$$\hat{\mathbf{H}} = [\mathbf{y}_0\mathbf{x}_0^H + \mathbf{F}^H\mathbf{y}_1\mathbf{x}_1^H + \cdots + (\mathbf{F}^{L-1})^H\mathbf{y}_{L-1}\mathbf{x}_{L-1}^H]. \qquad (13)$$

Substituting $\hat{\mathbf{H}}$ to (12) and varying candidate values of frequency offsets, it is found that minimizing (12) is equivalent to maximizing (14)

$$\begin{aligned}\Lambda(\tilde{\mathbf{F}}) &= \Lambda(\Delta\tilde{f}_1, \Delta\tilde{f}_2, \cdots, \Delta\tilde{f}_{N_r})\\ &= \mathbf{Re}\left[\sum_{b=1}^{L-1}\sum_{a=0}^{L-1-b}(\mathbf{x}_a^H\mathbf{x}_{a+b})\mathbf{y}_{a+b}^H\tilde{\mathbf{F}}^b\mathbf{y}_a\right].\end{aligned} \qquad (14)$$

From(14), the estimator for frequency offsets is obtained by

$$\hat{\mathbf{f}} = \texttt{arg}\ \max_{\tilde{\mathbf{f}}}\ \left\{\mathbf{Re}\left[\sum_{b=1}^{L-1}\sum_{a=0}^{L-1-b}(\mathbf{x}_a^H\mathbf{x}_{a+b})\mathbf{y}_{a+b}^H\tilde{\mathbf{F}}^b\mathbf{y}_a\right]\right\} \qquad (15)$$

where $\mathbf{f} = [\Delta f_1, \Delta f_2, \cdots, \Delta f_{N_r}]^T$.

The ML estimator (15) gives optimal performances, but requires grid search in the N_r dimensional space. The number of examined grid points increases exponentially with respect to the number of parameters, N_r. Therefore, the computational load to estimate frequency offsets is extremely high in this system.

3.2. Approximated suboptimal estimation using Periodic and Orthogonal TSs

To cope with computation load problem, the suboptimal estimation scheme should be needed. In this subsection, approximated ML-based estimation is proposed using periodic and orthogonal codes.

In the suboptimal scheme, to decouple the one receiver frequency from other frequencies, the partial derivatives are used although the solution point is not a unique stationary point. In(15), the estimated $\hat{\mathbf{f}}$ is the maximum point of the likelihood function $\Lambda(\tilde{\mathbf{f}})$ of (14). It is obvious that $\hat{\mathbf{f}}$ is the stationary point. If we take the partial derivatives of $\Lambda(\tilde{\mathbf{f}})$ with respect to $\Delta\tilde{f}_m$ and set the result equal to zero at this point $\Delta\hat{f}_m, m = 1, 2, \cdots, N_r$

$$\mathbf{Im}\ \left\{\sum_{b=1}^{L-1}\sum_{a=0}^{L-1-b}b(\mathbf{x}_a^H\mathbf{x}_{a+b})y_{a+b}^*(m)y_a(m)e^{j2\pi\Delta\hat{f}_m bT_s}\right\} = 0. \qquad (16)$$

For the proposal of the low computational approach, the periodic orthogonal sequences as transmitted codes is introduced. In other words, the rearranged sequence $\mathbf{X} = [\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_{L-1}]$ is composed of identical orthogonal submatrices of size $N_t \times N_t$

$$\mathbf{X} = [\mathbf{C}\quad \mathbf{C}\quad \cdots\quad \mathbf{C}], \qquad (17)$$

where $\mathbf{C}$ is $N_t \times N_t$ orthogonal matrix. For the periodic orthogonal codes, $\mathbf{x}_a^H\mathbf{x}_{a+b}$ is 1 if $b = N_t, 2N_t, \cdots, (L/N_t - 1)N_t$ and 0 otherwise. Using this fact, (16)is approximated as

$$\mathbf{Im}\ \left\{[\sum_{b=1}^{L-1}\sum_{a=0}^{L-1-b}b(\mathbf{x}_a^H\mathbf{x}_{a+b})y_{a+b}^*(m)y_a(m)]e^{j\pi\Delta\hat{f}_m LT_s}\right\} \simeq 0 \qquad (18)$$

for small normalized frequency offset $\Delta f_m T_s$ and high SNR. The frequency offset estimator obtained by (18) is

$$\Delta\hat{f}_m \simeq -\frac{1}{\pi LT_s}\left\{\sum_{b=1}^{L-1}\sum_{a=0}^{L-1-b}b(\mathbf{x}_a^H\mathbf{x}_{a+b})y_{a+b}^*(m)y_a(m)\right\} \qquad (19)$$

As shown in (19), the estimator is composed of products of $(\mathbf{x}_a^H\mathbf{x}_{a+b})$ and $y_{a+b}^*(m)y_a(m)$. When $(\mathbf{x}_a^H\mathbf{x}_{a+b})$ is precomputed at each receiver, the approximated scheme requires less computational load compared to the ML estimator. Moreover, in (19), the estimator for each receiver's frequency offset is computed using known transmitted signal and only the own received sinal. This makes it possible to estimate frequency offset at each receiver site independently before the data gathering for the coherent processing whereas in the ML estimation scheme all frequency offsets should be found simultaneously within N_t dimensional grid.

For small normalized frequency and high SNR, (18) is almost equivalent to (16). This means the approximated approach nearly achieves the optimal ML performance except the separation error. In this approach, the acquisition range is related to the length of the used code

$$|\Delta f| < \frac{1}{LT_s}. \qquad (20)$$

Unless the true frequency offset is confined within this interval, the suboptimal approach gives ambiguous results. When the number of snapshots increases, the error performance of the estimation is getting better but the detectable frequency offset range is limited.

4. SIMULATION RESULTS

In this section, the simulation results are shown to illustrate the performances of the frequency offset finding schemes. In overall numerical examples, two transmitters and two receivers (i.e., $N_t = 2$, $N_r = 2$) system are considered. The transmitted periodic orthogonal sequence is composed of the following orthogonal matrix.

$$\mathbf{C} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \qquad (21)$$

Fig.1 shows the average estimates obtained by suboptimal approach versus normalized frequency offset. This shows the performance of the algorithm and the ambiguous characteristic. In this example, SNR=20dB and L=20 are used. As shown in this result, the acquisition region is determined by the number of snapshots, i.e., $|\Delta fT_s| < 1/20$.

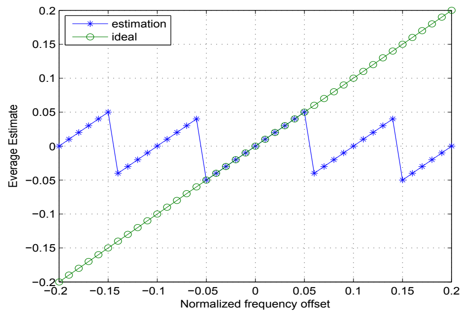


Fig. 1. Average frequency estimates of the approximated scheme versus $\Delta f_1 T_s$ for L=20

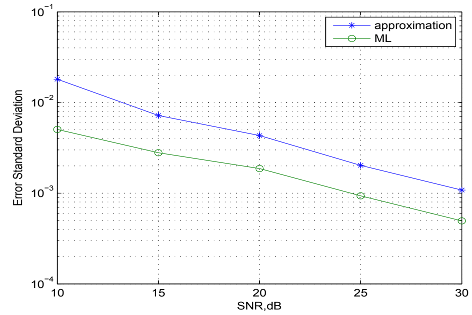


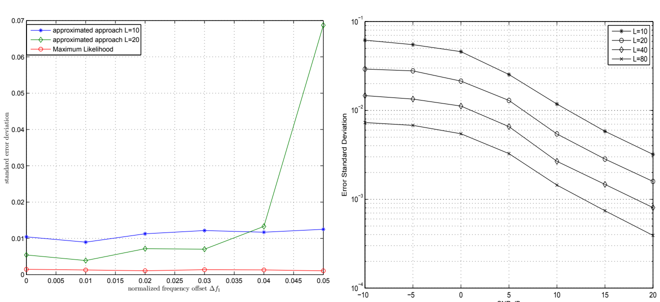
Fig. 3. Error standard deviation corresponding to the SNR

The next simulation depicts the dependency of the error performance on the number of snapshots. Fig.2 (a) gives the standard deviation of errors $\{E[\Delta \hat{f} T_s - \Delta f T_s]^2\}^{1/2}$ versus normalized frequency offset with two observation length L=10 and L=20. This denotes the larger number of snapshots induces the smaller errors close to the ML level within its acquisition region. However, as the frequency offset goes to outside of the permitted region the performance degrade appears because of the ambiguous characteristic. In Fig.2 (b), the error behavior of the suboptimal scheme corresponding to the SNR is illustrated for the various code length when the frequency offset Δf is zero. It is obvious that the large number of snapshot gives better performance.

The difference of the error performance between the ML and suboptimal method is displayed in Fig. 3. In this simulation, used code length is 10 and the true frequency offset is set to be zero. The results show that the errors of the suboptimal scheme can not be reduced to the ML level even if the SNR is high. The error performance degradation is due to the separation of the estimator for each receiver in approximated approach.

5. CONCLUSIONS

This paper presents the modeling scheme which explains the received data for the coherent processing in multi-static system. when the delayed transmission and sampling is employed using true time delay multi-static signal model looks like for MIMO digital communication systems. Based on the derived model, the frequency offset is estimated to overcome one of the key drawback of the multi-static radar. Joint ML solution of the channel response related to target amplitude and the all receiver frequency offsets is presented. Moreover, the suboptimal scheme which requires low complexity and makes it possible to estimate each receiver's offset independently, is provided. The simulation results shows the performance of ML and approximated schemes using the orthogonal sequences. The later method has extremely smaller complexity but somewhat higher error induced by the separation effect than the ML approach. The long term goal of this effort is the development of the algorithm to find frequency offsets and existing doppler of target simultaneously.



(a) (b)

Fig. 2. Error performance of the proposed scheme for the various code length.

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