POLARIZATION OPTIMIZATION FOR SCATTERING ESTIMATION IN HEAVY CLUTTER

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ABSTRACT

Controlling the polarization information in transmitted waveforms enables improving the performance of radar systems. We consider the design of optimal polarization for the estimation of target scattering embedded in heavy clutter. The goal is to minimize the mean squared error of the scattering estimation subject to an average radar pulse power constraint. We show that such a problem is equivalent to the optimal design of a radar sensing matrix that contains the polarization information. We formulate the optimal design as a nonlinear optimization problem which can be recast it in a convex form and is thus efficiently solvable by semi-definite programming (SDP). We compare the sensing performance of the optimally selected polarization over the conventional approaches. Our numerical results indicate that a significant amount of power gain is achieved in the target scattering estimation through such an optimal design.

Index Terms— Radar polarimetry, adaptive estimation, scattering matrices, optimization methods

1. INTRODUCTION

Advances in digital signal processing and computing technology have resulted in the emergence of increasingly adaptive radar systems. It is clear that such radar systems have more robust performance by adapting their sensing patterns (or more specifically waveforms) to the operation scenarios (such as target, clutter etc.). Adaptive waveform design has attracted a lot of attention recently [1, 2, 3]. In this paper we design the optimal waveform polarization for the estimation of target scattering in an environment with heavy clutter. We consider the optimal transmit and receive polarization for a polarimetric radar by adapting to the target and clutter polarimetric characteristics for enhanced sensing performance.

Polarimetric information of the radar targets reveals target details such as geometrical structure, shape, reflectivity, and orientation. Radar polarimetry can be used not only for target classification but also for enhancing target detection and estimation performance, *i.e.*, resolving targets in a clutter environment [2, 4]. To obtain the target polarimetric scattering information, polarimetric radar systems which transmit waveforms with both horizontal (H) and vertical (V) orientations have been developed and adopted in various applications [5]. Such systems alternately switch between the two orthogonal polarizations at both the transmit and receive sides, and thus result in four combinations of transmit and receive polarizations: HH, HV, VH, and VV.

In modern radar systems, any polarization on either transmission or reception can be synthesized by using the linear combinations of the H and V components. Thus, besides the four types of transmit/recieve combinations above, such radar can achieve any pair of transmit/recieve polarizations. Such flexibility greatly enhances the polarimetric sensing capability of the radar system. An an example, the exploration of adaptive polarization for polarimetric contrast enhancement has been widely studied in the synthetic aperture radar imaging [6, 7].

In this paper, we consider the radar waveform polarization optimization and power scheduling in the estimation of target scattering in heavy clutter. We cast such a problem as the optimal design of the radar sensing matrix that is determined by the radar transmit/receive polarization and waveform power levels. The optimal design of sensing matrix for a linear Gaussian model was pursued in [8], which has an analytical solution and can be considered as a special case of our model when clutter is not present. Due to the coupling of the clutter with the transmit waveforms, the resulted optimization problem is highly nonlinear but can be recast as a semi-definite programming (SDP). This enables an efficient numerical solution which demonstrate that the optimally selected polarization has a few dB power gain over the conventional fixed polarization approaches.

Notation: A lower case letter (*e.g.*, a) denotes a scalar, a boldface/lowercase letter (*e.g.*, $\mathbf{a}$) denotes a vector, and a boldface/uppercase letter (*e.g.*, $\mathbf{A}$) denotes a matrix. In addition, Tr($\mathbf{A}$), $\mathbf{A}^T$, and $\mathbf{A}^H$ denote the trace, transpose, and Hermitian of $\mathbf{A}$ respectively. The letter $\mathbf{I}_n$ denotes an identity

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matrix of size $n \times n$. For two matrices $\mathbf{B}$ and $\mathbf{C}$, the relation $\mathbf{B} \succeq \mathbf{C}$ means that $\mathbf{B} - \mathbf{C}$ is positive semi-definite.

2. PROBLEM FORMULATION

Consider a polarized waveform transmitted from a radar transmitter. When the transmitted waveform encounters a target or the clutter in the far field, another field is returned and received by the radar receiver. The two electric fields are related to each other by means of the target or clutter scattering matrices [9].

Specifically we assume that the radar transmits a polarized waveform $\mathbf{s}(t) = \sqrt{P}\boldsymbol{\xi}s(t) = [\xi_h, \xi_v]^H s(t)$ where $\boldsymbol{\xi}$ is the transmit polarization vector, $s(t)$ is the pulse shape, and P is the transmit power. It is assumed that $||\boldsymbol{\xi}|| = 1$ and the pulse has unitary energy, *i.e.*, $||s^2|| = \int_{-\infty}^{\infty} |s(t)|^2 dt = 1$. In addition, we assume that the receive antenna has polarization $\boldsymbol{\eta} = [\eta_h, \eta_v]^H$ with $||\boldsymbol{\eta}|| = 1$. After ignoring the target Doppler shift, we obtain that the complex envelope of the received signal at the radar receiver can be represented as [10, 2]

$$y(t) = \frac{g}{r^2}\sqrt{P}\,\boldsymbol{\eta}^H(\mathbf{S}_t + \mathbf{S}_c)\boldsymbol{\xi}\; s(t-\tau) + w(t). \qquad (1)$$

In the above equation, $\mathbf{S}_\mathrm{t}$ and $\mathbf{S}_\mathrm{c}$ are the target and clutter scattering matrices respectively, $w(t)$ is the white noise process, r is the distance from the target to the radar, τ is the delay resulted from waveform forward and backward propagation, and g is a constant depending on the radar system characteristics such as operating frequency, permittivity and permeability of free space, and antenna gain at the target illumination angle etc.

After performing a matched filtering on (1) and a normalization by absorbing the constant g/r^2 into $v(t)$, we obtain the following observation model:

$$y = \sqrt{P}\,\boldsymbol{\eta}^H(\mathbf{S}_t + \mathbf{S}_c)\boldsymbol{\xi} + v \qquad (2)$$

where v is white noise with variance σ_v^2. The scattering matrices $\mathbf{S}_\mathrm{t}$ and $\mathbf{S}_\mathrm{c}$ are represented by 2×2 S-matrices [10, 2], which describe completely the polarization transforming properties of the target and clutter. We assume that they have the following matrix representation

$$\mathbf{S}_\mathrm{t} = \begin{bmatrix} s^t_{\mathrm{hh}} & s^t_{\mathrm{hv}} \\ s^t_{\mathrm{vh}} & s^t_{\mathrm{vv}} \end{bmatrix}, \quad \mathbf{S}_\mathrm{c} = \begin{bmatrix} s^c_{\mathrm{hh}} & s^c_{\mathrm{hv}} \\ s^c_{\mathrm{vh}} & s^c_{\mathrm{vv}} \end{bmatrix}. \qquad (3)$$

Our goal is to estimate $\mathbf{S}_\mathrm{t}$ based on radar measurement y which includes returns from both the target and the clutter, which is further degraded by thermal and background noise. For notational convenience, we convert (2) into a linear observation model by vectorizing $\mathbf{S}_\mathrm{t}$ and $\mathbf{S}_\mathrm{t}$. Specifically, we introduce

$$\mathbf{x}_\mathrm{t} = \begin{bmatrix} s^t_{\mathrm{hh}} & s^t_{\mathrm{vv}} & s^t_{\mathrm{hv}} & s^t_{\mathrm{vh}} \end{bmatrix}^T$$
$$\mathbf{x}_\mathrm{c} = \begin{bmatrix} s^c_{\mathrm{hh}} & s^c_{\mathrm{vv}} & s^c_{\mathrm{hv}} & s^c_{\mathrm{vh}} \end{bmatrix}^T$$

and

$$\begin{aligned} \mathbf{a}(P;\boldsymbol{\xi},\boldsymbol{\eta}) &= \sqrt{P}\,\boldsymbol{p}(\boldsymbol{\xi},\boldsymbol{\eta}) \\ &\stackrel{\mathrm{def}}{=} \sqrt{P}\begin{bmatrix} \xi_\mathrm{h}\eta_\mathrm{h} & \xi_\mathrm{v}\eta_\mathrm{v} & \xi_\mathrm{h}\eta_\mathrm{v} & \xi_\mathrm{v}\eta_\mathrm{h} \end{bmatrix}^T. \end{aligned}$$

With the above notation, we can rewrite (2) as

$$y = \mathbf{a}(P;\boldsymbol{\xi},\boldsymbol{\eta})^T\mathbf{x}_\mathrm{t} + \mathbf{a}(P;\boldsymbol{\xi},\boldsymbol{\eta})^T\mathbf{x}_\mathrm{c} + v.$$

In addition, it is easy to see that

$$||\boldsymbol{p}(\xi,\eta)||^2 = ||\boldsymbol{\xi}||^2||\boldsymbol{\eta}||^2 = 1.$$

To estimate the full polarimetric information (*i.e.*, all component of $\mathbf{x}_\mathrm{t}$), multiple pulses of different polarizations need to be transmitted to obtain multiple measurements y. Suppose there are m pulses transmitted to measure $\mathbf{x}_\mathrm{t}$. We use $P(i)$, $\boldsymbol{\xi}(i)$, and $\boldsymbol{\eta}(i)$ to denote the power, transmit, and receive polarization of these pulses. The observation from these m pulses can thus be written as

$$\begin{aligned} y(i) = &\ \mathbf{a}(P(i);\boldsymbol{\xi}(i),\boldsymbol{\eta}(i))^T\mathbf{x}_\mathrm{t} + \mathbf{a}(P(i);\boldsymbol{\xi}(i),\boldsymbol{\eta}(i))^T\mathbf{x}_\mathrm{c} \\ &+ v(i), \qquad i = 1, 2, \ldots, m. \end{aligned} \qquad (4)$$

We assumed that during the period of the m pulses, both the target scattering $\mathbf{x}_\mathrm{t}$ and the clutter scattering $\mathbf{x}_\mathrm{c}$ remain unchanged.

Introducing vector notations $\mathbf{y} = [y_1, y_2, \ldots, y_m]^T$, $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_m]^T$ with $\mathbf{a}_i \stackrel{\mathrm{def}}{=} \mathbf{a}(\xi(i), \eta(i))$, and $\mathbf{v} = [v_1, v_2, \ldots, v_m]$, we obtain a matrix representation of the m measurements in (4)

$$\mathbf{y} = \mathbf{A}\mathbf{x}_\mathrm{t} + \mathbf{A}\mathbf{x}_\mathrm{c} + \mathbf{v}.$$

For both $\mathbf{x}_\mathrm{t}$ and $\mathbf{x}_\mathrm{c}$, we further assume they both have a complex Gaussian distribution with known covariance matrix $\mathbf{C}_\mathrm{t}$ and $\mathbf{C}_\mathrm{c}$ respectively. The details of characterizing target/clutter scattering by their covariance matrix can be found in [10]. It is then easy to calculate that the minimum MSE $\mathbf{D}$ of estimating $\mathbf{x}_t$ from $\mathbf{y}$ satisfies

$$\mathbf{D}^{-1} = \mathbf{C}_t^{-1} + \mathbf{A}^H(\mathbf{A}\mathbf{C}_c\mathbf{A}^H + \sigma_v^2\mathbf{I}_m)^{-1}\mathbf{A}. \qquad (5)$$

We also impose P as the average power constraint on the m transmitted signals. This leads to the following condition

$$\sum_{i=1}^{m} ||\mathbf{a}_i||^2 \le mP.$$

In the form of $\mathbf{A}$, the power constraint can be rewritten as

$$\sum_{i=1}^{m} ||\mathbf{a}_i||^2 = \mathrm{tr}(\mathbf{A}\mathbf{A}^H) \le mP. \qquad (6)$$

Therefore, to choose the optimal polarization and power scheduling to minimize the MSE of estimating $\mathbf{S}_\mathrm{t}$ subject to the average power constraint P using m diversely polarized pulses, we obtain the following optimization problem

$$\begin{aligned}
\min_{\mathbf{A},\mathbf{D}} \quad & \mathrm{tr}(\mathbf{D}) \\
\text{s.t.} \quad & \mathbf{D}^{-1} = \mathbf{C}_t^{-1} + \mathbf{A}^H(\mathbf{A}\mathbf{C}_c\mathbf{A}^H + \sigma_v^2\mathbf{I}_m)^{-1}\mathbf{A} \\
& \mathrm{tr}(\mathbf{A}\mathbf{A}^H) \leq mP \\
& \mathbf{D} \succeq 0
\end{aligned} \tag{7}$$

The above problem is apparently neither linear nor convex in $\mathbf{A}$ or $\mathbf{D}$. In the next section, we reformulate the above problem in a convex form to make it efficiently solvable.

3. CONVEX REFORMULATION

In this section we recast (7) in a convex form and then solve it numerically using existing solvers for convex optimization problems. Let us first introduce the following new variables

$$\begin{aligned}
\mathbf{B} &= \sigma_v^{-1}\mathbf{A}\mathbf{C}_c^{1/2} \\
\mathbf{C}_{t,1}^{-1} &= \mathbf{C}_c^{1/2}\mathbf{C}_t^{-1}\mathbf{C}_c^{1/2} \\
\mathbf{D}_1^{-1} &= \mathbf{C}_c^{1/2}\mathbf{D}^{-1}\mathbf{C}_c^{1/2}
\end{aligned}$$

In terms of the new variables, (5) becomes

$$\mathbf{D}_1^{-1} = \mathbf{C}_{t,1}^{-1} + \mathbf{B}^H(\mathbf{B}\mathbf{B}^H + \mathbf{I}_m)^{-1}\mathbf{B}. \tag{8}$$

Using the relation $\mathbf{B} = \sigma_v^{-1}\mathbf{A}\mathbf{C}_c^{1/2}$, we can also calculate

$$\begin{aligned}
\mathrm{tr}(\mathbf{A}\mathbf{A}^H) &= \mathrm{tr}(\sigma_v\mathbf{B}\mathbf{C}_c^{-1/2}\mathbf{C}_c^{-H/2}\mathbf{B}^H\sigma_v) \\
&= \sigma_v^2\,\mathrm{tr}(\mathbf{B}\mathbf{C}^{-1}\mathbf{B}^H) \\
&= \sigma_v^2\,\mathrm{tr}(\mathbf{C}^{-1}\mathbf{B}^H\mathbf{B}).
\end{aligned}$$

This leads to the following power constraint on $\mathbf{B}$ (c.f. (6)):

$$\mathrm{tr}(\mathbf{C}^{-1}\mathbf{B}^H\mathbf{B}) \leq mP/\sigma_v^2.$$

To further simplify (8), we apply the matrix inversion lemma and obtain

$$(\mathbf{I}_m + \mathbf{B}^H\mathbf{B})^{-1} = \mathbf{I}_4 - \mathbf{B}^H\left(\mathbf{I}_m + \mathbf{B}\mathbf{B}^H\right)^{-1}\mathbf{B}.$$

Therefore,

$$\begin{aligned}
\mathbf{D}_1^{-1} &= \mathbf{C}_{t,1}^{-1} + \mathbf{B}^H(\mathbf{B}\mathbf{B}^H + \mathbf{I}_m)^{-1}\mathbf{B} \\
&= \mathbf{C}_{t,1}^{-1} + \mathbf{I}_4 - (\mathbf{I}_4 + \mathbf{B}^H\mathbf{B})^{-1}.
\end{aligned} \tag{9}$$

Notice that $\mathbf{D}_1^{-1} = \mathbf{C}_c^{1/2}\mathbf{D}^{-1}\mathbf{C}_c^{1/2}$, which implies $\mathrm{tr}(\mathbf{D}) = \mathrm{tr}(\mathbf{C}_c^{1/2}\mathbf{D}_1\mathbf{C}_c^{1/2}) = \mathrm{tr}(\mathbf{D}_1\mathbf{C}_c)$. Therefore, in terms of $\mathbf{D}_1$ and $\mathbf{B}$, we can recast the optimization problem (7) as

$$\begin{aligned}
\min_{\mathbf{B},\,\mathbf{D}_1} \quad & \mathrm{tr}(\mathbf{D}_1\mathbf{C}_c) \\
\text{s.t.} \quad & \mathbf{D}_1^{-1} = \mathbf{I}_4 + \mathbf{C}_{t,1}^{-1} - (\mathbf{I}_4 + \mathbf{B}^H\mathbf{B})^{-1} \\
& \mathrm{tr}(\mathbf{C}_c^{-1}\mathbf{B}^H\mathbf{B}) \leq mP/\sigma_v^2 \\
& \mathbf{D}_1 \succeq 0
\end{aligned} \tag{10}$$

It is then natural to introduce a positive semi-definite matrix $\mathbf{R} \overset{\text{def}}{=} \mathbf{B}^H\mathbf{B}$. We further change the first constraint (10) into inequality[1]. Applying these change on (10) we obtain

$$\begin{aligned}
\min_{\mathbf{R},\,\mathbf{D}_1} \quad & \mathrm{tr}(\mathbf{D}_1\mathbf{C}_c) \\
\text{s.t.} \quad & \mathbf{D}_1^{-1} \preceq \mathbf{I}_4 + \mathbf{C}_{t,1}^{-1} - (\mathbf{I}_4 + \mathbf{R})^{-1} \\
& \mathrm{tr}(\mathbf{C}_c^{-1}\mathbf{R}) \leq mP/\sigma_v^2 \\
& \mathbf{R} \succeq 0, \quad \mathbf{D}_1 \succeq 0.
\end{aligned}$$

Introducing another auxiliary semidefinite matrix $\mathbf{S}$, we can write the first constraint equivalently as two inequalities:

$$\begin{aligned}
\mathbf{D}_1^{-1} &\preceq \mathbf{I}_4 + \mathbf{C}_{t,1}^{-1} - \mathbf{S} \\
\mathbf{S} &\succeq (\mathbf{I}_4 + \mathbf{B}^H\mathbf{B})^{-1}
\end{aligned}$$

By Schur's complement, the above two inequalities can be changed into the following convex form:

$$\begin{aligned}
\begin{bmatrix} \mathbf{D}_1 & \mathbf{I}_4 \\ \mathbf{I}_4 & \mathbf{I}_4 + \mathbf{C}_{t,1}^{-1} - \mathbf{S} \end{bmatrix} &\succeq 0 \\
\begin{bmatrix} \mathbf{S} & \mathbf{I}_4 \\ \mathbf{I}_4 & \mathbf{I}_4 + \mathbf{R} \end{bmatrix} &\succeq 0
\end{aligned}$$

Eventually we obtain the following convex programming

$$\begin{aligned}
\min_{\mathbf{R},\mathbf{S},\mathbf{D}_1} \quad & \mathrm{tr}(\mathbf{D}_1\mathbf{C}_c) \\
\text{s.t.} \quad & \begin{bmatrix} \mathbf{D}_1 & \mathbf{I}_4 \\ \mathbf{I}_4 & \mathbf{I}_4 + \mathbf{C}_{t,1}^{-1} - \mathbf{S} \end{bmatrix} \succeq 0 \\
& \begin{bmatrix} \mathbf{S} & \mathbf{I}_4 \\ \mathbf{I}_4 & \mathbf{I}_4 + \mathbf{R} \end{bmatrix} \succeq 0 \\
& \mathrm{tr}(\mathbf{C}_c^{-1}\mathbf{R}) \leq mP/\sigma_v^2 \\
& \mathbf{R} \succeq 0
\end{aligned} \tag{11}$$

Problem (11) is a semidefinite programming (SDP) [11]. SDP is a special class of convex optimization problem, and therefore enjoys all the advantages of convexity. There are well-developed numerical methods to solve a general convex optimization problem, among which the most well known one is the interior point method. In the numerical example, we adopt an optimization toolbox: SeDuMi[2] [12] to solve the SDP formulated in (11).

We show a numerical example to demonstrate the advantage of the polarization optimization over convectional approach with fixed H and V polarizations. In the conventional system, the transmit polarization and receive polarization are either H or V, *i.e.*, $\boldsymbol{\xi} = [1, 0]^T$ or $[0, 1]^T$, and $\boldsymbol{\eta} = [1, 0]^T$ or

[1] Notice that this does not change the solution since at the optimal solution, the equality holds, which can be proved by complementary slackness theorem [11].

[2] SeDuMi, which stands for Self-Dual-Minimisation, is a software package that solves optimization problems over symmetric cones using the primal-dual interior-point methods.

$[0,1]^T$. With the assumption of equal power for each pulse, this results in the following options for $\mathbf{a}(i)$ (rows of $\mathbf{A}$):

$$\begin{aligned}
\mathbf{a}_1^{(c)} &= \sqrt{P}\,\boldsymbol{p}([1,0]^T,[1,0]^T) = \sqrt{P}\,[1,0,0,0]^T \\
\mathbf{a}_2^{(c)} &= \sqrt{P}\,\boldsymbol{p}([0,1]^T,[1,0]^T) = \sqrt{P}\,[0,1,0,0]^T \\
\mathbf{a}_3^{(c)} &= \sqrt{P}\,\boldsymbol{p}([1,0]^T,[0,1]^T) = \sqrt{P}\,[0,0,1,0]^T \\
\mathbf{a}_4^{(c)} &= \sqrt{P}\,\boldsymbol{p}([0,1]^T,[0,1]^T) = \sqrt{P}\,[0,0,0,1]^T
\end{aligned} \tag{12}$$

Further, we choose the following covariance matrices for the target and clutter

$$\mathbf{C}_\mathrm{t} = \begin{bmatrix} 0.14 & 0.05+0.23i & 0.11-0.09i & 0.13+0.05i \\ 0.05-0.23i & 0.44 & -0.15-0.28i & 0.13-0.25i \\ 0.11+0.09i & -0.15+0.28i & 0.31 & 0.16+0.18i \\ 0.13-0.05i & 0.13+0.25i & 0.16-0.18i & 0.21 \end{bmatrix}$$

$$\mathbf{C}_\mathrm{c} = \begin{bmatrix} 6.00 & -1.46-3.97i & 1.66-0.77i & 0.01-1.55i \\ -1.46+3.97i & 4.19 & 1.47+0.43i & -1.52+1.25i \\ 1.66+0.77i & 1.47-0.43i & 3.05 & -3.55-1.54i \\ 0.01+1.55i & -1.52-1.25i & -3.55+1.54i & 6.89 \end{bmatrix}$$

Fig. 1 plots the MSE performance of estimating $\mathbf{S}_\mathrm{t}$ based on two schemes: (i) optimally designed $\mathbf{a}(i)$, and (ii) conventional $\mathbf{a}(i)$ from (12). The signal to noise ratio (SNR) is defined to be P/σ_v^2. We plot two cases by taking $m = 4$ or $m = 16$. As can be seen, the optimally designed $\mathbf{a}(i)$ based on polarization selection and power scheduling leads to a power gain of 3–6 dBs.

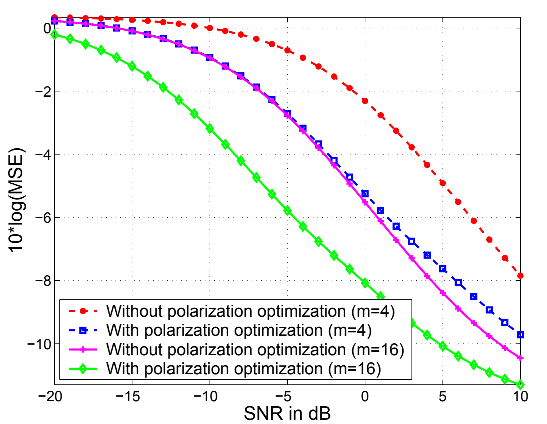

Fig. 1. MSE performance comparison for the estimation of the scattering matrix with or without polarization optimization.

4. CONCLUSION

We investigated the polarization optimization and power scheduling for the estimation of a target scattering matrix in a heavy clutter environment. An average power constraint for radar pulses was assumed. We cast this problem as a nonlinear optimization problem for the optimal design of a radar sensing matrix, which is further reformulated into a convex form and is thus numerically easily solvable. The numerical results demonstrate that by carefully choosing the transmit/receive polarizations and pulse power levels, clutter interference can be efficiently suppressed.

In this work we proposed a one-step optimization to select the optimal polarizations and power levels for all pulses. It is expected that additional performance gain can be achieved if such optimization is done sequentially on a pulse-by-pulse basis by using the most currently acquired information. In addition, to make the radar polarimetric sensing more efficient, multi-dimensional information of the incoming EM field at the radar receiver can be simultaneously measured using, *e.g.*, EM vector sensors [4]. Such a strategy will lead to additional constraints on the design of the sensing matrix $\mathbf{A}$. We will explore these topics in our future research.

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