INVESTIGATION OF SIGNAL DEMODULATION OF MACH-ZEHNDER INTERFEROMETRIC FIBER OPTIC SENSOR WITH 3×3 COUPLER

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ABSTRACT

Fiber optic sensors have been used widely during the past two decades. Demodulation module is very important in the system. The performance of the system may be suffered significantly if the demodulation algorithm is not properly carried out. The demodulation algorithm for a Mach-Zehnder interferometric fiber acoustic sensor with 3×3 coupler is investigated in this paper. The mathematical expression of the demodulation error is given under the condition that the splitting ratio of the three arms is not even. Based on that, a method to rectify the unbalance is proposed, which shows good performance in simulations.

Index Terms—fiber optic sensor, demodulation, coupler

1. INTRODUCTION

Over the past twenty years, two major product revolutions have taken place due to the growth of the optoelectronics and fiber optic communications industries. In parallel with these developments, fiber optic sensor technology has been a major user of technology associated with the optoelectronic and fiber optic communication industry [1-6].

Signal demodulation is a key technology to fiber optic sensors. There are several interferometric configurations that are commonly used in fiber optic sensing applications. Perhaps the simplest configuration is the Mach-Zehnder interferometer, shown in figure 1, where the light propagates in one direction from the source through the interferometer to the detectors. The two arms of the interferometer are typically named signal arm and reference arm. In many transducers the reference arm is shielded from the environment, and only the signal arm is exposed to the measurand. This is not a requirement, and in some transducers both arms are used for sensing in either differential or push-pull configurations to double the sensitivity of the transducer or for gradiometer applications. Due to the low loss of the optical fiber, the sensor interferometer can be a few meters or several tens of kilometers away, with little impact on the performance of the sensor [7].

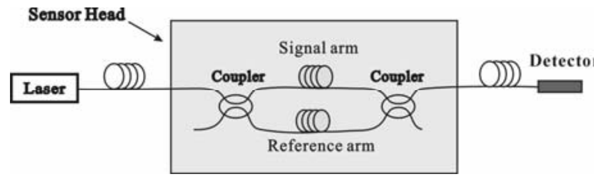

Figure 1. Fiber optic Mach-Zehnder interferometer configuration.

We can construct 3×3 coupling detector by replacing the output coupler of the common Mach-Zehnder interferometer with a 3×3 coupler, shown in figure 2. This increases the sensitivity by a factor of three compared to modulating a single fiber arm.

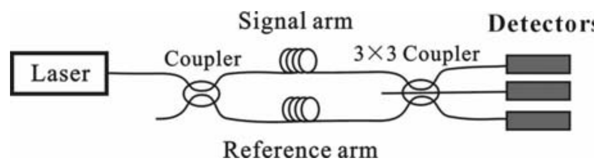

Figure 2. 3×3 coupling detector configuration.

2. REVIEW OF DEMODULATION FOR A 3×3 COUPLING FIBER OPTIC SENSOR

For a typical 3×3 coupler, the three output signals have 120° phase delay between each other. Ideally the splitting ratio among each arm is *1:1:1*. Outputs of the three arms are processed simultaneously in the calculation of demodulation. The three outputs can be written as

$$\begin{aligned} V1 &= D + E\cos(\phi - 2\pi/3) \\ V2 &= D + E\cos(\phi) \\ V3 &= D + E\cos(\phi + 2\pi/3) \end{aligned} \quad . \tag{1}$$

where ϕ is the measured signal. D and E are parameters defined by the system. Demodulation is to determine ϕ from the output $V1$, $V2$ and $V3$. The demodulation scheme is given in Figure 3. The steps of calculation is as follows.

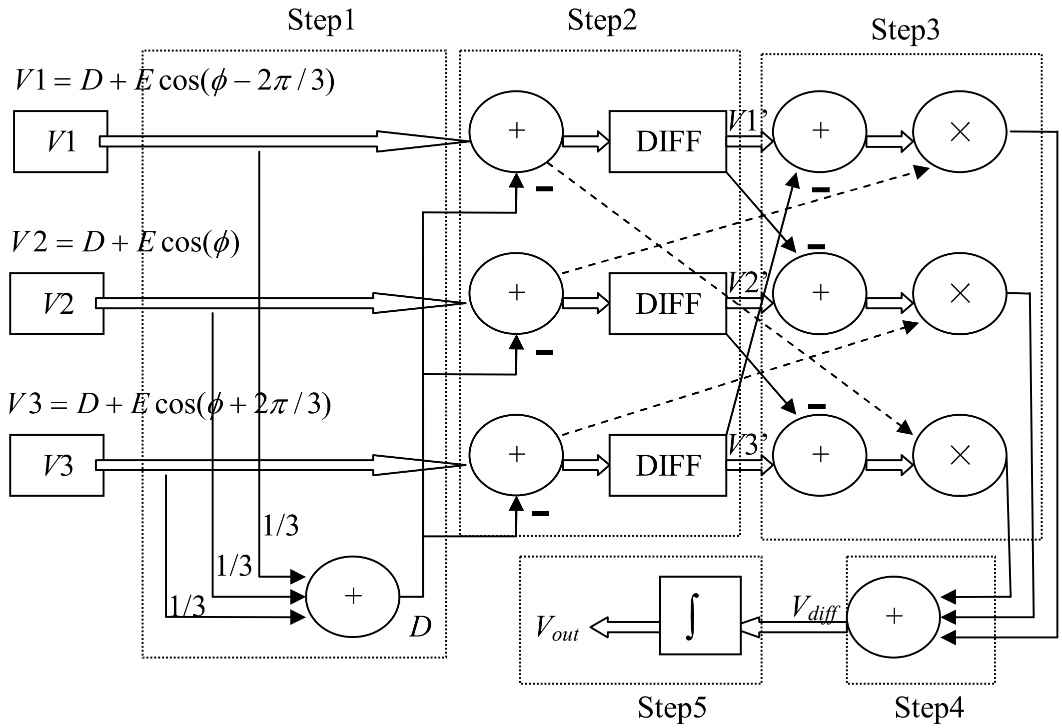

Figure 3. The demodulation process of 3×3 coupling

Step 1. Calculate D via

$$D = (V1 + V2 + V3)/3, \tag{2}$$

Step 2. Differentiate the equations (in (1)) with respect to "t",

$$\begin{aligned} V1' &= -E\cdot\dot{\phi}\cdot\sin(\phi - 2\pi/3) \\ V2' &= -E\cdot\dot{\phi}\cdot\sin(\phi) \\ V3' &= -E\cdot\dot{\phi}\cdot\sin(\phi + 2\pi/3) \end{aligned}, \tag{3}$$

Step 3. Make cross multiplication as follows,

$$\begin{aligned} (V1 - D)(V3' - V2') &= \sqrt{3}E^2\cdot\dot{\phi}\cdot\cos^2(\phi - 2\pi/3) \\ (V2 - D)(V1' - V3') &= \sqrt{3}E^2\cdot\dot{\phi}\cdot\cos^2(\phi) \\ (V3 - D)(V2' - V1') &= \sqrt{3}E^2\cdot\dot{\phi}\cdot\cos^2(\phi + 2\pi/3) \end{aligned}, \tag{4}$$

Step 4. Sum up the three results in Step 3,

$$V_{diff} = \frac{3\sqrt{3}}{2}E^2\dot{\phi}, \tag{5}$$

Step 5. Integrate V_{diff} then we get the final result

$$V_{out} = \frac{3\sqrt{3}}{2}E^2\phi\cdot \tag{6}$$

3. ERROR EXPRESSION DUE TO UNBALANCE AND A PROPOSED METHOD TO ELIMINATE ERROR

In fact, the splitting ratio of *1:1:1* could not be always guaranteed. We have to investigate if the demodulation algorithm still works well when the splitting ratio is not equal to 1.

The splitting ratio is assumed as *α:1:β*. The three outputs can be written as

$$\begin{aligned} V1 &= (D + E\cos(\phi - 2\pi/3))\alpha \\ V2 &= D + E\cos(\phi) \\ V3 &= (D + E\cos(\phi + 2\pi/3))\beta \end{aligned}, \tag{8}$$

Applying the demodulation algorithm given in Figure 3，we can get the demodulation result

$$V_{out} = \frac{\sqrt{3}}{2}E^2(\alpha + \beta + \alpha\beta)\phi + A\int\sin(\phi + \theta)\phi' dt, \tag{9}$$

where

$$\begin{aligned} A &= DE\sqrt{\frac{9}{4}(\alpha - \beta)^2 + \frac{3}{4}(\alpha + \beta - 2)^2} \\ tg\theta &= \frac{\sqrt{3}(\alpha - \beta)}{\alpha + \beta - 2} \end{aligned}, \tag{10}$$

We can see that (9) is composed of two parts. The first one is the signal multiplied by a constant. The other is the demodulation error.

A method is proposed subsequently to eliminate the demodulation error, which is summarized as follows.

1) Sampling the data from the three arms. The samples should be big enough.
2) Searching the maximum and minimum of *V1, V2* and *V3* in (8) within the total sample record.

$$V1_{\max} = \alpha(D+E), V2_{\max} = D+E, V3_{\max} = \beta(D+E)$$
$$V1_{\min} = \alpha(D-E), V2_{\min} = D-E, V3_{\min} = \beta(D+E) \quad (11)$$

D, E, α and *β* can be derived easily from the above formula.

3) Using $1/\alpha$ and $1/\beta$ to multiply $V1$ and $V3$ in (8) respectively, the three signals can be written as

$$\begin{aligned} V1 &= (\alpha D + \alpha E\cos(\phi - 2\pi/3)) \times 1/\alpha \\ V2 &= D + E\cos(\phi) \\ V3 &= (\beta D + \beta E\cos(\phi + 2\pi/3)) \times 1/\beta \end{aligned} \quad , \quad (12)$$

Then by 3×3 coupling demodulation algorithm above-mentioned we can get the demodulation result

$$V_{out} = \frac{3\sqrt{3}}{2} E^2 \phi \, . \quad (13)$$

Actually, Step 3) can also be taken placed of by the following operations. After removing D from $V1$, $V2$ and $V3$ in (8), the three outputs can be written as

$$\begin{aligned} V1 &= \alpha E\cos(\phi - 2\pi/3) \\ V2 &= E\cos(\phi) \\ V3 &= \beta E\cos(\phi + 2\pi/3) \end{aligned} \quad , \quad (14)$$

Then, the demodulation result is

$$V_{out} = \frac{\sqrt{3}}{2} E^2 (\alpha + \beta + \alpha\beta)\phi \; . \quad (15)$$

4. SIMULATIONS

Suppose the measured signal ϕ is a linear swept-frequency cosine signal, which starts at DC, the upper frequency is 1000Hz at t=0.02 sec. The sampling rate is 48 kHz. The splitting ratio of the three arms is assumed *5:1:0.8*. The waveform of the signal and the demodulation result are given in Figure 4. The solid line represents the signal and the dotted line represents the demodulation result.

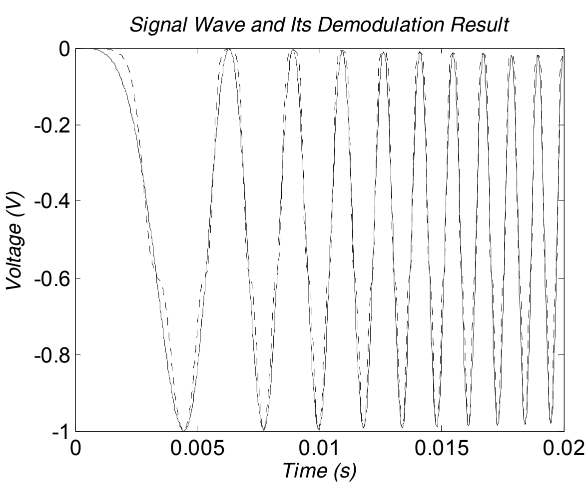

Figure 4. Normalized Signal wave and its demodulation result when the splitting ratio of the three arms is *5:1:0.8*

From Figure 4, we can find that the unbalance of the three arms may bring big error to the demodulation result. An variable *Err(α,β)* is defined to quantify how the demodulation error varying with splitting ratio.

$$Err(\alpha,\beta) = \frac{\sum[X(\alpha,\beta) - Y]^2}{\sum Y^2} , \quad (14)$$

where *X(α,β)* denotes the normalized demodulation result, *Y* denotes the normalized signal.

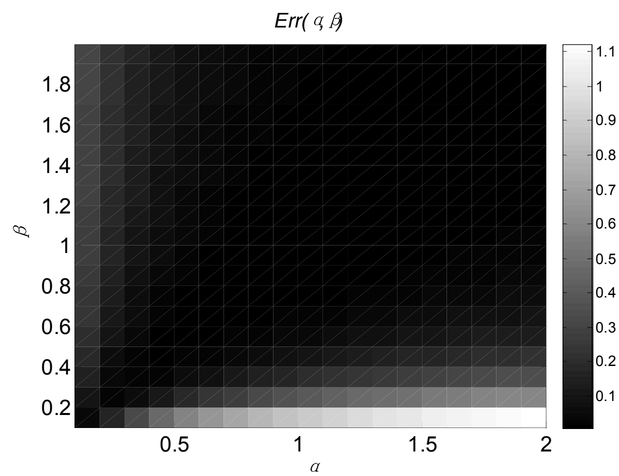

Figure 5. Distribution of *Err(α,β)* within *α*~[0.1,2] and *β*~[0.1,2]

The distribution of *Err(α,β)* within the area of *α*~[0.1,2] and *β*~[0.1,2] is given in Figure 5. Obviously, the error is more sensitive to *α* than *β*. At the same time, the error is decreasing when *α* is closing to *β*, and the error is increasing when the difference between *α* and *β* goes big.

The method to eliminate the demodulation error is used as follows.

1) Sampling 1000 points of data from the three arms.
2) Searching the maximum and minimum of *V1, V2* and *V3* in (8) within the samples, we get,

$$\begin{aligned} V1_{\max} &= 10.5, V1_{\min} = 0.5 \\ V2_{\max} &= 2.1, V2_{\min} = 0.11001 \\ V3_{\max} &= 1.68, V3_{\min} = 0.080044 \end{aligned} \quad , \quad (17)$$

D, α and *β* can be derived easily from the above formula.

$$\begin{aligned} D &= (V2_{\max} + V2_{\min})/2 \approx 1.105 \\ \alpha &= V1_{\max}/V2_{\max} = V1_{\min}/V2_{\min} = 5 \\ \beta &= V3_{\max}/V2_{\max} = V3_{\min}/V2_{\min} = 0.8 \end{aligned} \quad , \quad (18)$$

3) Using $1/\alpha$=0.2 and $1/\beta$=1.25 to multiply $V1$ and $V3$ respectively, the splitting ratio is modified to *1:1:1*. Then apply the demodulation algorithm as given in Figure 3. The difference between the demodulation result and the real signal is given in Figure 6.

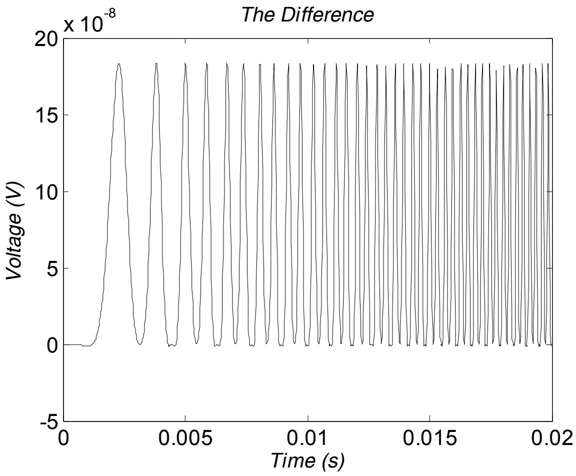

Figure 6. Different between the demodulation result when the splitting ratio of the three arms is fixed to *1:1:1*

From Figure 6, we can see that the difference of the signal and its demodulation result is quite small. The total demodulation error is calculated as.

$$Err = \frac{\sum [X-Y]^2}{\sum Y^2} = 2.4 \times 10^{-14}$$

4) Use another way for demodulation. Remove *D* from *V*1, *V*2 and *V*3. The difference between the signal and the demodulation result is given in Figure 7.

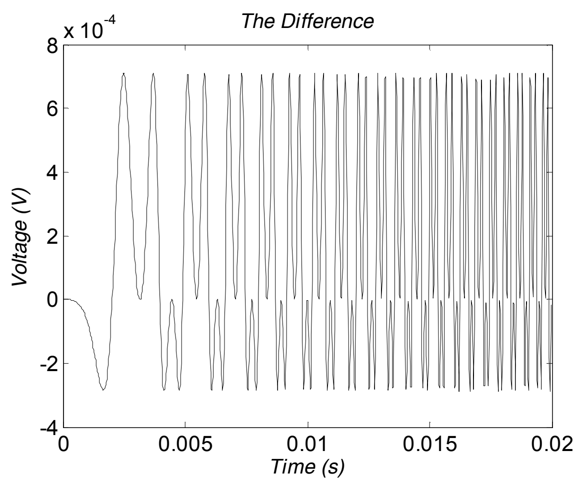

Figure 7. The difference of the signal and the fixed demodulation result when remove *D* from *V*1, *V*2 and *V*3

The difference between the signal and its demodulation result is quite small. The total demodulation error is calculated as.

$$Err = \frac{\sum [X-Y]^2}{\sum Y^2} = 3.5 \times 10^{-7}$$

Obviously, the total error is much bigger than that obtained in Step 3). Therefore, we may prefer the method in Step 3) to that in Step 4).

5. CONCLUSIONS

The demodulation of a 3×3 coupling fiber optic sensor is reviewed in this paper. In practice, the splitting ratio of the three arms may not be 1 exactly. Based on this background, this paper investigates how the demodulation error varies with unbalance. Some conclusions are obtained as follows.

1) Big error would arise when the splitting ratio is not equivalent to 1.
2) The error can be eliminated by compensating the three outputs from the arms.
3) The proposed method to fix the error is very helpful to improve the practicability of 3×3 coupling Mach-Zehnder interferimetic fiber optic sensor.

6. ACKNOWLEDGE

This paper is supported by Natural Sciences Funding of China (No. 10674149).

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