# A ROBUST NOISE INDEPENDENT METHOD TO ESTIMATE OUT OF FOCUS BLUR

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# ABSTRACT

Lens defocus causes image blurring. Restoring these kind of images is an interesting research field. The first and most important step of restoring images is blur identification. Since now, many researchers have presented methods to estimate out of focus blur and restore original images, but most of these methods suffer the additive noise. In this paper we have presented a method to estimate out of focus blur precisely, such that it does not depend on the noise level. This method uses two rough estimates of blur function parameters, then it calculates the real parameter using an iterative algorithm. Wiener filter, Bessel function and some mathematical relations properties are the bases of this method. Experimental results showed our method gives high precision.

*Keywords*— Blur identification, Out of focus, Wiener Filter, Bessel function.

## 1. INTRODUCTION

In most cases a blurring system is modeled as follows [2] [1]:

$$g(i,j) = h(i,j) \bigotimes f(i,j) + n(i,j) \tag{1}$$

In this equation g, f, n and h are observed image, original image, additive noise(usually Gaussian) and PSF of blurring system, respectively. Regarding to equation (1) to restore the original image we should estimate PSF. The procedure to find d is called blur identification.

Most of the time, the restoration problem is reviewed in frequency domain. Equation (2) shows the frequency domain version of equation (1).

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$
(2)

In this equation, G,H,F and N show the frequency response of observed image, blurring function, original image and additive noise respectively.

Out of focus blurring is the most famous blurring function that occurs frequently in images. In an imaging system focusing has an important role, because defocusing the lens causes the captured image lost its sharpness and some other feature. Therefore an analysis module of an Auto Focusing subsystem at image formation system should determine whether an input image is focused or not.

Many researchers have presented methods to estimate out focus blur since now, but most of these methods do not work precisely in image with high level of noise. The method presented in [3], tries to determine the exact location of blurred edges using LSF (Line Spread Function) analysis in spatial domain, using this information the out of focus blur is estimated. Another method that uses image LSF is presented at [7]. In the method presented at [7], also, the power spectrum equalization (PSE) restoration filter in image form is presented which is used to restore original image. The method presented in [6] uses block based edge classification to find amount of out of focus blur. This method works mainly on low and median frequency therefore sharp details are not improved in restoration process. There are some other methods that work in frequency domain; edge information is employed to find out of focus blur in [4]; however, extended DCT approach were used for edge detection of Bayer pattern in this paper. Using ML method [8] and wavelet transform[5] is also presented in literature to estimate out of focus blur.

All of this methods have constraints and limitation. For example ML method needs priori information about blur parameter or methods presented at [5] [4] need all objects in front of a back ground have sharp edges. Existing at least one edge pattern is addressed in [6]. Also, there is a limit of out of focus amount in the method presented in [6].

In this paper, we have presented a method to estimate out of focus blur using the properties of wiener filter and Bessel function. Our method has no constraints and it can work in each noise level. To do this at first we use two rough estimates of blur parameter and then by using an iterative algorithm, the real parameter is identified. The amount of noise will be estimated in this method properly. We have supposed the noise model is Gaussian with zero mean.

The rest of paper is organized as follows: In section 2 we present blur model and its parameter. In section 3 the blur estimation method is presented. Experimental results are introduced in section 4 and finally we have concluded our conclusion in section 5.



Fig. 1. The frequency response of out focus blur with R = 2.5 pixels

## 2. OUT OF FOCUS BLUR MODEL

In most cases, the out of focus blur is modeled as [3][1]:

$$h(x,y) = \begin{cases} 0 & if\sqrt{x^2 + y^2} > R\\ \frac{1}{\pi R^2} & if\sqrt{x^2 + y^2} \le R \end{cases}$$
(3)

in this equation R is radius of COC( Circle of Confusion). It has been shown in [9] that an accurate and complex physical model does not result in significantly restoration than this geometric model. Regarding to equation (3) to find blurring function it is significant to find R. The frequency response of equation (3) which is called OTF (Optical Transfer Function) is defined by equation (4) that is based on a Bessel function of the first kind [10]:

$$H(u,v) = \left[\frac{J_1(R\sqrt{u^2 + v^2})}{R\sqrt{u^2 + v^2}}\right]$$
(4)

Where J is the Bessel function of first kind and R is radius of uniform disk. The Bessel function is defined as:

$$J_{\alpha}(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha}$$
(5)

The  $\Gamma$  function is defined as:

$$\Gamma(Z) = \int_0^\infty t^{Z-1} e^{-t} \tag{6}$$

when Z is a complex number. If Z was a real function then definition of  $\Gamma$  is:

$$\Gamma(Z) = (Z - 1)! \tag{7}$$

Figure 1 shows the frequency response of equation (3) with specified radius and figures 2,3 show a blurred image using out of focus blurring and its frequency response.

#### 3. BLUR ESTIMATION METHOD

Regarding to equation (3) to estimate blurring function, it is enough to estimate the R parameter that is radius of COC. To do it, we need some mathematical observation. Suppose



Fig. 2. (a)-Camera man image that is degraded using out of focus blur with parameter R = 10 pixels and no additive noise(b)-frequency response of (a).



Fig. 3. (a)-Lena image that is degraded using out of focus blur with parameter R = 12 pixels and additive noise  $(SNR = 40 \ dB)$  (b)-frequency response of (a).

that the radius of COC in equation (3) is R. If we consider another blurring function that its parameter is  $R_1$ , then regarding to equation (4) we can conclude:

$$H_1(u,v) = \left[\frac{J_1(R_1\sqrt{u^2 + v^2})}{R_1\sqrt{u^2 + v^2}}\right]$$
(8)

Using equations (4) and (8) we can conclude that:

$$\frac{H(u,v)}{H_1(u,v)} = \frac{R}{R_1} \left[ \frac{J_1(R\sqrt{u^2 + v^2})}{J_1(R_1\sqrt{u^2 + v^2})} \right]$$
(9)

The equation (9) concludes the following equation at the frequency center:

$$\lim_{(u,v)\to(0,0)}\frac{H(u,v)}{H_1(u,v)} = \frac{R}{R_1}$$
(10)

At now, we are going to restore degraded image using wiener filter. The wiener filter equation in frequency domain looks like the following equation:

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + k}$$
(11)

or

$$W(u,v) = \frac{1}{H(u,v)} * \frac{|H(u,v)|^2}{|H(u,v)|^2 + K^2}$$
(12)

In these equations W(u, v) shows where filter in frequency domain, H(u, v) shows the frequency response of degradation function and K is a constant that models signal to noise ratio. To restore a degraded image; it is enough to use the following equation:

$$F'(u,v) = G(u,v)W(u,v)$$
 (13)

F', G and W show restored image frequency response, observed image frequency response and wiener filter frequency domain, respectively. Because observed image is already available to restore image, we need to estimate W(u, v) that is based on H(u, v). Therefore, it is enough to find frequency response of degradation function (H(u, v))to restore image.

At first, we suppose that we use a arbitrary COC radius( $R_a$ ) to create a degraded function( $H_a$ ). If we use  $H_a$  to restore original image, then we can conclude that:

$$F_a(u,v) = G(u,v) \cdot \frac{H_a^*(u,v)}{|H_a(u,v)|^2 + K}$$
(14)

By regarding equations (14) and (2) we can conclude that:

$$F_{a}(u,v) = (F(u,v).H(u,v) + N(u,v)).\frac{H_{a}^{*}(u,v)}{|H_{a}(u,v)|^{2} + K}$$
(15)

If we consider the frequency center of the restored image (that is available now) and if we assume that additive noise average is zero, then we can conclude the following equation:

$$F_a(0,0) = \frac{H_a^*(0,0).H(0,0).F(0,0)}{|H_a(0,0)|^2 + K}$$
(16)

In this equation H(0,0), F(0,0),  $F_a(0,0)$  and K are unknown parameters and  $H_a(0,0)$  is known parameter. If we restore degraded image by using another degradation function with arbitrary radius  $R_b$ , we can conclude the following equation in a same way:

$$F_b(0,0) = \frac{H_b^*(0,0).H(0,0).F(0,0)}{\left|H_b(0,0)\right|^2 + K}$$
(17)

By using equations (16) and (17) we have:

$$K = \frac{F_a(0,0) \cdot |H_a(0,0)|^2 \cdot H_b^*(0,0) - F_b(0,0) \cdot |H_b(0,0)|^2 \cdot H_a^*(0,0)}{F_b(0,0) \cdot H_a^*(0,0) - F_a(0,0) \cdot H_b^*(0,0)}$$
(18)

Regarding to equations (18),(9),(10) and (8) we can conclude:

$$K = R^2 \frac{F_a(0,0)R_a^2 R_b - R_b^2 R_a F_b(0,0)}{F_b(0,0)R_a - F_a(0,0)R_b}$$
(19)

At now, we are going to find the degradation function. Regarding to equation (16) and (8) we can conclude:

$$F(0,0) = F_a(0,0)(1+K)$$
(20)



Fig. 4. Barbara picture which was degraded by R = 6 and  $SNR = 35 \ dB$  pixels. Estimated parameter using our algorithm was R = 6.04 pixels.

In this equation F is real original image,  $F_a$  shows the frequency response of restored image using  $H_a$  degradation function, and K is noise level that can be computed using equation (19). Therefore, to find R it is enough to find K. But the value of F(0,0) which is the mean of original image is not known, precisely. To find K and using it to calculate R we used a simple iterative algorithm. To present this algorithm, the equation F'(0,0) is defined in a same manner with equation (20):

$$F'(0,0) = F_b(0,0)(1+K)$$
(21)

The estimation algorithm works as follow:

- 1. Use the average of gray levels of restored images using  $H_a$  and  $H_b$  as the first seed for F(0,0) and F'(0,0) and call them  $F_s$  and  $F'_s$  respectively.
- 2. Calculate F(0,0) and F'(0,0) using equation (20) and (21).
- 3. if  $F(0,0) F'(0,0) \le \epsilon$  return K.

4. 
$$F_s = F_s + STEP_s$$

5. goto 2.

If we suppose that the image under process is 8 bit gray level image, the max value of F(0,0) will be 255. Therefore, if above algorithm does not converge when  $F_s \geq 255$ , we rerun algorithm and the step 4 changes as:

$$F_s = F_s - STEP$$

The STEP value is arbitrary and if it is smaller then algorithm precision will be higher. The value we used for STEP was 0.2. By using the value of K extracted using this algorithm we can calculate R regarding to equation (19).

### 4. EXPERIMENTAL RESULTS

To validate our method we degraded some standard images like Lena, Baboon, Barbara, etc then we have added additive



Fig. 5. (a)-Baboon picture which was degraded by R = 7 and  $SNR = 50 \ dB$  pixels. (b)- Restored image of (a) using wiener filter and our algorithm.

Real R	Average of Estimated values
4	4.1
6	5.92
8	8.05
10	10.11
12	12.05
14	13.87

 Table 1.
 Average value of some estimated parameter against real ones.

noise to these images. To add additive noise we used a random variable to declare noise variance. The set of these images built our test engine. The resolution of test images was  $256 \times 256$ .Experimental results showed that our algorithm worked greatly. The calculated mean error for this method using 40 different degraded images were about 2.5% which is a great result. In our test bed we had images with low SNR. The SNR of images was between 20 dB to 60 dB. Our algorithm was success in high level noisy images, also. Figure 4 shows a blurred image and its corresponding estimation. Figure 5 shows a degraded image and its restoration result was obtained using wiener filter; the blur parameter was estimated using our method. Table 1 shows some real parameters and their estimated values using our algorithm.

### 5. CONCLUSION

In this paper we presented a precise, robust and noise independnt method to estimate the out of focus blur function. In spite of other presented method since now, our method has no constraints and it works on all defocused images. This method estimates precisely blur parameters in noisy image with low *SNRs* because it can estimate noise parameter precisely.

To test our method we used some degraded images that additive noise was added to them. The type of image degradation was known and we used our method to estimate its parameter; the experimental results were great. In future work we are going to develop a method that can identify the degradation type before blur identification.

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