A DETECTION ALGORITHM FOR ZERO-QUANTIZED DCT COEFFICIENTS IN JPEG

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ABSTRACT

The discrete cosine transform (DCT) is widely used in image/video coding standards. However, since most DCT coefficients will be quantized to zeros, a large number of redundant computations are introduced. This paper presents an early detection algorithm to predict zero-quantized DCT coefficients for fast JPEG encoding. Based on the theoretical analysis for 2-D DCT and quantization in JPEG standard, we derive a sufficient condition under which each quantized coefficient becomes zero. Finally, the transform of the zero-quantized coefficients is omitted. Experimental results show that the proposed algorithm can significantly reduce the redundant computations and speed up the image encoding. Moreover, it doesn't cause any performance degradation. Computational reduction also implies longer battery lifetime and energy economy for digital applications.

Index Terms— discrete cosine transform (DCT), JPEG, image coding, computational complexity

1. INTRODUCTION

The discrete cosine transform (DCT) performs very close to the statistically optimum Karhunen-Loeve transform (KLT) in terms of compression efficiency [1], so it has been widely used in speech and image/video compression. Traditionally, the objective in image coding has been the high compression performance, which is usually achieved at the cost of increasing computational complexity. However, as most portable devices such as mobile phones are still suffering from the lack of computational power and energy-consumption constraints, there is significant interest and research in reducing the computations for fast encoding.

Many algorithms have been developed for fast calculation of DCT. These algorithms can be classified into direct and indirect algorithms. The direct algorithms generally have a regular structure, which reduces the implementation complexity [2]. On the other hand, indirect algorithms exploit the relationship between DCT and other transforms. These algorithms include the calculation of DCT through Hartley [3], polynomial transform [4] and Poisson Equation [5]. Both direct and indirect algorithms can speed up the calculation of DCT by utilizing more efficient structure. However, they do not reduce the redundant computations.

As the structure for calculation of DCT is optimized, more efforts are focused on reducing the redundant computations of DCT coefficients. Most of these effects are on motion-compensated DCT blocks [6]-[7] and significant reductions are obtained. However, they cannot be directly applied to the normal DCT in JPEG and intra block in MPEG. Y. Nishida proposed a zero-value prediction for fast DCT calculation [8] in 2003. If two consecutively zero elements are produced during the DCT operation, the remaining transform is skipped. Although this method can reduce the total computations by 29% for DCT, the visual quality is degraded.

In what follows we describe an early detection algorithm to skip redundant DCT and quantization without any quality degradation. Although the proposed model is implemented based on the 8×8 DCT in JPEG, it can be widely used on other DCT based image/video standards. As a result, high prediction efficiency and good computational savings are achieved by the proposed model.

The rest of this paper is organized as follows. The sufficient condition for zero-quantized DCT coefficient is mathematically analyzed in Section 2. Section 3 proposes the early detection algorithm for fast JPEG encoding. The experimental results are presented in Section 4. Finally, Section 5 concludes this paper.

2. ANALYSIS OF 2-D DCT AND QUANTIZATION

In this paper, we mainly consider the 8×8 2-D DCT which is widely used in JPEG and MPEG standards. If we define $f(x,y)$ as the pixel value, $0 \le x,y \le 7$, the DCT coefficient $F(u,v)$, $0 \le u,v \le 7$, is computed by

$$F(u,v) = \frac{c(u)c(v)}{4}\sum_{x=0}^{7}\sum_{y=0}^{7} f(x,y)\cos\frac{(2x+1)u\pi}{16}\cos\frac{(2y+1)v\pi}{16} \quad (1)$$

where $c(u), c(v) = 1/\sqrt{2}$, for $u,v = 0$, and $c(u)$, $c(v) = 1$, otherwise.

As DCT is a linear and separate transform, we can calculate the 8×8 2-D DCT in the row-column order. The row-wise transform of eight point DCT is defined as

$$F(u,y) = \frac{c(u)}{2}\sum_{x=0}^{7} f(x,y)\cos\frac{(2x+1)u\pi}{16} \quad (2)$$

where $\forall u,y \in \{0,1,\ldots,7\}$ and $F(u,y)$ is the DCT coefficient after the row transform.

TABLE I THRESHOLDS OF ZERO QUANTIZED ROW-WISE 1-D DCT COEFFICIENTS ($0 \le y \le 7$)

Threshold	DCT Coefficient (u, y)
$T_1(y) = \dfrac{2\alpha}{\cos(\pi/16)}$	$u = 1, 3, 5, 7$
$T_2(y) = \dfrac{2\alpha}{\cos(\pi/8)}$	$u = 2, 6$
$T_3(y) = 2\sqrt{2}\alpha$	$u = 4$

Then, the column-wise DCT after the row transform is

$$F(u,v) = \frac{c(v)}{2}\sum_{y=0}^{7} F(u,y)\cos\frac{(2y+1)v\pi}{16} \quad (3)$$

$$\forall u, v \in \{0,1,\ldots,7\}$$

Similarly, we decompose the 8×8 quantization matrix $Q(u,v)$ into the following format

$$Q(u,v) \approx \alpha \times Q^f(u,v) \quad (4)$$

if we define

$$\alpha = \left\lfloor \sqrt{min\{Q(u,v)\}} \right\rfloor \quad \text{and} \quad Q^f(u,v) = \lfloor Q(u,v)/\alpha \rfloor \quad (5)$$

$$\forall u, v \in \{0,1,\ldots,7\}$$

where $\alpha, Q^f(u,v)$ are the quantization parameters used to quantize the DCT coefficients after the transform in each stage in the row-column order. And $\lfloor x \rfloor$ denotes the nearest integer less than or equal to x.

In this way, the quantization step is integrated into the 2-D DCT. Firstly, the row-wise DCT is performed and then quantized by α. Secondly, the column-wise DCT is computed and quantized by $Q^f(u,v)$. As α is small enough, it does not cause any information loss compared to the standard DCT and quantization approach in JPEG.

Therefore, the DCT coefficient after the row transform will be quantized to zero if such condition holds true

$$F(u,y) < \alpha \quad (6)$$

And the 2-D DCT coefficient will be quantized to zero if

$$F(u,v) < Q^f(u,v) \quad (7)$$

3. PROPOSED EARLY DETECTION ALGORITHM

3.1 Proposed early detection model

Since the 2-D DCT can be calculated separately, we first consider the row-wise 1-D DCT. If $\bar{f}(y)$ is the mean value of the eight pixels in each row and $f^r(x,y)$ is the residual pixel value, we define

$$\bar{f}(y) = \tfrac{1}{8}\textstyle\sum_{x=0}^{7} f(x,y), \quad f^r(x,y) = f(x,y) - \bar{f}(y) \quad (8)$$

Then, each DCT coefficient can be computed by the mean value and the eight residual pixel values as

$$F(u,y) = \begin{cases} 2\sqrt{2}\bar{f}(y) & for \ \ u = 0 \\ \dfrac{c(u)}{2}\displaystyle\sum_{x=0}^{7} f^r(x,y) cos\frac{(2x+1)u\pi}{16} & otherwise \end{cases} \quad (9)$$

where (10) gives the full proof of the deduction process.

In addition, the sum of absolute difference $SAD(y)$ of the eight residual pixels in each row is defined as

$$SAD(y) = \sum_{x=0}^{7} |f^r(x,y)| \quad (11)$$

From (9) and (11), the coefficient $F(u,y)$ is bounded by

$$F(u,y) \le \frac{c(u)}{2} max\left\{\left|cos\frac{(2x+1)u\pi}{16}\right|\right\} \times SAD(y) \quad (12)$$

$$for \ u \ne 0$$

So, $F(u,y)$ can be predicted as zero if

$$SAD(y) \le \frac{2\alpha}{c(u) max\left\{\left|cos\frac{(2x+1)u\pi}{16}\right|\right\}} \qquad for\, u \ne 0 \quad (13)$$

Therefore, we can predict $F(u,y)$ as zero by comparing $SAD(y)$ with the threshold in (13). Each DCT coefficient is bounded relying on the frequency position that affects the maximum value of the cosine function. As a result, the thresholds to determine zero-quantized DCT coefficients are listed in Table I.

$$F(u,y) = \frac{c(u)}{2}\sum_{x=0}^{7} f(x,y) cos\frac{(2x+1)u\pi}{16} = \frac{c(u)}{2}\sum_{x=0}^{7}\left[f^r(x,y) + \bar{f}(y)\right] cos\frac{(2x+1)u\pi}{16}$$

$$= \frac{c(u)}{2}\sum_{x=0}^{7} f^r(x,y) cos\frac{(2x+1)u\pi}{16} + \frac{c(u)}{2}\bar{f}(y)\sum_{x=0}^{7} cos\frac{(2x+1)u\pi}{16} \quad (10)$$

Since

$$\sum_{x=0}^{7} f^r(x,y) = 0 \quad for\, u = 0 \qquad and \qquad \sum_{x=0}^{7} cos\frac{(2x+1)u\pi}{16} = 0 \quad for\, u \ne 0$$

Thus, (9) is proved.

TABLE II THRESHOLDS OF ZERO QUANTIZED 2-D DCT COEFFICIENTS ($0 \le u \le 7$)

Threshold	DCT Coefficient (u, v)
$T_1(u) = \frac{2Q^f(u,v)}{\cos(\pi/16)}$	$v = 1,3,5,7$
$T_2(u) = \frac{2Q^f(u,v)}{\cos(\pi/8)}$	$v = 2,6$
$T_3(u) = 2\sqrt{2}Q^f(u,v)$	$v = 4$

Similarly, we continue to decompose the 1-D DCT coefficient $F(u,y)$ into a series of mean values $\bar{F}(u)$ and residuals $F^r(u,y)$ as (8). The 2-D DCT coefficient $F(u,v)$ will be predicted as zero if

$$SAD(u) \le \frac{2Q^f(u,v)}{c(v)max\left\{\left|cos\frac{(2y+1)v\pi}{16}\right|\right\}} \quad for\, v \ne 0 \quad (14)$$

where

$$SAD(u) = \sum_{y=0}^{7} |F^r(u,y)| \quad (15)$$

The threshold for each 2-D DCT coefficient $F(u,v)$ to be quantized to zero is listed in Table II.

Theoretically, the DCT coefficients can be most likely predicted as zeros in the following two situations. One, if all the eight values are very close to zeros (e.g., high frequency coefficients). Two, the variation is small enough. Fig. 1 gives an example based on Couple image. Eight 1-D DCT coefficients after the row transform at $u = 0$ are shown in (a). Although these coefficients are large, the residuals are very small. (b) shows the eight high frequency coefficients at $u = 7$ and they are similar to the residual values in (a). Therefore, all the 2-D DCT coefficients will be predicted as zeros without taking the column-wise transform.

3.2 Implementation of proposed detection model

Based on the thresholds in Table I and II, we propose an algorithm to perform the 2-D DCT computations in the row-column order. Table III shows the implementation on the row-wise stage. Take the row transform for example, if $SAD(y) \le T_1(y)$, we only compute the first coefficient on each row. Otherwise if $SAD(y) \ge T_3(y)$, all transform and quantization are required. For the column-wise transform, since the quantization $Q^f(u,v)$ is usually non-uniform, each coefficient has to be compared with its own threshold to decide to skip the transform or not.

As for the 2-D DCT implementation, we utilize the row-column approach and butterfly-flow structure. Since we can predict DCT coefficients as zeros in advance, the DCT computations can be skipped. For quantization, we just omit this step if the coefficients are set to zeros.

4. EXPERIMENTAL RESULTS

TABLE III IMPLEMENTATION OF PROPOSED ALGORITHM ON ROW-WISE TRANSFORM

Type	Condition	Implementation
1	$SAD \le T_1(y)$	only the first coefficient
2	$T_1(y) \le SAD \le T_2(y)$	only coefficients 0,1,3,5,7
3	$T_2(y) \le SAD \le T_3(y)$	only coefficients 0,1,2,3,5,6,7
4	$T_3(y) \le SAD$	all coefficients

	210	211	210	209	210	212	210	211
210	0	1	0	-1	0	2	0	1

(a)

	0	-1	0	-1	0	0	1	0
0	0	-1	0	-1	0	0	1	0

(b)

Fig. 1 Example of DCT coefficients to be predicted as zeros

In order to evaluate the performance of the proposed algorithm, a series of experiments were performed with JPEG. Four benchmark images (512×512) are tested. All the simulations are running on a PC with Intel Pentium 3.2G and 1.5Gbytes of RAM. The quantization strategy is in accordance with [9] where two quantization tables are used for luminance transform and chrominance transform. Moreover, a scaling factor p is used to get various size of compressed bit stream with different quality.

4.1Computational reduction of DCT and quantization

Firstly, we will study the computational complexity of the proposed model. The comparison of the complexity about DCT and quantization between the proposed model and the JPEG encoder are illustrated in Table IV. The required computational cost for the proposed model is

$$C = \frac{T_d}{T_d^o} \times 100\% \quad (16)$$

where T_d and T_d^o are the required encoding time of DCT and quantization for the proposed model and the baseline codec. It is obvious that the proposed algorithm can effectively reduce redundant computations and achieve better performance in terms of computational cost. In general, the average computations of 2-D DCT have been decreased by 10~50%, although the extent is different with different texture and different quantization.

4.2 False acceptance rate and false rejection rate

As two important evaluation parameters, the false acceptance rate (FAR) and the false rejection rate (FRR) are provided to evaluate the proposed analytical model. Normally, the smaller the FAR, the less the video quality degrades and the smaller the FRR, the more efficient the predictive model. Therefore, it is desirable to have both small FAR and FRR for an efficient predictive model.

From the experimental results, the proposed model has a zero FAR, which in turn validates our mathematical analysis in Section 2. So we only list the FRR in Table V for different images. Averagely, the proposed algorithm can predict 60% of the zero-quantized coefficients.

4.3 Video quality and encoding time comparison

Finally, we will study the visual quality and the encoding time of the proposed algorithm compared to the original codec. The visual quality is measured by the Peak Signal to Noise Ratio (PSNR), no visual degradation is observed for the proposed model. This is exactly in accordance with FAR. Fig.2 shows the entire encoding time of the proposed model. The encoding time reduction ∇T is defined as

$$\nabla T = \frac{T}{T_{org}} \times 100\% \quad (17)$$

where T and T_{org} are the entire encoding time of the proposed model and the JPEG encoder.

From Fig.2, it is obvious that our analytical model achieves better real-time performance than original codec. This validates that the proposed model can reduce the computational complexity of the encoder, which is more practical for real-time applications and portable devices.

5. CONLUSION

This paper proposes a detection algorithm to predict zero-quantized DCT coefficients for fast JPEG encoding. Based on the mathematical analysis, we derive a sufficient condition under which each DCT coefficient is quantized to zero. Finally, the transform of the zero-quantized coefficients is skipped. Experimental results show that the proposed model can significantly improve the encoding efficiency without visual degradation and outperforms the method in [8]. Moreover, it can be directly applied to other existing 2-D DCT schemes. Potential applications could be for portable digital devices with restrict battery lifetime and other areas with real-time requirement.

6. ACKNOWLEDGEMENT

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TABLE IV COMPUTATIONAL REDUCTION OF DCT AND QUANTIZATION (%)

Image	QP (p)	Row	Column	2-D DCT
Baboon	0.9	99.86	85.65	92.82
	1.6	99.74	62.17	81.15
	2.5	99.56	45.99	73.21
	3.6	99.32	36.50	68.26
Tree	0.9	98.98	58.68	78.98
	1.6	98.30	41.22	70.22
	2.5	97.46	31.88	65.42
	3.6	96.73	26.62	61.98
Airplane	0.9	87.28	51.55	69.79
	1.6	85.26	37.89	61.65
	2.5	83.77	28.94	56.53
	3.6	82.69	23.17	53.19
Couple	0.9	90.40	32.35	61.57
	1.6	86.70	24.70	55.96
	2.5	83.25	20.55	52.32
	3.6	80.50	18.03	49.89

TABLE V STASTICAL RESULTS OF FRR (%)

Image	QP (p)	Row	Column	2-D DCT
Baboon	0.9	98.65	78.86	80.87
	1.6	98.07	53.69	58.52
	2.5	97.47	39.00	46.16
	3.6	96.83	31.54	40.43
Tree	0.9	94.11	48.45	56.58
	1.6	92.36	33.42	45.32
	2.5	90.55	26.25	40.76
	3.6	89.39	22.66	39.04
Airplane	0.9	62.17	42.51	48.12
	1.6	62.35	32.30	41.30
	2.5	62.62	25.21	36.94
	3.6	62.74	20.63	34.28
Couple	0.9	75.08	25.79	40.43
	1.6	71.37	20.62	37.31
	2.5	68.51	17.75	35.78
	3.6	66.40	16.04	34.80

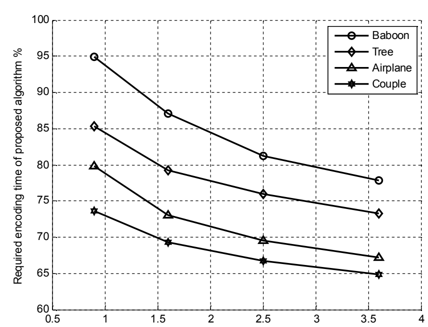

Fig.2 Required encoding time of the proposed detection algorithm compared to the baseline JPEG encoder

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