DETERMINING THE PARAMETERS IN REGULARIZED SUPER-RESOLUTION RECONSTRUCTION

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ABSTRACT

We derive a novel method to determine the parameters for regularized super-resolution problems. The proposed approach relies on the Joint Maximum a Posteriori (JMAP) estimation technique. The classical JMAP technique provides solutions at low computational cost, but it may be unstable and presents multiple local minima. We propose to stabilize the JMAP estimation, while achieving a cost function with an unique global solution, by assuming a gamma prior distribution for the hyperparameters. The resulting fidelity is similar to the quality provided by the best methods such as the Evidence, which are computationally expensive. Experimental results illustrate the low complexity and stability of the proposed method.

***Index Terms*—** Super-resolution, regularization, Bayesian estimation, JMAP.

1. INTRODUCTION

Research on super-resolution (SR) methods dates back to the 90's when the authors in [1] employed Fourier domain methods. Since then, many approaches have been proposed, including projections onto convex sets (POCS) [2], non-uniform interpolation [3] and iterative back-projection [4]. Regularized SR approaches based on maximum a posteriori (MAP) and regularized least squares appeared in [5, 6]. In general, regularized approaches minimize a cost function composed by the residual associated with the estimated high-resolution (HR) frame plus another term, called the prior term, used to regularize the problem.

One of the difficulties in SR is the existence of motion error between frames caused by imprecise motion estimation or by occlusion of objects moving in the scene. Motion error reduces the effectiveness of SR methods and generates some artifacts in the estimated HR image [7, 8]. To overcome this problem, [5, 6] propose to weight the residuals independently. However, the choice of proper weighting values is a difficult problem. In practice, weighting values as well as the regularization parameter have to be estimated from the data, which increases the complexity of the problem. The joint determination of weights and the regularization parameter, simply called multi-parameter problem in this paper, is addressed in [7, 8]. A statistical method for the problem, called Evidence, is proposed in [9]. The method is stable and provides good quality results. However it is computationally demanding for general SR problems and applies only to block-circulant matrices. Gradient-based methods [7, 8] estimate the HR frame and the parameters at each iteration. These methods have been shown to be stable and are, in general, faster than Evidence but the quality of the estimated frame is inferior.

We address the determination of parameters for the traditional regularized SR problem with multiple parameters. The regularized SR algorithm is reviewed in Section 2. In Section 3, the parameters estimation problem is addressed using the Joint Maximum a Posteriori (JMAP) estimation technique [10]. The classical JMAP approach, which assumes uniform density for the hyperparameters is, in general, unstable [10]. To circumvent this, we assume a gamma probability density for the hyperparameters in order to produce a stable algorithm with a unique global solution. Numerical experiments presented Section 4 illustrate that the proposed method provides HR frames with the same quality as classical methods such as Evidence [9] but at low computational cost. Section 5 concludes this paper.

2. REVIEW OF REGULARIZED SR METHODS

Traditional regularized SR algorithms [6, 5] produce a single HR frame from a sequence of LR frames by solving the problem:

$$\hat{\mathbf{f}}_k = \arg\min_{\mathbf{f}_k} \sum_{j=1}^{L} \alpha_{j,k} \|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2 + \lambda_k \|\mathbf{R}\mathbf{f}_k\|_2^2 \qquad (1)$$

Here, $\mathbf{g}_j$ is a vector of size $N \times 1$ that contains pixel information in lexicographic order of the LR frame, captured at instant j, $\mathbf{f}_k$, of size $M \times 1$, $M \geq N$, contains pixel information of the HR image captured at instant k, and $\mathbf{C}_{j,k} = \mathbf{D}_j\mathbf{M}_{j,k}$. Matrix $\mathbf{D}_j$, of size $N \times M$, models the acquisition process applied to the HR image $\mathbf{f}_j$ involving blurring and subsampling. Matrix $\mathbf{M}_{j,k}$, of size $M \times M$, represents the motion transformation, or warping. It can be generated either from a discretized continuous motion operator, where a parametric motion is assumed, or from a discrete motion vector field [11]. Matrix $\mathbf{R}$, of size $P \times M$, represents the prior term, introduced to enforce an unique and stable solution to (1). It is usually a discrete differential operator. The regularization parameter, λ_k, dictates the influence of the prior term in the solution.

In these algorithms it is assumed that the error level on the data is different for each frame, specially due to different levels of motion error in each frame. Thus the residuals related to the LR frames are weighted individually by $\alpha_{j,k}$ in (1). The $\alpha_{j,k}$ values tend to be small with the temporal distance between frames, $|j-k|$, since the frame similarity decreases with the distance.

3. PARAMETERS ESTIMATION METHOD

This section describes the proposed approach to estimate the parameters based on the joint maximum a posteriori (JMAP) estimation.

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JMAP is a Bayesian estimator that focus on the estimation of the HR images and the parameters together [12].

3.1. Classical JMAP

The classical JMAP estimative is given as:

$$\hat{\mathbf{f}}_k, \hat{\boldsymbol{\theta}}_k, \hat{\beta}_k = \arg\max_{\mathbf{f}_k,\boldsymbol{\theta}_k,\beta_k} \rho(\mathbf{f}_k, \boldsymbol{\theta}_k, \beta_k|\boldsymbol{g}) \tag{2}$$

where $\rho(\mathbf{f}_k, \boldsymbol{\theta}_k, \beta_k|\boldsymbol{g})$ is the posterior density, $\boldsymbol{g} = [\mathbf{g}_1^T, \dots, \mathbf{g}_L^T]^T$ are the LR frames, $\mathbf{f}_k$ is the HR image, $\boldsymbol{\theta}_k = [\theta_{1,k}, \dots, \theta_{L,k}]$ are the data hyperparameters, and β_k is the hyperparameter of the image prior density, and

$$\rho(\mathbf{f}_k, \boldsymbol{\theta}_k, \beta_k|\boldsymbol{g}) \propto \left[\prod_{j=1}^{L} \rho(\mathbf{g}_j|\mathbf{f}_k, \theta_{j,k})\rho(\theta_{j,k})\right] \rho(\mathbf{f}_k|\beta_k)\rho(\beta_k) \tag{3}$$

Functions $\rho(\theta_{j,k})$ and $\rho(\beta_k)$ are the prior densities assigned to the hyperparameters, also known as hyperpriors [9, 12]. The data density, $\rho(\mathbf{g}_j|\mathbf{f}_k, \theta_{j,k})$, and the image prior density, $\rho(\mathbf{f}_k|\beta_k)$, are the same as used in MAP estimation. Let us assume the following Gaussian densities

$$\rho(\mathbf{g}_j|\mathbf{f}_k, \theta_{j,k}) = \frac{1}{(2\pi\theta_{j,k})^{N/2}} e^{-\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2/(2\theta_{j,k})} \tag{4}$$

where $\theta_{j,k}$, in this case, is the variance of the LR frame error, and

$$\rho(\mathbf{f}_k|\beta_k) = \frac{1}{(2\pi\beta_k)^{M/2}} e^{-\|\mathbf{R}\mathbf{f}_k\|_2^2/(2\beta_k)} \tag{5}$$

In this work each $\theta_{j,k}$ and β_k are assumed independent of each other.

In MAP estimation the hyperparameters are assumed to have fixed values [12], whereas in JMAP estimation, the hyperparameters are random values that need to be estimated from the data as well as the HR image. Thus, in the same way that an image prior is needed for the estimation of the HR image, the hyperpriors are needed for the estimation of the hyperparameters. If one assumes uniform densities for the hyperpriors as in [9, 10], the values are equiprobable, therefore $\rho(\theta_{j,k}) \propto cte$ and $\rho(\beta_k) \propto cte$, for $0 < \theta_{j,k}, \beta_k < \infty$. The JMAP estimation with these hyperpriors becomes:

$$\hat{\mathbf{f}}_k, \hat{\boldsymbol{\theta}}_k, \hat{\beta}_k = \arg\min_{\mathbf{f}_k,\boldsymbol{\theta}_k,\beta_k} \sum_{j=1}^{L} \frac{\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2}{2\theta_{j,k}} + \sum_{j=1}^{L} \frac{N}{2}\ln\theta_{j,k} + \frac{\|\mathbf{R}\mathbf{f}_k\|_2^2}{2\beta_k} + \frac{M}{2}\ln\beta_k + cte \tag{6}$$

From (6) it is possible to find the hyperparameter for fixed $\mathbf{f}_k$, differentiating (6) with respect to the hyperparameters and setting the result to zero. This leads to the following closed form solutions:

$$\hat{\theta}_{j,k} = \frac{\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2}{N}, \quad \hat{\beta}_k = \frac{\|\mathbf{R}\mathbf{f}_k\|_2^2}{M}. \tag{7}$$

for the data hyperparameters and for the image hyperparameter, respectively. By substituting (7) into equation (6), leads to:

$$\hat{\mathbf{f}}_k = \arg\min_{\mathbf{f}_k} \sum_{j=1}^{L} \ln(\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2) + \frac{M}{LN}\ln(\|\mathbf{R}\mathbf{f}_k\|_2^2). \tag{8}$$

The minimizer in (8) is shown to be the solution of

$$\sum_{j=1}^{L} \alpha_{j,k}\mathbf{C}_{j,k}^T\mathbf{C}_{j,k}\mathbf{f}_k + \lambda_k\mathbf{R}^T\mathbf{R}\mathbf{f}_k = \sum_{j=1}^{L} \alpha_{j,k}\mathbf{C}_{j,k}^T\mathbf{g}_j \tag{9}$$

where λ_k and $\alpha_{j,k}$ are:

$$\lambda_k = \frac{M}{N}\frac{\|\mathbf{g}_k - \mathbf{D}_k\mathbf{f}_k\|_2^2}{\|\mathbf{R}\mathbf{f}_k\|_2^2}, \quad \alpha_{j,k} = \frac{\|\mathbf{g}_k - \mathbf{D}_k\mathbf{f}_k\|_2^2}{\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2} \tag{10}$$

The cost function in (8) is non-convex and the estimation unstable [9, 12, 7]. It requires proper constraining to avoid divergence. In the Bayesian statistical sense, constrains can be expressed by defining proper hyperparameter priors [12, 9]. When employing uniform densities, as done in the classical JMAP, the hyperparameters are not properly constrained and generate unstable estimates. More restrictive hyperpriors, on the other hand, lead to a stable estimative and a globally convex problem with a unique minimum.

3.2. Proposed Method

In the JMAP method, the density of the data or the prior density of the images is connected with the density of its respective hyperparameter. For example, the image prior, $\rho(\mathbf{f}_k|\beta_k)$, may enforce that the HR image is smooth, constraining the estimative to smooth images. The associated hyperparameter, β_k, defines "how smooth" is the resulting image. However, when an uniform density is assigned to the hyperparameter, as $\rho(\beta_k) \propto cte$, then it is implicitly assumed that an oversmooth image, like a constant intensity value image, when $\beta_k \to 0$, is as likely to occur as a noisy image, like the one produced by a completely unregularized estimation, when $\beta_k \to \infty$. A good hyperprior density should prevent the hyperparameter to reach very extreme values. The desired prior density for the hyperparameters needs to enforce positive values and provide low probability for very low or very high values. Among several candidates, the gamma density, with specific parameters so as to make it similar to the chi-squared density, has been shown practical and theoretical advantages over the alternatives.

The gamma densities for the hyperparameters are given by

$$\rho(\theta_{j,k}) = \frac{\theta_{j,k}^{(a_{j,k}-1)} b_{j,k}^{-a_{j,k}}}{\Gamma(a_{j,k})} e^{-\frac{\theta_{j,k}}{b_{j,k}}}, \quad \rho(\beta_k) = \frac{\beta_k^{(c-1)} d^{-c}}{\Gamma(c)} e^{-\frac{\beta_k}{d}} \tag{11}$$

where $a_{j,k}$ and c are the scale factors, $b_{j,k}$ and d are the shape factors, and $\Gamma(x)$ is the gamma function. Also, $E\{\theta_{j,k}\} = a_{j,k}b_{j,k}$, $\text{var}\{\theta_{j,k}\} = a_{j,k}b_{j,k}^2$, $E\{\beta_k\} = cd$ and $\text{var}\{\beta_k\} = cd^2$.

Substituting the gamma densities in equation (2) leads to:

$$\hat{\mathbf{f}}_k, \hat{\boldsymbol{\theta}}_k, \hat{\beta}_k = \arg\min_{\mathbf{f}_k,\boldsymbol{\theta}_k,\beta_k} \sum_{j=1}^{L} \left[\frac{\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2}{2\theta_{j,k}} + \left(\frac{N}{2} - a_{j,k} + 1\right)\ln\theta_{j,k} + \frac{\theta_{j,k}}{b_{j,k}}\right] + \frac{\|\mathbf{R}\mathbf{f}_k\|_2^2}{2\beta_k} + \left(\frac{M}{2} - c + 1\right)\ln\beta_k + \frac{\beta_k}{d} + cte \tag{12}$$

Note that when $a_{j,k} = N/2 + 1$ and $c = M/2 + 1$, the gamma density has nearly the same shape as the chi-squared density. These values for $a_{j,k}$ and c will be used in our development, they provide a necessary condition to achieve a globally convex problem. The $b_{j,k}$ and d will be replaced by expressions involving the expected values of the hyperparameters, namely $b_{j,k} = E\{\theta_{j,k}\}/a_{j,k} = m_{\theta_{j,k}}/a_{j,k}$ and $d = E\{\beta_k\}/c = m_{\beta_k}/c$. Assigning the mentioned values for $a_{j,k}$, $b_{j,k}$, c, and d, and applying some algebra, equation (12) reduces to:

$$\hat{\mathbf{f}}_k, \hat{\boldsymbol{\theta}}_k, \hat{\beta}_k = \arg\min_{\mathbf{f}_k, \boldsymbol{\theta}_k, \beta_k} \sum_{j=1}^{L} \frac{\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2^2}{2\theta_{j,k}} + \frac{\theta_{j,k}(N+2)}{2m_{\theta_{j,k}}} + \frac{\|\mathbf{R}\mathbf{f}_k\|_2^2}{2\beta_k} + \frac{\beta_k(M+2)}{2m_{\beta_k}} \quad (13)$$

Differentiating equation (13) with respect to the hyperparameters, for fixed $\mathbf{f}$, leads to the following estimative

$$\hat{\theta}_{j,k} = \sqrt{m_{\theta_{j,k}}} \frac{\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2}{\sqrt{N+2}}, \quad \hat{\beta}_k = \sqrt{m_{\beta_k}} \frac{\|\mathbf{R}\mathbf{f}_k\|_2}{\sqrt{M+2}}. \quad (14)$$

Substituting the results in (14) into (13), gives

$$\hat{\mathbf{f}}_k = \arg\min_{\mathbf{f}_k} \sum_{j=1}^{L} \gamma_{j,k} \|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2 + \mu_k \|\mathbf{R}\mathbf{f}_k\|_2 \quad (15)$$

where

$$\mu_k = \sqrt{\frac{m_{\theta_{k,k}}(M+2)}{m_{\beta_k}(N+2)}}, \quad \gamma_{j,k} = \sqrt{\frac{m_{\theta_{k,k}}}{m_{\theta_{j,k}}}} \quad (16)$$

Considering the gradient of the cost function in (15), the solution of this optimization problem is found when

$$\sum_{j=1}^{L} \alpha_{j,k} \mathbf{C}_{j,k}^T \mathbf{C}_{j,k} \mathbf{f}_k + \lambda_k \mathbf{R}^T \mathbf{R} \mathbf{f}_k = \sum_{j=1}^{L} \alpha_{j,k} \mathbf{C}_{j,k}^T \mathbf{g}_j \quad (17)$$

where the parameters are defined by

$$\alpha_{j,k} = \gamma_{j,k} \frac{\|\mathbf{g}_k - \mathbf{D}_k\mathbf{f}_k\|_2}{\|\mathbf{g}_j - \mathbf{C}_{j,k}\mathbf{f}_k\|_2}, \quad \lambda_k = \mu_k \frac{\|\mathbf{g}_k - \mathbf{D}_k\mathbf{f}_k\|_2}{\|\mathbf{R}_k\mathbf{f}_k\|_2} \quad (18)$$

The values of $\gamma_{j,k}$ and μ_k can be chosen from average values, as in (16), or from an analysis of the estimation error which gives

$$\gamma_{j,k} = 1/(1 + |j-k|), \quad \mu_k = \sqrt{tr(\mathbf{D}_k^T\mathbf{D}_k)}/\sqrt{tr(\mathbf{R}^T\mathbf{R})} \quad (19)$$

This choice will be carefully addressed in a further paper.

The proposed method involves a convex cost function with a unique minimum. Also, equation (15) can be minimized using fast methods as the Non-Linear Conjugated Gradient (NL-CG) [13].

4. EXPERIMENTS

The following experiment evaluates the performance of the methods in finding the parameters for the SR algorithms discussed in this paper. Given a HR image sequence, with known or previously estimated motion, the simulated acquisition process was performed, employing the average of a squared area of $R \times R$ pixels with subsampling factor of R, where $R = 2, 3$, and an additive white Gaussian noise with variance adjusted to achieve a fixed SNR [1]. Two situations were considered: high acquisition noise, with SNR$_A$=20dB and medium noise, with SNR$_A$=30dB. These noise levels are the typical levels found in commercial image sensors[2] [14].

The quality of the HR sequence recovered with the parameters found by a particular method is measured in terms of SNR [15]. Computational effort of each method was evaluated by considering the time it takes for convergence, where convergence is assumed to be reached when the improvement in quality is less 10^{-2} dB. This procedure was repeated using 20 random noisy realizations for each noise level. The entire experiment was repeated for each image sequence of a total of 6 different image sequences. In some of the sequences, the motion was artificially generated without considering occlusions in the scene, whereas in other sequences, which are from real video sequences, the motion was estimated using the optical flow method [16]. In this case, linear interpolated versions of the LR images were employed. The estimated motion vectors are not completely reliable in this case, therefore, occlusions and motion errors occur in several places in the sequence. In this evaluation, the procedure of detection and removal of the occlusion regions was not considered in order to evaluate the performance of the methods in finding the best weighting values.

The methods used to find the parameters are mentioned below.

K-HE - A deterministic method proposed in [8].

EVID - The statistical method Evidence, proposed in [9]. To apply this method to non block-circulant matrices, the trace of the inverse matrix is statistically estimated.

JMAP - The classical JMAP approach [10] as equation (8), using GC to find the HR images with (7) to update the parameters.

PROP - Proposed method with minimization using GC-NL.

All these methods are iterative. The Conjugate Gradient method is used in EVID, JMAP and in the proposed method to find the HR images. K-HE is limited to the Gradient method. The same initial conditions are considered: the initial HR image is a null image, and the initial parameters is randomly chosen from 10^{-6} to 10^{6}. In addition to these methods, the results obtained by the following predetermined parameter were also compared:

KNOWN - Employs the MAP estimative where the parameters are known a priori. Since the noise and the original HR images are known in the experiments, the hyperparameters are computed without difficulties. This method is used as reference only, since it cannot be used in practice.

The average quality of estimated images, its standard deviation and the relative computational time (with respect to KNOWN) resulting from the respective methods are shown in Table 1. Some visual results are shown in the Figure 1.

Table 1 shows that the quality obtained by the proposed method is similar to the results of KNOWN. Moreover, the results illustrate the low computational cost provided by the proposed method. The performance of the proposed method was superior than the K-HE, recently proposed for the multi-parameter traditional SR. One can notice the instability of the classical JMAP by its high STD results.

Table 1. SNR average in dB, standard deviation (STD), and relative computational time (CT) for multi-parameter SR algorithm

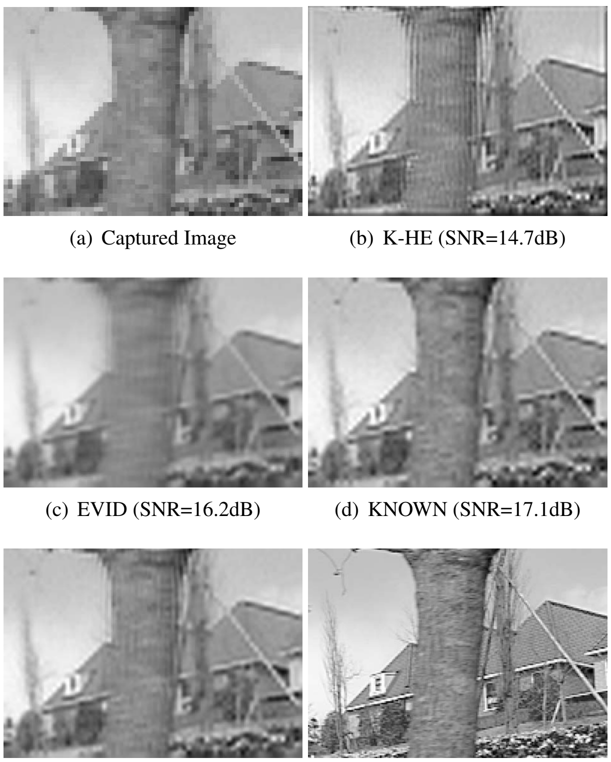
Method	R=2 SNR$_A$=20dB			R=3 SNR$_A$=30dB		
	SNR	STD	CT	SNR	STD	CT
K-HE	16.5	1.9	20.6	14.0	2.4	46.0
EVID	20.8	1.6	90.2	19.3	1.1	466.1
JMAP	15.3	6.1	66.5	11.6	3.6	84.3
KNOWN	22.0	0.6	1.0	20.9	1.1	1.0
PROP	22.7	0.9	1.8	21.2	1.9	2.7

Figure 1 illustrates the performance of the methods in controlling the weighting in order to avoid the distortions caused by the

[1]The acquisition SNR is defined as SNR$_A = 10\log_{10}(\sigma^2_{\mathbf{Df}}/\sigma^2_{\boldsymbol{\eta}})$, where $\sigma^2_{\mathbf{Df}}$ is a LR noise free sequence variance and $\sigma^2_{\boldsymbol{\eta}}$ is the noise variance.

[2]Typical acquisition SNR may vary from 10dB to 40dB, depending on the exposure [14].

large motion errors. One can see that the results of the proposed method were very similar to the results of KNOWN. Also, in this example, one can see that the distortions caused by the occlusions were not completely removed, but they were significantly attenuated. The complete removal of these distortions requires the use of a robust SR method [17], or an occlusion and motion error detection with a removal procedure [18].



(a) Captured Image (b) K-HE (SNR=14.7dB)

(c) EVID (SNR=16.2dB) (d) KNOWN (SNR=17.1dB)

(e) PROP (SNR=17.2dB) (f) Original Image

Fig. 1. Visual results from an image of the sequence Flower Garden, with R=3 and SNR$_A$=30dB.

5. CONCLUSIONS

In this paper, a technique to determine the parameters for super-resolution methods is proposed. The problem of parameters estimation has been addressed with the Bayesian theory, using Joint Maximum a Posteriori (JMAP) estimation. A gamma density is proposed for the hyperparameters in order to provide a globally convex cost function, resulting in a unique solution. The proposed method provides very low computational cost and produces estimated images with the same quality as the ones provided by the best classical methods. We provide a set of experiments to illustrate the superior efficiency and stability of the proposed technique when compared with other competing methods.

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