CONFIDENCE BASED OPTICAL FLOW ALGORITHM FOR HIGH RELIABILITY

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ABSTRACT

Estimation of optical flow is an important topic to provide motion information for motion analysis. This paper addresses an effective confidence based optical flow algorithm. It considers the bidirectional symmetry of forward and backward flow to compute the confidence measure for each flow estimate. According to the confidence, the reliable flow estimates have greater contribution to local averages while unreliable estimates are suppressed. The errors cannot be propagated. Since the image-driven and flow-driven discontinuity preserving methods have complementary advantages and limitations, we propose a region based method combining these two types of methods to preserve motion boundaries. Experiments on typical sequences have successfully demonstrated the validity of the proposed algorithm.

***Index Terms*—** Confidence, optical flow, smoothness, reliability

1. INTRODUCTION

Motion analysis is an important research topic for many digital video applications. Estimation of optical flow (2-D velocity field) has been widely used to provide motion information for motion analysis. Various algorithms have been proposed to compute optical flow [1]-[5]. These algorithms can be classified into the global methods which can yield dense optical flow fields due to the filling-in effect [5] and the local methods which usually yield sparse flow fields. Among the global methods, the pioneering work by H&S [1] is probably the most popular algorithm because of its simplicity and reasonable performance. However, it has the limitation of noise sensitivity. A significant improvement has been achieved in estimation of optical flow by some modification versions of the H&S algorithm. For example, Bruhn et al. [5] proposed a Combined Local-Global strategy (CLG) that combines the density of H&S method with the robustness of L&K [2]. In general, estimated optical flow is forward flow which moves a point from a location in frame t to some location in frame t+1. On the other hand, backward optical flow which moves a point from a location in frame t+1 to some location in frame t is not considered. However, it has been neglected that backward and forward flow can assist each other to improve accuracy of motion estimation.

This paper proposes a novel bidirectional strategy combing forward and backward optical flow to estimate optical flow accurately. It extends H&S algorithm. We have known the global methods have the advantage of the filling-in effect, i.e., at a location where the image gradient is zero, the flow estimate is usually the average of the flow estimates in the neighborhood of this location. In the conventional algorithms, flow estimate at each location is usually expected to have the same reliability and the same weight. Thus, the effect of the averaging is isotropic. It is clear that whether dense or sparse flow cannot have the same reliability for all pixels. Under this situation, estimate errors of flow estimates at neighboring locations can be propagated into local averages. It causes the unreliability of estimated optical flow. It is natural that optical flow estimates with higher reliability should have greater contributions than those with lower reliability to the averages. This requires greater weights should be assigned to flow estimates with high reliability while smaller weights to flow estimates with low reliability, which need to be suppressed in order to prevent the estimate errors from propagating. Thus, the effect of averaging is anisotropic. We introduce the confidence measure as this type of the adaptive weights. This paper combines forward and backward optical flow to compute the confidence measure at each location.

Local velocity averages indicate the influence of the smoothness constraints and implement the filling-in effect. However, the averages make true motion boundaries blurred. To cope with this problem, the image-driven and flow-driven methods have been proposed. The image-driven methods suppress smoothing at image boundaries. It may preserve some image boundaries as false flow boundaries where the smoothness effect is reduced. On the other hand, the flow-driven methods can preserve true flow boundaries. However, it is difficult to estimate precise motion boundaries. We have seen that both image-driven and flow-driven methods have complementary advantages and limitations. So this paper proposes a region-based discontinuity preserving method combining both image-driven and flow-driven methods.

2. PROPOSED OPTICAL FLOW ALGORITHM

H&S algorithm is based on the brightness constancy model (BCM), i.e., the brightness of each point is constant along its motion trajectory in an image sequence. Let $I(x,y,t)$ denote the brightness of a point (x,y) in the image at time t. The forward BCM (FBCM) and backward BCM (BBCM) at time t are respectively given as

$$I(x,y,t)=I(x+u_{\mathrm{f}},y+v_{\mathrm{f}},t+1) \tag{1}$$

and

$$I(x,y,t+1)=I(x+u_{\mathrm{b}},y+v_{\mathrm{b}},t) \tag{2}$$

where u and v are the horizontal and vertical components of optical flow respectively, and the subscripts f and b denote the forward and backward optical flow respectively. We expand $I(x+u_{\mathrm{f}},y+v_{\mathrm{f}},t+1)$ and $I(x+u_{\mathrm{b}},y+v_{\mathrm{b}},t)$ around the points (x,y,t) and $(x,y,t+1)$ by Taylor series expansion, respectively. FBCM and BBCM lead to the optical flow constraints (OFCs) (3) and (4) by eliminating the second and higher order terms in the above expansions, respectively.

$$\varepsilon_{\mathrm{f}}=u_{\mathrm{f}}I_x^0+v_{\mathrm{f}}I_y^0+I_t^{0,\mathrm{f}}=0 \tag{3}$$

and

$$\varepsilon_{\mathrm{b}}=u_{\mathrm{b}}I_x^1+v_{\mathrm{b}}I_y^1-I_t^{1,\mathrm{b}}=0 \tag{4}$$

where I_x^w, I_y^w, and I_t^w are the partial derivatives of I^w with respect to x, y, and t at time w ($w=0,1$), respectively, and the superscripts 0 and 1 denote time t and $(t+1)$, respectively. These two OFCs are the ill-posed equations. Then, assuming the smooth flow field, the forward optical flow or backward optical flow $\boldsymbol{v}_\ell$ is computed by minimizing the corresponding total energy

$$E_\ell=\iint_{x,y}\left(\varepsilon_\ell^2+\alpha^2\left((\frac{du_\ell}{dx})^2+(\frac{du_\ell}{dy})^2\right)+\alpha^2\left((\frac{dv_\ell}{dx})^2+(\frac{dv_\ell}{dy})^2\right)\right)dxdy \tag{5}$$

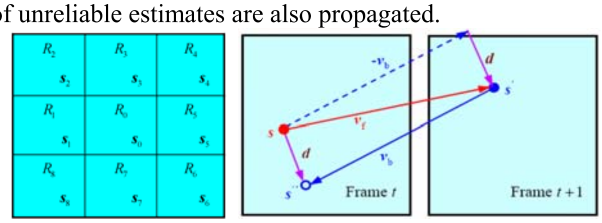
where the coefficient α is the smoothness weight, and the subscript ℓ ($\ell=\mathrm{f},\mathrm{b}$) represents the forward or backward optical flow. Then, we adopt the same method as H&S algorithm to solve the optical flow $(u_{\mathrm{f}},v_{\mathrm{f}})$ and $(u_{\mathrm{b}},v_{\mathrm{b}})$. Thus, the optical flow $\boldsymbol{v}_\ell$ can be obtained iteratively. At (n+1)th iteration,

$$\begin{cases} u_\ell^{n+1}=\bar{u}_\ell^n-I_x^w\dfrac{I_x^w\bar{u}_\ell^n+I_y^w\bar{v}_\ell^n+\lambda I_t^{w,\ell}}{\alpha^2+(I_x^w)^2+(I_y^w)^2} \\ v_\ell^{n+1}=\bar{v}_\ell^n-I_y^w\dfrac{I_x^w\bar{u}_\ell^n+I_y^w\bar{v}_\ell^n+\lambda I_t^{w,\ell}}{\alpha^2+(I_x^w)^2+(I_y^w)^2} \end{cases} \tag{6}$$

where $\bar{u}$ and $\bar{v}$ are the averages of the horizontal and vertical estimates in the neighborhood of each position, respectively, and $w=0$ if $\ell=\mathrm{f}$; otherwise 1. In (6), $\lambda=1$ if $\ell=\mathrm{f}$; otherwise -1. From (6), the forward and backward optical flow take advantage of the partial derivatives of the images at times t and $t+1$, respectively. The local averages indicate the influence of the smoothness constraints. It leads to the filling-in effect. Without preserving motion discontinuity, the local average $\bar{\boldsymbol{v}}_\ell^n(\boldsymbol{s}_0)=\left(\bar{u}_\ell^n(\boldsymbol{s}_0),\bar{v}_\ell^n(\boldsymbol{s}_0)\right)$ at one location $\boldsymbol{s}_0=(x,y)$ is defined as

$$\bar{\boldsymbol{v}}_\ell^n(\boldsymbol{s}_0)=\left(\sum_{\boldsymbol{s}_i\in N(\boldsymbol{s}_0)}R_\ell^n(\boldsymbol{s}_i)\boldsymbol{v}_\ell^n(\boldsymbol{s}_i)\right)\Big/\left(\sum_{\boldsymbol{s}_i\in N(\boldsymbol{s}_0)}R_\ell^n(\boldsymbol{s}_i)\right) \tag{7}$$

where $N(\boldsymbol{s}_0)$ denotes the pre-defined neighborhood of the location $\boldsymbol{s}_0$, and $R_\ell^n(\boldsymbol{s}_i)$ is the confidence measure which indicates the reliability of estimated optical flow $\boldsymbol{v}_\ell^n$ at this neighbor $\boldsymbol{s}_i$. Fig.1 (a) shows the corresponding mean template. The conventional algorithms only estimate forward optical flow. Furthermore, the confidence measure for each estimate is replaced by the constant weight. Horn and Schinck use the suitable weight $\omega_{s_i}=2$ for non-diagonal neighbors and $\omega_{s_i}=1$ for diagonal neighbors. It is clear that this type of the mean template cannot identify unreliable velocity estimates. The smoothness (average) can reduce the influence of unreliable estimates while the estimation errors of unreliable estimates are also propagated.



(a) (b)

Fig. 1 (a) Proposed mean template. (b) Illustration of the deviation of forward and corresponding backward flow.

We assign an adaptive confidence weight to each estimate. The confidence $R_\ell^n(\boldsymbol{s},t)$ of a flow estimate $\boldsymbol{v}_\ell^n(\boldsymbol{s},t)$ at location $\boldsymbol{s}$ at iteration n is defined as

$$R_\ell^n(\boldsymbol{s},t)=\exp\left(-\frac{\left\|\boldsymbol{v}_\ell^n(\boldsymbol{s},t)+\boldsymbol{v}_{\ell^-}^n(\boldsymbol{s}+\boldsymbol{v}_\ell^n(\boldsymbol{s},t),t+1)\right\|}{\left(\left\|\boldsymbol{v}_\ell^n(\boldsymbol{s},t)\right\|+\left\|\boldsymbol{v}_{\ell^-}^n(\boldsymbol{s}+\boldsymbol{v}_\ell^n(\boldsymbol{s},t),t+1)\right\|\right)/2+\beta}\right) \tag{8}$$

where β prevents the denominator from equaling zero, and $\ell^-=\mathrm{b}$ if $\ell=\mathrm{f}$; otherwise f. Under the ideal situation, $\boldsymbol{v}_\ell(\boldsymbol{s},t)=-\boldsymbol{v}_{\ell^-}(\boldsymbol{s}+\boldsymbol{v}_\ell(\boldsymbol{s},t),t+1)$. We call this relationship as the bidirectional symmetry, e.g., the forward flow moves a point p from location $\boldsymbol{s}$ to $\boldsymbol{s}'$ while the corresponding backward flow moves this point p from location $\boldsymbol{s}'$ to $\boldsymbol{s}$. When the estimation error occurs, the deviation $\left\|\boldsymbol{d}_\ell^n(\boldsymbol{s},t)\right\|=\left\|\boldsymbol{v}_\ell^n(\boldsymbol{s},t)+\boldsymbol{v}_{\ell^-}^n(\boldsymbol{s}+\boldsymbol{v}_\ell^n(\boldsymbol{s},t),t+1)\right\|\neq0$. As an example, Fig.1 (b) illustrates this deviation. The reliability measure $R_\ell^n(\boldsymbol{s},t)$ increases monotonically as $\left\|\boldsymbol{d}_\ell^n(\boldsymbol{s},t)\right\|$ decreases. The greater the measure $R_\ell^n(\boldsymbol{s},t)$, the more reliable the flow estimate at this location $\boldsymbol{s}$.

From (7), local averages are mainly derived from more reliable estimates. The more reliable the estimates are, the greater contributions they have to the averages. Those estimates with lower reliability are suppressed so that their estimation errors cannot be propagated. It makes the filling-in effect more reliable and more accurate by rejecting unreliable estimates. Furthermore, our mean template can utilize all estimates in this template if the pre-defined neighborhood includes the central location in (7) since it considers their reliability, as shown in Fig.1 (a).

3. PROPOSED DISCONTINUITY-PRESERVING METHOD

In general, motion boundaries lie on some boundaries of spatial regions which have the coherence in motion. At motion boundaries, the smoothness is no longer valid. To handle motion boundaries, our discontinuity-preserving method (DP) be performed by changing the way that local averages are calculated in the neighborhood since the smoothness embodies on local averages.

The watershed algorithm [7] is used to divide each frame into non-overlapping homogeneous regions with closed and precise boundaries. The template $\mathrm{N}(\boldsymbol{x})$ is centered at the point $\boldsymbol{x}=(x,y)$, as shown in Fig.2. The points in each segmented spatial region are firstly merged and labeled with the same symbol, and the points in variant connected regions are labeled by different symbols. Assume that there are D different labels within the region $\mathrm{N}(\boldsymbol{x})$. Let J_i be the ith spatial region within $\mathrm{N}(\boldsymbol{x})$, and L_i denote the corresponding label. Without loss of generality, assume J_0 is the spatial region the point $\boldsymbol{x}$ belongs to. $A(L_i)$ is the number of the estimates within the region J_i. The local average $\bar{\boldsymbol{v}}(\boldsymbol{x})$ is computed by processing all regions within $\mathrm{N}(\boldsymbol{x})$ as follow:

$$\bar{\boldsymbol{v}}(\boldsymbol{x})=\left(\sum_{i=0}^{D}\left(\sum_{\boldsymbol{x}\in J_i}\boldsymbol{v}(\boldsymbol{x})R(\boldsymbol{x})\right)g(i)\right)\Bigg/\left(\sum_{i=0}^{D}\left(\sum_{\boldsymbol{x}\in J_i}R(\boldsymbol{x})\right)g(i)\right) \tag{9}$$

where $g(i)$ is the controlling function to determine whether the region J_i has contribution to the local average, which is defined by

$$g(i)=\begin{cases}1 & \text{if } \left|C(i)-\left(\sum_{x\in J_i}\boldsymbol{v}(x)R(x)\right)\Big/\left(\sum_{x\in J_i}R(x)\right)\right|<T_r\left|C(i)\right| \\ 0 & \text{otherwise}\end{cases} \tag{10}$$

where T_r is the threshold constant and $C(i)$ is defined as

$$C(i)=\left(\sum_{k=0}^{i-1}\left(\sum_{\boldsymbol{x}\in J_k}\boldsymbol{v}(\boldsymbol{x})R(\boldsymbol{x})\right)g(k)\right)\Bigg/\left(\sum_{k=0}^{i-1}\left(\sum_{\boldsymbol{x}\in J_k}R(\boldsymbol{x})\right)g(k)\right). \tag{11}$$

We initialize the $C(0)$ to the average of the flow estimates within the region J_0. After processing all the regions, if a much smaller number of the estimates are used to compute the local average, it is easily influenced by outliers. To further improve the effect of smoothness, we add the unprocessed estimates in the 8-neighborhood of the point $\boldsymbol{x}$ in computation of the local average $\bar{\boldsymbol{v}}(\boldsymbol{x})$.

Fig.2 illustrates the proposed method. In this case, there are four spatial regions (different colors) with the different labels. The velocity estimates within the region with the label L_0 are close to those within the region with the label L_1. They have the same motion coherence and belong to the same motion region. The regions with L_2 and L_3 have similar velocity estimates which are different from those within the regions with L_0 and L_1. Thus, the estimates within the regions with L_0 and L_1 are merged together to compute the local average. It stops the smoothness from propagating across the true motion boundary (red boundary) and performs the smoothness across the false motion boundary (blue boundary).

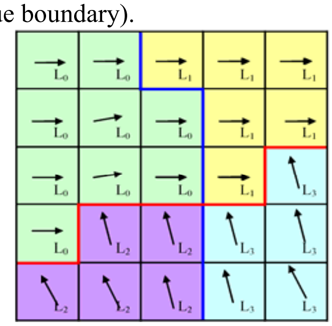


Fig. 2 Illustration of the proposed discontinuity-preserving method.

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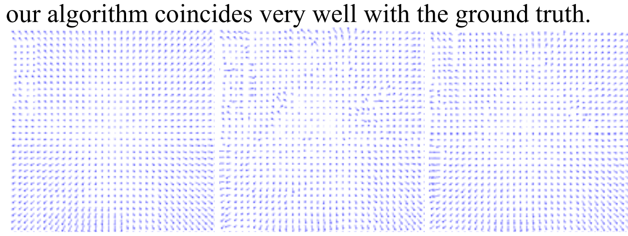
4. EXPERIMENTAL RESULTS

We evaluate our algorithm on three synthetic sequences with different motion types [4], where the ground truth data is available. The number of iteration is 200 for all the algorithms.

Fig.3 shows our first experiment on the Office sequence where divergent motion is dominating. The flow field (Fig.3 (b)) with the H&S algorithm is disorderly in some regions since estimation errors can be propagated. The flow field (Fig.3 (c)) with our algorithm without DP coincides well with the ground truth field (Fig.3 (a)). Table 1 shows the quantitative evaluation of these algorithms. As can be seen, the proposed algorithm outperforms the same type of the other algorithms with respect to the average angular error (AAE) [8] under the full density (100%). It achieves the best AAE of 7.5°. In addition, we compare our algorithm with the typical LK algorithm [2]. LK algorithm leads to a parse flow field (73%) due to no filling-in effect. Table 1 further confirms the observation in Fig.3 that the result of our algorithm coincides very well with the ground truth.



(a) (b) (c)

Fig.3 (a) Ground truth field (sampled and amplified by 4 times) for frame 5 of the Office sequence. (b) Estimated optical flow using the H&S algorithm ($\alpha=20$). (c) Ditto for the proposed algorithm ($\alpha=20$).

Table 1: Quantitative comparison for frame 5 of the Office sequence

Technique	AAE (°)	Density (%)
H&S [1]	12.3	100
I&L (No OR) [6]	14.3	100
LK [2]	10.1	73
2D-CLG-linear [5]	8.7	100
Proposed (No DP)	7.5	100

Table 2 gives the objective comparison of the performances of several algorithms on the street sequence. In this case, though AAE of B&W algorithm [5] is smaller than that of the proposed algorithm with DP, we observe the optical flow with the proposed algorithm (Fig.4 (a)) on the interesting detail (the car) looks better than that with B&W algorithm (Fig.4 (b)).

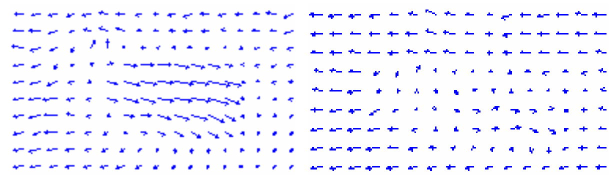
The third experiment on the rotating sphere sequence demonstrates the effectiveness of the DP method. As can be seen in Fig.5 (a), the flow estimates on the rotating sphere are propagated into the non-moving background regions and the smoothness constraint blurs the motion boundary. The proposed DP method can reduce this propagation, as shown in Fig.5 (b), though some parts of the motion boundary deviate from the corresponding parts of the real boundary. The deviation is mainly caused by the camouflage problem, i.e., the intensities of some parts of the moving object are much similar to those of the background. This segmentation error leads to the blurred motion boundary parts. It can be reduced by further improving spatial segmentation accuracy. The quantitative evaluation is given in Table 2. From Table 2, the DP method can further decrease AAE. The best result is achieved by the proposed algorithm with DP (5.1°) under the full density (100%).

5. CONCLUSION

This paper addresses an effective optical flow algorithm based on the confidence which takes advantage of the bidirectional symmetry of optical flow. The proposed algorithm improves the reliability and accuracy of optical flow by suppressing unreliable flow estimates. It leads to the reliable filling-in effect. The proposed discontinuity preserving method can effectively stop the smoothness from propagating across motion boundaries. The experiments show the results of the proposed algorithm coincide very well with the ground truth. Compared with several reference algorithms, the proposed algorithm successfully achieves the excellent results with the full density.

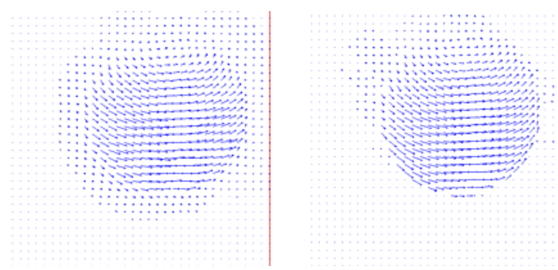
Table 2: Quantitative comparison of several algorithms

Technique	Street		Sphere	
	AAE (°)	Density	AAE (°)	Density
H&S	11.8	100%	12.2	100%
I&L (No OR)	10.7	100%	10.1	100%
I&L (OR)	---	---	6.9	100%
LK	8.9	75%	5.7	56%
2D-CLG-linear	6.1	100%	6.5	100%
Proposed(No DP)	9.3	100%	5.1	100%
Proposed (DP)	8.7	100%	3.9	100%



(a) (b)

Fig.4 (a) Optical flow with our algorithm on the interesting region. (b) Ditto for BW algorithm.



(a) (b)

Fig.5 Illustration of the DP method. (a) Optical flow without DP. (b) Optical flow with DP.

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