MULTI-TARGET TRACKING BASED ON KLD MIXTURE PARTICLE FILTER WITH RADIAL BASIS FUNCTION SUPPORT

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ABSTRACT

The two major difficulties associated with practical real time multi-target tracking are accuracy and speed. A new technique is proposed for multi-target tracking based on multi-modal particle filter with fast tracking capability and improved accuracy. The speed in tracking is achieved by a KLD sampling stage while the accuracy is improved by an additional stage that uses radial basis functions (RBF) for interpolating the sparse particles. Test results of the proposed multi-target tracking approach on both synthetic and real video data demonstrate the improved performance.

Index Terms— Mixture Particle Filter, Kullback-Leibler Divergence, KLD Sampling, Radial Basis Functions.

1. INTRODUCTION

Accurate and fast multi-target tracking is amongst the fundamental challenges in visual computing. Although particle filters [1] are very successful and accurate in tracking a wide range of challenging non-linear and non-Gaussian models, they are poor in consistently maintaining multiple modes in the target distributions that arises due to presence of multiple objects. To address this issue of maintaining the multi-modality of the target distribution, a technique called mixture particle filter [2] was proposed which models the target distribution as a non-parametric mixture model by distinguishing each target distribution and evolving each of them individually.

The number of particles used to estimate the state is a deciding factor in the accuracy and speed of the particle filter. The particle filter used in tracking the dynamic states become numerically intensive as the number of particles increases, although in general the accuracy of estimation increases with the number of particles used. Issue of improving accuracy of estimate without increasing number of particles needs to be addressed. Finding a solution to this problem becomes more imminent especially in the case of mixture particle filter where multiple modes of the density due to the underlying targets would naturally demand a larger number of particles and hence lending mixture particle filter computationally intensive and sluggish.

This paper introduces a strategy by adopting Kullback-Leibler divergence (KLD) sampling [5, 6] which makes the mixture particle filter less intensive. Further, the accuracy is preserved or improved through re-sampling the particles based on radial basis functions (RBF) [4]. Synthetic data and real video experiments are presented to demonstrate the improved performance of the proposed technique.

The basic idea of the KLD based mixture particle filter is presented in Section 2. Section 3 explains accuracy improvement of the estimate with radial basis function support. Then the proposed tracking approach is given in Section 4. Results for synthetic data and real video data are presented in Section 5. Section 6 concludes the paper.

2. KLD BASED MIXTURE PARTICLE FILTER

The basic idea behind the KLD based mixture particle filter is invoking KLD sampling to adapt the cardinality of the particle set during tracking.

In the mixture particle filter, individual modes in the posterior distribution representing multiple targets are clustered out using standard clustering techniques. At every time instant the overall prediction distribution is obtained by computing prediction distribution for each of the components individually. Overall weight set is obtained by normalizing over all weight sets of individual components.

To capture multi-modality the posterior is formulated as an M-component mixture model as follows:

$$P\left(x_t \mid z_t\right) = \sum_{m=1}^{M} \pi_{m,t} P_m\left(x_t \mid z_t\right) \tag{1}$$

where

$$P_m\left(x_t \mid z_t\right) = \frac{P\left(z_t \mid x_t\right) P_m\left(x_t \mid z_{t-1}\right)}{\int P\left(z_t \mid x_t, z_{t-1}\right) P_m\left(x_t \mid z_{t-1}\right) dx_t} \tag{2}$$

$$\pi_{m,t} = \frac{\pi_{m,t-1} P_m\left(z_t \mid z_{t-1}\right)}{\sum_{n=1}^{M} \pi_{n,t-1} P_n\left(z_t \mid z_{t-1}\right)} \tag{3}$$

To improve the speed performance by regulating the number of particles in each time instant, KLD sampling [5,6] is used so that the number of particles is made adaptive such that the divergence between real posterior and estimated posterior is bounded by a specified limit. The particle subsets associated with each of the individual targets are subjected to KLD sampling. This enhances the speed performance of mixture particle filter as only minimal and optimum number of particles will be kept to ensure desired accuracy.

3. MEAN ESTIMATION USING RADIAL BASIS FUNCTIONS

A typical particle filter provides a way of performing recursive Bayesian estimation using a set of particles associated with the underlying system. The state set and its corresponding weight set can be used to estimate the mean state. In the proposed approach, at this stage a new set of states within the range of existing state set of the particle set is generated from the same proposal distribution used in particle filter. Also, new states are chosen such that they are concentrated around particles with large weights. The cardinality of new state set is larger than the existing set. The corresponding weights associated with the new state set including the old state set can be obtained by interpolation of existing particle set using radial basis functions [4] with an appropriate basis function and smoothness. The underlying posterior distribution $s(x)$ is estimated from particle weights W at distinct states given by particle states X of the existing particle set.

Suppose $g(x)$ is a real valued function, we try to approximate $g(x)$ by $s(x)$ given values of $g = \{g_1, g_2, \ldots, g_N\}$ at discrete $X = \{x_1, x_2, \ldots, x_N\}$. The underlying function $s(x)$ is formulated as a radial basis function of the form,

$$s(x) = p(x) + \sum_{i=1}^{N} \lambda_i \psi\left(|x - x_i|\right), \quad x \in \mathbb{R}^d \qquad (4)$$

where p is a polynomial of degree at most k, λ_i is a real-valued weight, $|.|$ is the Euclidean norm, ψ is an appropriate basis function. Given the interpolant values g the weights and the coefficients that give p in terms of the basis are found such that $s(x)$ satisfies,

$$s(x_i) = g_i \quad i = 1, 2, \ldots, N \qquad (5)$$

This gives a closed form representation of underlying function $s(x)$. In a similar fashion we try to formulate the underlying posterior density given the weights and the states, forming the particle set, representing g and X respectively by $s(x)$. Now from the closed form representation of the posterior density $s(x)$, weights associated with the new state set, are generated as discussed earlier, is obtained. Estimate of the mean state using the new, larger particle set is better than that estimated with the original particle set.

This can be shown with an example of a simple problem of integration. Let P be the probability measure defined over variable space x. We are interested in estimating expectation of a function of interest f over x with respect to the probability density P,

$$E_p(f(x)) = \int_X f(x) P(x) dx \qquad (6)$$

Since the integral cannot be evaluated analytically, Monte Carlo methods are used. An estimate is obtained by generating N samples according to P. The empirical mean of f is given as,

$$\hat{E}_N(f; X) := \frac{1}{N} \sum_{i=1}^{N} f(x_i) \quad where \quad x \sim P(X) \qquad (7)$$

Now a new set of variable samples is generated, distributed according to P, with the same range as the existing set. The cardinality of the new sample set along with the existing set is $M >> N$. RBF interpolation is used on f to obtain the underlying function $s(x)$ of f from which the function values are calculated for the new state set. The new empirical mean of f obtained by using the values of the function f for the new sample set along with that of the old is obtained as,

$$\hat{E}_M(f; X) := \frac{1}{M} \sum_{i=1}^{M} f(x_i) \quad where \quad M >> N \qquad (8)$$

It can be show that,

$$Prob\left(X : \left|\hat{E}_N(f;X) - E_P(f(x))\right| > \delta_{\min}\right) > Prob\left(X : \left|\hat{E}_M(f;X) - E_P(f(x))\right| > \delta_{\min}\right) \qquad (9)$$

Bounds [7, 8] exits such that,

$$Prob\left(X : \left|\hat{E}_N(f;X) - E_P(f(x))\right| > \delta_{\min}\right) \le q(N, \delta_{\min}) \qquad (10)$$

Similarly, $Prob\left(X : \left|\hat{E}_M(f;X) - E_P(f(x))\right| > \delta_{\min}\right) \le q(M, \delta_{\min})$ (11)

The bounds are given by,

$$q(D, \delta_{\min}) = \begin{cases} \dfrac{1}{4 D \delta_{\min}^2} & Bernoulli\ Bound \\ 2 e^{-2 D \delta_{\min}^2} & Chernoff\ Bound \\ \dfrac{var(f)}{D \delta_{\min}^2} & Chebychev\ Bound \end{cases} \qquad (12)$$

Since $M >> N$ we have,

$$q(M, \delta_{\min}) << q(N, \delta_{\min}) \qquad (13)$$

Then we have,

$$Prob\left(X : \left|\hat{E}_N(f;X) - E_P(f(x))\right| > \delta_{\min}\right) >> Prob\left(X : \left|\hat{E}_M(f;X) - E_P(f(x))\right| > \delta_{\min}\right) \qquad (14)$$

Thus, consistently the probability that the difference between true mean and the mean estimated with N particles exceeds a bound, is far greater than the probability that the difference between true mean and the mean estimated with M particles, obtained by RBF interpolation, exceeds the same bound. From this we can infer that the mean estimated by RBF interpolation is more accurate.

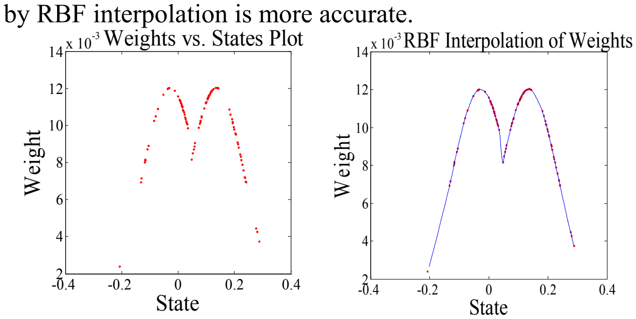

Fig. 1. Weights vs. states plot of Initial Particle set (left) and overall Particle set after RBF interpolation.

An illustration of RBF interpolation is shown in Fig 1. The plot on left in Fig 1. shows the weights vs. states plot with N elements of dual-object scenario and that on right shows weights vs. states plot with $M >> N$ elements obtained by RBF interpolation with appropriate basis function and smoothness from a simulation run of regular particle filter.

4. PROPOSED MULTI-TARGET TRACKING

Summarizing the ideas put forth in Sections 2 and 3, we propose a multi-tracking approach using Mixture Particle filter which incorporates KLD sampling and RBF interpolation technique. The proposed KLD mixture particle filter with RBF support is presented in Table 1, with steps explained in detail in the following paragraphs.

Step 1: KLD Sampling: The state space is downsized to the calculated cardinality n_χ and samples are generated from suitably chosen proposal distribution q which depends on the previous state and present measurement.

Table 1. Proposed KLD-MPF with RBF support algorithm.

Inputs: $S_{t-1} = \left\{\left\langle x_{t-1}^i, w_{t-1}^i \right\rangle \mid i = 1, \ldots, N\right\}$, observation z_t, bounds e, δ, minimum number of samples $n_{\chi_{\min}}$

$S_t = 0, n = 0, n_\chi = 0, k = 0$

do

1. Sampling the states $x_t^n \sim q\left(x_t \mid x_{t-1}^n, z_t\right)$ $n \in I_m$
2. Re-weighting using likelihood function
   a. $w_t^n = \dfrac{\tilde{w}_t^n}{\sum_{j \in I_m} \tilde{w}_t^j}$, $\tilde{w}_t^n = \dfrac{w_{t-1}^n P\left(z_t \mid x_t^n\right) P\left(x_t^n \mid x_{t-1}^n\right)}{q\left(x_t^n \mid x_{t-1}^n, z_t\right)}$
   b. $\pi_{m,t} = \dfrac{\pi_{m,t-1} \tilde{w}_{m,t}}{\sum_{n=1}^{M} \pi_{n,t-1} \tilde{w}_{n,t}}$, $\tilde{w}_{m,t} = \sum_{i \in I_m} \tilde{w}_{m,t}^i$
3. Update particle size n_χ using KLD transformation

   if x_t^n falls into empty bin b **then**

   $k = k + 1$

   b = mark bin as non-empty

   if $n \geq n_{\chi_{\min}}$ **then**

   $n_\chi = \dfrac{k-1}{2e}\left\{1 - \dfrac{2}{9(k-1)} + \sqrt{\dfrac{2}{9(k-1)}} z_{1-\delta}\right\}^3$

   end if

   end if

   $n = n + 1$

while ($n < n_\chi$ and $n < n_{\chi_{\min}}$)

4. Normalize the weights and form the state-space
   $w_t = \dfrac{w_t}{\sum_{i=1}^{n} w_t^i}$ and x_t
5. Estimation using radial basis function interpolation
   RBF interpolated particle set $S_t = \left\{\left\langle x_t^i, w_t^i \right\rangle \mid i = 1, \ldots, M\right\}$
   $\left\langle x_t \right\rangle = \sum_{i=1}^{M} w_t^i x_t^i$ where $M>>N$ and w_t normalized
6. Formulate the particle set
   $S_t = \left\{\left\langle x_t^i, w_t^i \right\rangle \mid i = 1, \ldots, N\right\}$

return S_t

Step 2: Re-weighting: The particle weights w_t and the mixture component weights $\Pi_t = \{\pi_{m,t}\}_{m=1}^{M}$ are calculated using steps 2a. and 2b. The likelihood $P(z_t \mid x_t)$ is calculated using a suitable likelihood function $\Phi\left(x_t, z_t\right)$.

Step 3: Particle Set Cardinality Calculation: The number of particles used to estimate the state is a deciding factor in the accuracy and speed of the particle filter. Relation between the number of particles used and the accuracy of the estimates is determined by a metric called Kullback-Leibler divergence which measures how best the MC estimate of the posterior density can match the true posterior density. The smaller the divergence value, the better is the match. Thus, the problem of achieving an optimal tradeoff between accuracy and speed reduces to the problem of determining number of particles at each iteration of particle filter such that, with a probability the error between true posterior and posterior estimate is minimum. Suppose that we have two distributions p and q, KLD [5] is defined as,

$$KLD(p, q) = \sum_X p(x) \log\left(\frac{p(x)}{q(x)}\right) \tag{15}$$

KLD is always positive and zero if the distributions are identical. Equation (16) gives the number of particles n that guarantees with probability $1\text{-}\delta$ that KLD is less than e.

$$n = \frac{k-1}{2e}\left\{1 - \frac{2}{9(k-1)} + \sqrt{\frac{2}{9(k-1)}} z_{1-\delta}\right\}^3 \tag{16}$$

where $z_{1-\delta}$ is the upper $1\text{-}\delta$ quantile of the standard normal distribution and k is the number of bins of the MC posterior density estimate with support.

Step 4: The weights of the particles are then normalized.

Step 5: RBF interpolation of the particle set according to Section 3.

Step 6: The particle set is formulated. Mean state can be computed from this particle set with size $M>>N$ using,

$$\left\langle X_t \right\rangle = \sum_{i=1}^{M} w_t^i X_t^i \tag{17}$$

Only initial N samples prior to RBF interpolation are propagated in to the next iteration of mixture particle filter.

5. RESULTS

The proposed approach is systematically evaluated and compared using simulated data. The performance is also evaluated on real video sequences.

Consider a point moving in 1-D. A true path of the point is generated based on a non-linear motion model, and non-Gaussian noise is then added to the actual path to simulate a noisy measurement of the actual path. Then we evaluate our KLD-MPF with RBF support algorithm and compare against regular MPF [3] and KLD-MPF to track the true path using the noisy path as measurement for two targets. Fig. 2. illustrates this comparison. The speed and accuracy performance from this comparison is tabulated in Table 2 which shows that KLD-MPF with RBF support outperforms regular MPF with sufficient accuracy and speed performance.

Next, we apply KLD-MPF with RBF support on real video sequence. This is intended to verify the claim of gain in speed of tracking while maintaining accuracy of multiple tracked objects.

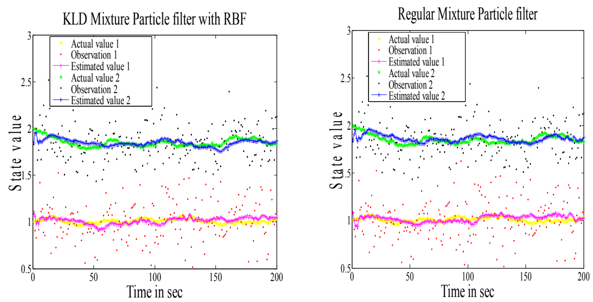

Fig. 2. Left: KLD-MPF with RBF support; Right: MPF.

Table 2. Time elapsed and MSE accuracy with KLD-MPF with RBF support, KLD-MPF and regular MPF.

KLD-MPF with RBF			KLD-MPF			Regular MPF		
Time (secs)	Track1 MSE x10^{-4}	Track2 MSE x10^{-4}	Time (secs)	Track1 MSE x10^{-4}	Track2 MSE x10^{-4}	Time (secs)	Track1 MSE x10^{-4}	Track2 MSE x10^{-4}
5.482	17	30	2.765	25	33	30.05	26	37
5.125	7.792	17	2.735	17	29	29.66	6.715	14
4.640	9	40	2.703	16	64	30.09	11	40
5.453	18	21	2.703	21	27	29.83	19	25
5.531	19	20	2.734	23	41	29.87	21	21
4.619	29	18	2.781	49	20	29.97	27	18
5.720	26	12	2.750	28	12	29.53	27	14

The performance of the algorithm on real video shows that with an initial particle size of 2000 per object and 8 objects, KLD-MPF with RBF support algorithm is consistently 5 times faster than regular MPF without loss in accuracy as seen from Fig. 3. The elapsed time performance over 24 frame (1 sec) sequence is in Table 3. Tracking runs on real hockey video and a dual-object surveillance video with is shown in Fig. 3. Experiments were performed on Matlab on a non-dedicated 1.7GHz Pentium processor with 1GB RAM.

Table 3. Time elapsed for KLD-MPF with RBF support and regular MPF on multi-object hockey sequence.

KLD-MPF RBF	Regular MPF	
Time (secs)	Time (secs)	Speed Improvement
49.8270	258.6860	5.1968
49.7170	260.9200	5.2481
50.0640	257.0480	5.1343
52.0470	258.1420	5.0569

6. CONCLUSIONS

In conclusion, along with a tracking speed improvement the KLD-MPF with RBF support outperforms regular MPF with preserved or improved accuracy promising an efficient real time tracking system. KLD sampling is introduced in to the mixture particle filter which improves speed performance tremendously. Further, radial basis function tweak is used to improve the accuracy of the estimate without any overhead in the propagation iterations of mixture particle filter. Since the number of objects tracked is many, the KLD-MPF with RBF support provides a practical tracking solution with very good accuracy with limited number of particles per object.

7. REFERENCES

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Fig. 3. KLD-MPF with RBF support tracking on dual-object video (top) & multi-object hockey sequence (bottom).