# SIMULTANEOUS RD-OPTIMIZED RATE CONTROL AND VIDEO DE-NOISING

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## ABSTRACT

In this paper, we propose a simultaneous rate control and video de-noising algorithm based on rate distortion optimization. According to our previous works [1] [2], video de-noising can be performed by using rate distortion optimization with a lower bound quantization parameter (QP) constraint, where the lower bound QP is determined by the noise variance. Then, we find that the macroblock level rate control method in H.264 can be seen as an approximate solution of a rate distortion optimization problem with a specified rate distortion function. Based on these two studies, we integrate the video de-noising problem and rate control problem to a rate distortion optimization problem. We show the convexity of the problem and derive the optimal solution. To reduce the complexity, we propose to use a suboptimal solution based on simply thresholding. Some experiments are conducted to demonstrate the efficiency and effectiveness of the proposed method.

***Index Terms*—** Video De-noising, Rate Control, Rate Distortion Optimization

## 1. INTRODUCTION

The rate control technique in the video coding scheme is used to control the output bit rate of the video encoder according to the network condition. However, the traditional rate control methods are usually specially designed for the input video sequences that are noise-free [3] [4] [5]. When the input video sequences are corrupted by noise, these well-designed rate control methods may have some problems, e.g. they may allocate the bit rate to the noisy blocks, which results in poor PSNR performance of the reconstruction. Moreover, the visual quality of output reconstructed video sequences may be noisy. In order to address the rate control problem for the video sequences partially or fully corrupted by Gaussian noise as shown in Figure 1, in this paper, we propose a simultaneous rate control and video de-noising algorithm using rate distortion optimization.

(a) (b)

**Fig. 1**: Video Sequences Partially or Fully Corrupted by Noise.

## 2. DE-NOISING PROBLEM FORMULATION

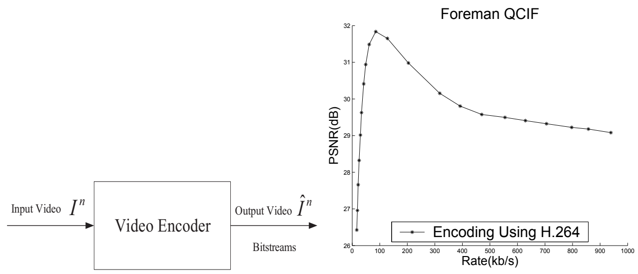
The video corrupted by additive Gaussian noise can be formulated as following:

$$I^n = I + n \tag{1}$$

where $I = [I_1, I_2...I_k, I_{k+1}...I_m]^T$ is the original (ideal) video while $I_k$ is the $k$-th frame, $n = [n_1, n_2...n_k, n_{k+1}...n_m]^T$ is the additive Gaussian noise, and $I^n = [I_1^n, I_2^n...I_k^n, I_{k+1}^n...I_m^n]^T$ is the noisy observation of the video. We assume that each frame have $N$ pixels. In this case, $I_k$, $n_k$, $I_k^n$ are all length-$N$ vectors.

The video denoising problem is designed to find a estimation $\hat{I}$ from the original video based on the noisy observation $I^n$. From our previous work [1], we know that the denoising problem can be regraded as a rate distortion optimization problem under some appropriate assumptions, which means that we are able to perform video denoising through video encoders. However, directly inputting the noisy video to the video encoder (Figure 2(a)), we may generate bitstreams with high bitrate but poor PSNR performance as shown Figure 2(b). In order to avoid this case, we should carefully choose the parameters of the video encoder.

In a traditional block-based motion compensated video encoder, the rate-distortion optimization model for a mac-




(a) (b)

**Fig. 2**: (a): Video Encoder; (b) Rate Distortion Performance of H.264 For The Noisy Video.

roblock $p$ is defined as follows:

$$\begin{aligned} S_p^* &= \arg\min_{S_p} D(p, S_p) + \lambda(Q) R_p(S_p) \\ \hat{I}_p &= \hat{I}_p^n(S_p^*) \end{aligned} \quad (2)$$

while $S_p = \{$*mode*, *motion vectors*, *reference index*, *quantization parameter*$\}$ denotes the vector of coding decisions for the macroblock $p$. Among these, the most important parameter is quantization parameter $QP$ since using too small $QP$ will output the bitstream with high bitrate but poor PSNR performance as shown in Figure 2(b). In the following, we will describe how to generate the lower bound quantization parameter $QP_{lowbound}$. And the $QP$ which is smaller than $QP_{lowbound}$ will output unreasonable bitstreams with a higher bit rate as well as larger distortion. On the contrary, any $QP$ which is larger than $QP_{lowbound}$ will generate reasonable bitstreams with de-noised reconstruction at low bitrate.

## 3. LOWER BOUND QUANTIZATION PARAMETER

For de-noising purpose, we want to minimize the distortion between the original data $I$ and the reconstructed version $\hat{I}^n$. We assume that minimizing over the whole frame is equivalent to minimizing over each marcoblock (MB) within the frame independently, ignoring any dependence among MBs. Let us define the reconstructed distortion $E(D_p)$ for MB $p$ as the expectation of the square difference between the original pixel $I_i$ and the reconstructed one $\hat{I}_i^n$. According to our previous work [2], the corresponding quantization parameter $QP_0$ that minimizes $E(D_p)$ is:

$$QP_0 = \begin{cases} \sqrt{\omega(\theta\sigma_n + \epsilon)^2}, & for\ H.263 \\ 3\log_2[\omega(\theta\sigma_n + \epsilon)^2] + 12, & for\ H.264 \end{cases} \quad (3)$$

where $\omega$, $\theta$, and $\epsilon$ are parameters. And $\sigma_n$ is the square root of the noise variance.

In this process of getting $QP_0$, we do not involve the role of bit rate. Therefore, we can treat this $QP_0$ as a lower bound quantization parameter. Any reasonable $QP$ should be larger than $QP_0$ with a smaller bit rate. The $QP$ which is smaller than $QP_0$ will have a larger bit rate as well as larger distortion, which is not reasonable. Here, let us define the lower bound quantization parameter as $QP_{lowbound}$, and

$$QP_{lowbound} = \begin{cases} \sqrt{\omega(\theta\sigma_n + \epsilon)^2}, & for\ H.263 \\ 3\log_2[\omega(\theta\sigma_n + \epsilon)^2] + 12, & for\ H.264 \end{cases} \quad (4)$$

## 4. MB LEVEL RATE CONTROL IN H.264

In the following, we briefly review the MB level rate control method in H.264. For detailed information, please refer to [5]. The MB level rate control method in H.264 mainly contains three steps:

*1):* Predict the MADs of the remaining MBs in the current frame by Eqn. (5) using the actual MADs of the co-located MBs in the reference frame.

$$MAD(i,t) = a_1 MAD(i,t-1) + a_2 \quad (5)$$

where $a_1$ and $a_2$ are two parameters of the MAD prediction models.

*2):* Compute the target bits $R(i)$ of current MB $i$ by Eqn. (6) and the header bits by a linear model [5].

$$R(i) = R_t(i) \frac{MAD^2(i,t)}{\sum_{k=i}^{N_{MB}} MAD^2(k,t)} \quad (6)$$

where $R_t(i)$ denotes the number of the remain bits for the remaining MBs in the current frame, and the initial value of $R_t(i)$ is $R_t$, the bits allocated to current frame. $N_{MB}$ is the total number of MB in current frame.

*3):* Compute the quantization parameter of current MB using the quadratic R-D model [3].

## 5. APPROXIMATE OPTIMIZED MB LEVEL RATE ALLOCATION MODEL FOR H.264

Let us define the distortion measure of current MB $i$ as a function of $MAD(i,t)$ and $R(i)$ as: $D(i) = \alpha \frac{MAD^4(i,t)}{R(i)}$. For the MB level rate allocation problem, we want to minimize the overall distortion of current frame under the constraint that the overall bitrate is equal to $R_t$. And the problem can be formulated as follows:

$$\min_{R(i)} \sum_{i=1}^{N_{MB}} \alpha \frac{MAD^4(i,t)}{R(i)}, \qquad s.t. \sum_{i=1}^{N_{MB}} R(i) = R_t \quad (7)$$

where $MAD(i,t)$ can be predicted by Eqn. (5).

Obviously, the optimization problem above is a convex optimization problem. The optimal solution can be generated by using Lagrangian optimization as:

$$R^\star(i) = R_t \frac{MAD^2(i,t)}{\sum_{k=1}^{N_{MB}} MAD^2(k,t)} \quad (8)$$

Since the number of the remain bits for the remaining MBs in the current frame $R_t(i) = R_t - \sum_{k=1}^{i-1} R^\star(k)$, we have:

$$R^\star(i) = R_t(i) \frac{MAD^2(i,t)}{\sum_{k=i}^{N_{MB}} MAD^2(k,t)} \tag{9}$$

Compared with Eqn. (6) and (9), we can see that the MB level rate control method in H.264 is the same as the optimal solution of the optimization problem in Eqn. (7). Unfortunately, they are actually different since the parameters $a_1$ and $a_2$ in Eqn. (5) are updated after encoding every MB. However, we can treat the MB level rate control method in H.264 as an approximate solution of the problem in Eqn. (7).

## 6. PROPOSED SIMULTANEOUS RD-OPTIMIZED RATE CONTROL AND DE-NOISING METHOD

According to our discussion in Section 2. and 3., in order to perform video de-noising using rate distortion optimization, the quantization parameter $QP$ should be set to be larger than $QP_{lowbound}$. And according to the discussion in Section 4. and 5., rate control can be achieved by solving the optimization problem in Eqn. (7). Therefore, we can integrate the de-noising problem and rate control problem by solving the following optimization problem:

$$\min_{R(i)} \sum_{i=1}^{N_{MB}} \alpha \frac{MAD^4(i,t)}{R(i)}$$
$$s.t. \sum_{i=1}^{N_{MB}} R(i) = R_t; \;\; R(i) \leq R(QP_{lowbound}(i)). \tag{10}$$

where $R(QP_{lowbound}(i))$ can be obtained using the quadratic R-D model [3].

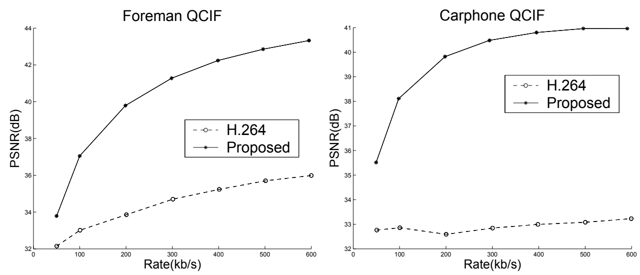
Obviously, the optimization problem above is also a convex optimization problem. The optimal solution can be generated by solving the KKT conditions [6]:

$$R^\star(i) = \mathbf{min}[R(QP_{lowbound}(i)), \sqrt{\frac{a}{\lambda^\star}} MAD^2(i,t)] \tag{11}$$

where $\sum_{i=1}^{N_{MB}} R^\star(i) = R_t$.

We are able to find the optimal solution through an iterative form of finding $\lambda^\star$. However, since it is too complex, we use a suboptimal solution instead. We use an approximate Lagrangian parameter $\tilde{\lambda} = (\frac{\sqrt{\alpha}\sum_{i=1}^{N_{MB}} MAD^2(i,t)}{R_t})^2$, which is actually the optimal Lagrangian parameter of the optimization problem in Eqn. (7). And the suboptimal solution $\tilde{R}(i)$ is:

$$\tilde{R}(i) = \mathbf{min}[R(QP_{lowbound}(i)), \frac{R_t(i) MAD^2(i,t)}{\sum_{k=i}^{N_{MB}} MAD^2(k,t)}] \tag{12}$$




(a) (b)

**Fig. 3**: Rate distortion performance comparison.

Let us define the total distortion of current frame as a function of the rate $R(i)$ and corresponding $\lambda$ as:

$$D(R(i), \lambda) = \sum_{i=1}^{N_{MB}} \alpha \frac{MAD^4(i,t)}{R(i)} \tag{13}$$

Then, we can evaluate the performance of using $\tilde{R}(i)$ and $\tilde{\lambda}$ by comparing $D(\tilde{R}(i), \tilde{\lambda})$ with $D(R^\star(i), \lambda^\star)$:

$$\frac{D(\tilde{R}(i), \tilde{\lambda})}{D(R^\star(i), \lambda^\star)} = \frac{\sum_{i=1}^{N_{MB}} \frac{\alpha MAD^4(i,t)}{\tilde{R}(i)}}{\sum_{i=1}^{N_{MB}} \frac{\alpha MAD^4(i,t)}{R^\star(i)}} \tag{14}$$

For the ideal case that $MAD(i,t) = MAD(j,t)$ for all $i$,$j$; $\sqrt{\frac{a}{\lambda^\star}} MAD^2(i,t) > R(QP_{lowbound}(i))$ for $i \leq k$ and $\sqrt{\frac{a}{\lambda^\star}} MAD^2(i,t) \leq R(QP_{lowbound}(i))$ for $k < i \leq N_{MB}$, and $R(QP_{lowbound}(i)) = \frac{R_t}{\eta N_{MB}}$ for $i \leq k$, we can get:

$$\left. \frac{D(\tilde{R}(i), \tilde{\lambda}) - D(R^\star(i), \lambda^\star)}{D(R^\star(i), \lambda^\star)} \right|_{ideal\_case} = \frac{\eta - 1}{\eta} \frac{k}{N_{MB} - k} \tag{15}$$

## 7. EXPERIMENTAL RESULTS

The proposed simultaneous rate control and video de-noising algorithm is evaluated based on modified H.264 JM 9.0. In order to simulate the noising process, the clean video sequences are first manually distorted by adding Gaussian noise. We assume that one slice contains one row MBs and each slice has a possibility $\rho$ to be corrupted by Gaussian noise and possibility $1 - \rho$ to be error-free. Then, the noisy videos are input to the video encoder. The video sequences "*Foreman*" and "*Carphone*" in QCIF format are tested. In all our experiments, the parameters $\omega$, $\theta$ and $\epsilon$ are set to be 1.694, 1.049, and 0.445 respectively [2]. In the following, we only show the results of $\sigma_n^2 = 100$ and $\rho = 0.4$. Similar results are obtained using different $\sigma_n^2$ and $\rho$. The PSNR is computed by comparing with the original video sequence. We compare the proposed method with the rate control method in H.264 [5].

(a) (b) (c) (d)

(e) (f) (g) (h)

**Fig. 4**: Subjective quality comparison for "*Foreman*" and "*Carphone*": (a)(e) original frames; (b)(f) noisy frames with spatially varying noise variance; (c)(g) reconstructed frames using H.264, where the corresponding average (bitrate kb/s, PSNR dB) are (301.26, 34.70) and (300.27, 32.85); (d)(h) reconstructed frames using the proposed method, where the corresponding average (bitrate kb/s, PSNR dB) are (298.42, 41.27) and (293.55, 40.48).

The rate distortion performance is shown in Figure 3. We can see that the PSNR peformance of the proposed method is much better than that of the method in H.264 [5], especially at high bit-rate situation. Up to 7.88 dB gain can be obtained. The subjective visual quality is also examined in Figure 4, where (a)(e) and (b)(f) are the original and noisy frames. (c)(g) are the results reconstructed by the rate control method in H.264 while (d)(h) are the results generated using the proposed method at bit-rate about 300kb/s. We can see that by using the proposed rate control method, not only the noise in the noisy slices is greatly reduced, but also the detail information in the error-free slices can be well-preserved. On the contrary, using the rate control method in H.264 will not only have a poor denoising performance for the noisy slices but also introduce serious distortion to the error-free slices. Notice that the artifacts not only come from the compression distortion of current frame but also from the distortion propagation of previous frames.

## 8. CONCLUSION

There are mainly three contributions of this paper. First, we showed that video de-noising can be achieved by using rate distortion optimization with a lower bound QP constraint. Second, we showed that the MB level rate control method in H.264 can be seen as an approximate solution of a rate distortion optimization problem with a specified rate distortion function. Third, we proposed a novel algorithm to simultaneously address the rate control and video de-noising problems for the noisy input. Experimental results showed that the proposed method outperforms the rate control method in H.264.

## 9. ACKNOWLEDGEMENT

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