SEGMENTATION OF THE LEFT VENTRICLE IN CARDIAC MR IMAGES USING GRAPH CUTS WITH PARAMETRIC SHAPE PRIORS

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ABSTRACT

The left ventricle in MR images presents many challenges for automated segmentation including poor contrast at desired tissue boundaries. Segmentation methods based on information from the image alone do not work well in such cases and additional constraints are necessary. In this paper, we propose a novel segmentation method that incorporates parametric shape priors, which do not require statistical training, to the graph cuts technique for robust and efficient segmentations of the left ventricle in cardiac images. We introduce novel terms accounting for shape prior/segmentation and shape prior/image fit to the graph cuts representation. The latter prevents a vicious cycle of bad segmentation/shape priors. We demonstrate the effectiveness of our method on real cardiac images with ground truth segmentations.

Index Terms— Left ventricle segmentation, shape prior, graph cuts, cardiac MRI, expectation maximization, Gaussian mixture model.

1. INTRODUCTION

Segmentation of cardiac structures in medical images leads to many applications, which can assist the diagnostics of cardiac diseases. The left ventricle is of special importance, because it pumps oxygenized blood away from the heart to the rest of the body. Manual segmentation of cardiac images by human experts can be time-consuming. Data segmented by human experts also tends to show inter- and intra-observer inconsistency. For these reasons, automated segmentation of the left ventricle in cardiac images is of great interest.

The left ventricle in MR images presents many challenges for automated segmentation. For example, desired tissue boundaries, such as those between the epicardium and the liver and those between the endocardium and the blood pool, can have poor contrast. On the other hand, strong contours may exist where boundaries are not desired [1]. Segmentation methods based on information from the image alone do not work well in such cases and additional constraints are necessary.

[2] and [3] proposed level set-based segmentation methods for cardiac images. In [4], a method combining edge, region and shape information is presented. These algorithms require a large amount of training cases, which can be difficult to obtain and time consuming to process. In short-axis MR cardiac images, the left ventricle roughly resembles the shape of a donut. Therefore, simpler parametric shape priors, which do not require statistical training, may suffice.

The graph cuts-based segmentation method has recently become popular because it allows for a globally optimal efficient solution in an N-dimensional setting [5]. Despite its advantages, graph cuts cannot produce an accurate segmentation for objects with weak boundaries. There have been recent attempts to add a shape prior to the graph cuts segmentation technique. [6] presented a method that uses a fixed shape template aligned with the image by the user input. In [7], a model based method for left ventricle segmentation is presented. [8] proposed the usage of an elliptical prior. This method iteratively solves for the image segmentation and elliptical fitting problems and the elliptical shape is estimated as the best least square fit of the segmentation. Although, this method cannot correct for inaccurate segmentations and bad segmentations inevitably lead to bad elliptical priors.

In this paper, we propose a novel segmentation method that incorporates parametric shape priors, which do not require statistical training, to the graph cuts technique for robust and efficient segmentations of the left ventricle in cardiac images. We introduce novel terms accounting for shape prior/segmentation and shape prior/image fit to the graph cuts representation. The latter prevents a vicious cycle of bad segmentation/shape priors.

2. SEGMENTATION METHOD

In this section, we first give a brief summary of the graph cuts image segmentation framework. We then discuss our model for shape priors and representation of intensity probability distribution. We end by presenting our novel objective function and its optimization.

2.1. Graph Cuts

The basic graph cuts image segmentation framework is developed in [9]. The idea is as follows. An image is mapped onto a weighted undirected graph $G =< V, E >$ where each pixel is represented as a node $v \in V$ and each pair of neighboring pixels is linked by an edge $e \in E$ called an *n*-link. Two additional "terminal" nodes, the source s and the sink t, represent the object and the background. Every non-terminal node is linked to s and t through edges called *t*-links. A cut on the graph divides the nodes into two sets: one that is connect to the source s and one that is connect to the sink t. The cost of a cut is the sum of weights of all the edges severed by the cut. The formulation of the edge weights depends on the specific algorithm, but should be such that the minimum cut (i.e. the cut with the minimum cost) on the graph gives the ideal segmentation. There are numerous algorithms that can solve the minimum cut problem in polynomial time [5].

Graph cuts can be extended to multiple labels using α-expansion [5]. The α-expansion move assigns label α to an arbitrary set of pixels. In the case of multiple labels, exact minimum on the graph is generally NP-hard [10]. The α-expansion move generates a local minimum that is within a known factor of the global minimum when edge weights satisfy certain conditions [10].

2.2. Shape Representation

We model the left ventricle using two concentric circles. We denote the shape parameters as $\mathbf{w} = \{w_0, w_1, w_2, w_3\}$, where (w_0, w_1) is the center of the circles and w_2 and w_3 are respectively the radii for the inner circle (representing the endocardium boundary) and outer circle (representing the endocardium boundary). For convenience, we represent our shape prior with signed distance functions, where boundary pixels have the value of zero and inside and outside pixels are assigned negative and positive distances respectively. Our concentric circles are denoted as $\mathbf{U} = (\mathbf{U}_1, \mathbf{U}_2)^T$, where $\mathbf{U}^1$ and $\mathbf{U}^2$ are respectively the signed distance functions of the inner and outer circles in array forms.

2.3. Mixture of Gaussians

We model the intensity probability distribution of the image with a Gaussian mixture model (GMM) using one Gaussian for the blood pool, one Gaussian for the myocardium and $D - 1$ Gaussians for the background. GMM has been applied previously to graph cuts segmentation in [11]. Let $\underline{\theta} = \{\mu_d, \Sigma_d, \pi_d, d \in 0, ..., D\}$ denote the parameters for the GMM, where μ_d, Σ_d, π_d are respectively the mean, variance, and prior probability of Gaussian d with $d = 0$ being for the blood pool (BP), $d = 1$ for the myocardium (M) and $d = 2, ..., D$ for the background (B). We define a parameter $\mathbf{b} = \left(\left(b_1^{\mathrm{BP}}, b_1^{\mathrm{M}}, b_1^{\mathrm{B}}\right)^T, \ldots, \left(b_N^{\mathrm{BP}}, b_N^{\mathrm{M}}, b_N^{\mathrm{B}}\right)^T \right)$ (where $b_n^{\mathrm{BP}} \in \{0\}$, $b_n^{\mathrm{M}} \in \{1\}$ and $b_n^{\mathrm{B}} \in \{2, ..., D\}$) that assigns all N pixels in the image to one Gaussian belonging to the blood pool, one to the myocardium and one to the background.

2.4. Objective Function

Our problem can be formulated as: given a cardiac image in an array form $\mathbf{I} = (I_1, \ldots, I_N)$ with N pixels, we wish to (1) identify the shape parameters $\mathbf{w}$ that best match the left ventricle in the image, and (2) segment the image into blood pool (BP), myocardium (M) and background (B). We assign a label f_n to each pixel, where $f_n \in \{\mathrm{BP}, \mathrm{M}, \mathrm{B}\}$. Our segmentations are represented as label configurations $\mathbf{f} = (f_1, \ldots, f_N)$. The segmentation and shape fitting problems are inter-related and we solve them in an iterative manner.

We define the energy functional in equation (1) to guide the image segmentation and shape parameters calculation.

$$\begin{aligned} E(\mathbf{I}, \underline{\theta}, \mathbf{w}, \mathbf{f}, \mathbf{b}) = E_{\mathrm{D}}(\mathbf{I}, \underline{\theta}, \mathbf{f}, \mathbf{b}) + E_{\mathrm{N}}(\mathbf{I}, \mathbf{f}) \\ + E_{\mathrm{P}}(\mathbf{f}, \mathbf{w}) + E_{\mathrm{S}}(\mathbf{w}, \mathbf{I}). \end{aligned} \tag{1}$$

The first term measures how well the pixel labels and the GMM parameters fit the image given its intensities and can be written as

$$E_{\mathrm{D}}(\mathbf{I}, \underline{\theta}, \mathbf{f}, \mathbf{b}) = -\sum_n \log \mathrm{P}(I_n, b_n = d | \underline{\theta}), \tag{2}$$

where $b_n = b_n^{f_n}$. The second term measures the smoothness of the label configuration and follows the standard graph cuts formulation:

$$E_N(\mathbf{I}, \mathbf{f}) = \sum_{f_n \neq f_m, n, m \in C} \frac{1}{1 + (I_n - I_m)^2}, \tag{3}$$

where $n, m \in C$ denotes that pixel n and m are neighbors. The third term denotes the fitness between the current shape prior and the current segmentation and can be written as

$$E_{\mathrm{P}}(\mathbf{f}, \mathbf{w}) = \sum_n [(M_n^1 - U_n^1)^2 + (M_n^2 - U_n^2)^2], \tag{4}$$

where $M_n^1 = c_1$ when $f_n \neq \mathrm{BP}$ and $M_n^1 = -c_1$ when $f_n = \mathrm{BP}$. Here, $M_n^2 = c_2$ when $f_n = \mathrm{B}$ and $M_n^2 = -c_2$ when $f_n \neq \mathrm{B}$, and c_1, c_2 are positive constants. A penalty is applied for pixels classified differently by the segmentation and the shape prior through equation (4).

The last term measures how well the shape prior itself fits with the image through calculating the entropy of intensity distributions inside the blood pool, inside the myocardium and outside the left ventricle. This term can be written as

$$\begin{aligned} E_{\mathrm{S}}(\mathbf{w}, \mathbf{I}) = & -\sum_i [p_{\mathrm{BP}}(i) \log p_{\mathrm{BP}}(i) \\ & + p_{\mathrm{M}}(i) \log p_{\mathrm{M}}(i) + p_{\mathrm{B}}(i) \log p_{\mathrm{B}}(i)], \end{aligned} \tag{5}$$

where $p_{\mathrm{BP}}(i)$, $p_{\mathrm{M}}(i)$ and $p_{\mathrm{B}}(i)$ are respectively the probability for pixels inside the blood pool, inside the myocardium and outside the left ventricle to have intensity *i* and are calculated as $p_{\mathrm{BP}}(i) = \frac{N_{\mathrm{BP}}(i)}{A_{\mathrm{BP}}}$, $p_{\mathrm{M}}(i) = \frac{N_{\mathrm{M}}(i)}{A_{\mathrm{M}}}$, and $p_{\mathrm{B}}(i) = \frac{N_{\mathrm{B}}(i)}{A_{\mathrm{B}}}$. $N_{\mathrm{BP}}(i)$ and A_{BP} are the number of pixels with intensity *i* and the area inside the blood pool, $N_{\mathrm{M}}(i)$ and A_{M} are those for inside the myocardium, and $N_{\mathrm{B}}(i)$ and A_{B} are those outside the left ventricle. If we denote a heavyside step function with $H(\bullet)$ and a delta function with $\delta(\bullet)$, then

$$\begin{aligned}
A_{\mathrm{BP}} &= \sum_n H(-U_n^1), \\
A_{\mathrm{M}} &= \sum_n H(U_n^1)H(-U_n^2), \\
A_{\mathrm{B}} &= \sum_n H(U_n^2), \\
N_{\mathrm{BP}}(i) &= \sum_n H(-U_n^1)\delta(I_n - i), \\
N_{\mathrm{M}}(i) &= \sum_n H(U_n^1)H(-U_n^2)\delta(I_n - i), \\
N_{\mathrm{B}}(i) &= \sum_n H(U_n^2)\delta(I_n - i).
\end{aligned} \tag{6}$$

By adding the last term in equation (1), we prevent inaccurate segmentations from producing inaccurate shape priors.

2.5. Energy Minimization

We use an expectation maximization (EM) style approach to minimize the energy function presented in equation (1) and alternately update the GMM and shape parameters while fixing the segmentation (maximization step) and use the GMMs and the shape prior to facilitate the image segmentation (estimation step).

2.5.1. M(aximization) Step

We start by assigning three Gaussians to each pixel, one which minimizes the first term in equation (1) when the pixel belongs to the blood pool, one when the pixel belongs to the myocardium and one the background. Differentiating equation (2) w.r.t. μ_d, Σ_d and π_d, we get the update equations for the GMM parameters:

$$\mu_d = \frac{\sum_{b_n=d} I_n}{\sum_{b_n=d} 1}, \Sigma_d = \frac{\sum_{b_n=d}(I_n - \mu_d)^2}{\sum_{b_n=d} 1}, \pi_d = \frac{\sum_{b_n=d} 1}{\sum_n 1}. \tag{7}$$

We use a gradient descent optimization to update the shape parameters. The update equation for $\mathbf{w}$ is

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha(\nabla_{\mathbf{w}} E_{\mathrm{P}} + \nabla_{\mathbf{w}} E_{\mathrm{S}}), \tag{8}$$

where α is the step size for the gradient descent optimization and $\mathbf{w}_t$ and $\mathbf{w}_{t+1}$ are the parameters at time t and $t+1$ respectively. $\nabla_{\mathbf{w}} E_{\mathrm{P}}$ is calculated by differentiating (4):

$$\frac{\partial E_{\mathrm{P}}}{\partial w_k} = \sum_n [2(U_n^1 - M_n^1)\frac{\partial U_n^1}{\partial w_k} + 2(U_n^2 - M_n^2)\frac{\partial U_n^2}{\partial w_k}], \tag{9}$$

Here, $\nabla_{\mathbf{w}} U_n^1$ and $\nabla_{\mathbf{w}} U_n^2$ can be obtained easily using geometry. $\nabla_{\mathbf{w}} E_{\mathrm{S}}$ is calculated by differentiating (5):

$$\begin{aligned}
\frac{\partial E_{\mathrm{S}}}{\partial w_k} = &- \sum_i [\frac{1}{A_{BP}} Q_{BP} - \frac{1}{A_M} Q_M] \frac{\partial N_{BP}(i)}{\partial w_k} \\
&- \sum_i [\frac{1}{A_B} Q_B - \frac{1}{A_M} Q_M] \frac{\partial N_B(i)}{\partial w_k} \\
&+ \sum_i [\frac{N_{BP}(i)}{A_{BP}^2} Q_{BP} - \frac{N_M(i)}{A_M^2} Q_M] \frac{\partial A_{BP}(i)}{\partial w_k} \\
&+ \sum_i [\frac{N_B(i)}{A_B^2} Q_B - \frac{N_M(i)}{A_M^2} Q_M] \frac{\partial A_B(i)}{\partial w_k},
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
Q_{BP} &= (\log \frac{N_{BP}(i)}{A_{BP}} + 1) \\
Q_M &= (\log \frac{N_M(i)}{A_M} + 1) \ , \\
Q_B &= (\log \frac{N_B(i)}{A_B} + 1)
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
\frac{\partial A_{BP}}{\partial w_k} &= -\sum_n \delta(-U_n^1)\frac{\partial U_n^1}{\partial w_k} \\
\frac{\partial A_B}{\partial w_k} &= \sum_n \delta(U_n^2)\frac{\partial U_n^2}{\partial w_k} \\
\frac{\partial N_{BP}(i)}{\partial w_k} &= -\sum_n \delta(-U_n^1)\frac{\partial U_n^1}{\partial w_k}\delta(I_n - i) \\
\frac{\partial N_B(i)}{\partial w_k} &= \sum_n \delta(U_n^2)\frac{\partial U_n^2}{\partial w_k}\delta(I_n - i)
\end{aligned} . \tag{12}$$

2.5.2. E(stimation) Step

Since we have three labels, segmentation of the image consists of three stages based on the α-expansion. During each stage, we follow the graph cuts framework for segmentation and create a graph with nodes corresponding to pixels and two additional terminal nodes. The first and third terms in our energy function are applied to the graph as t-links. The second term is added as n-links between neighboring nodes. We use the max-flow [5] algorithm to find the minimum cut on this graph.

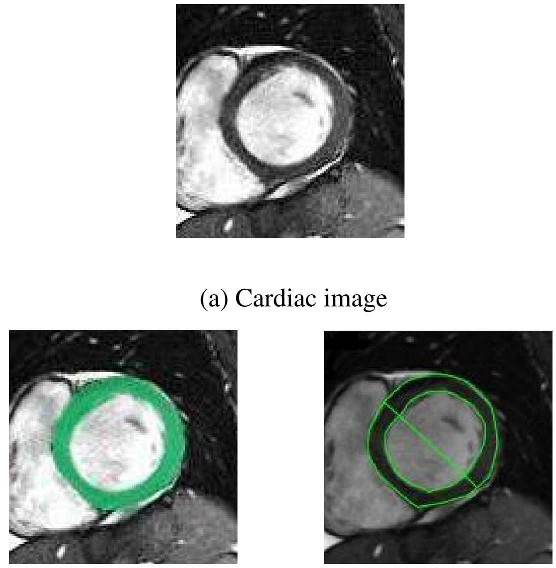
3. EXPERIMENTAL RESULTS

Fig. 1(b) shows the segmentation produced by our proposed method and Fig. 1(c) the ground truth segmentation for a slice of short axis cardiac image with poor boundary contrast (Fig.1(a)). The ground truth segmentation is produced manually by an expert. As visible from Fig. 1, our segmentation result is very similar to the ground truth.

4. CONCLUSIONS

The left ventricle in MR images presents many challenges for automated segmentation including poor contrast at desired tissue boundaries. Segmentation methods based on information from the image alone do not work well in such cases and additional constraints are necessary.

In this paper, we proposed a novel segmentation method that incorporates parametric shape priors, which do not require statistical training, to the graph cuts technique for robust and efficient segmentations of the left ventricle in cardiac images. We introduced novel terms accounting for shape prior/segmentation and shape prior/image fit to the graph cuts representation. We minimize the energy function through an EM-style approach and solve for the left ventricle segmentation and shape parameters calculation iteratively. The term accounting for shape prior/image fit in the objective function prevents a vicious cycle of bad segmentation/shape priors. We demonstrated the effectiveness of our method on real cardiac images with ground truth segmentations. We are currently working on quantitative analysis of results from our proposed method.



(a) Cardiac image

(b) Our segmentation (c) Ground truth segmentation

Fig. 1. Segmentation produced by our proposed method (b) and the ground truth segmentation (c) for a slice of short axis cardiac image with poor boundary contrast (a). In (b), the green area is the myocardium. In (c), the two circular green lines show respectively the endocardium and the epicardium boundaries. Visually, our segmentation result is very close to the ground truth.

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