ANTI-ALIAS IMAGE RECONSTRUCTION IN MAGNETIC RESONANCE IMAGING

Satoshi ITO, Shun NAKAMURA, Na LIU, Yoshifumi YAMADA

Faculty of Eng. Dept. of Information Sciences, Utsunomiya Univ., Utsunomiya, Japan

ABSTRACT

An image reconstruction technique that reduces aliasing artifacts by scalable image reconstruction in magnetic resonance imaging is proposed. The signal obtained by the phase-scrambling Fourier imaging technique can be transformed to the signal described in the Fresnel transform equation of the objects. Therefore, image reconstruction can be performed not only by inverse Fourier transform but also by inverse Fresnel transform. Image reconstruction by inverse Fresnel transform allows shrinking of images of rather wide scales. Thus, alias-free images can be reconstructed even from signals that produce aliasing artifacts by standard Fourier transform reconstruction. Simulation and experimental studies reveal that the proposed method can be used to produce anti-alias images.

Index Terms—Signal sampling, Magnetic resonance imaging, Image reconstruction, Antialiasing

1. INTRODUCTION

A number of methods have been proposed to remove aliasing artifacts, which are intended to accelerate the data acquisition time. When one receiver RF coil is used in data acquisition, we can remove the aliasing artifacts by using the correlation in k-space and time, which is referred to as *k-t* BLAST [1]. When more than one receiver RF coil can be used, aliasing artifacts can be removed using the sensitivity map of the RF coils that are used in data acquisition, which is referred to as parallel imaging. Currently, the best known are SMASH [2], SENSE [3], and GRAPPA [4]. These methods are quite effective in order to improve the temporal resolution in dynamic imaging or the data acquisition speed.

The signal obtained in the phase-scrambling Fourier imaging technique [5,6], in which a phase-scrambling pulse is added to the phase encoding step of conventional Fourier transform imaging, can be transformed to the signal described in the Fresnel transform equation of the objects. Therefore, image reconstruction can be performed not only by standard inverse Fourier transform but also by inverse Fresnel transform [7]. Image reconstruction by inverse Fresnel transform allows optional scaling of reconstructed images by using an adequate reconstruction parameter corresponding to the focal length in the optical image formation. Thus, alias-free images can be reconstructed even from signals that produce serious aliasing artifacts by scaling images, so as not to exceed the field-of-view. A scheme for re-scaling an image from the signal obtained in the phase-scrambling Fourier imaging technique is presented, and the effectiveness of the proposed method is demonstrated by simulation and experimental studies.

2. METHOD

2.1. NMR Signal in Phase-Scrambling Fourier Imaging Technique

Phase-Scrambling Fourier Transform (PSFT) imaging is a technique whereby a quadratic field gradient $\Delta B=b(x^2+y^2)$ is added to the pulse sequence of conventional FT imaging in synchronization with the field gradient for phase encoding [5,6]. Fig.1 shows the pulse sequence for the PSFT. The signal obtained in PSFT is given by Eq. (1),

$$v(k_x,k_y)=\iint\left[\rho(x,y)e^{-j\gamma b\tau\left(x^2+y^2\right)}\right]e^{-j\left(k_x x+k_y y\right)}dxdy \quad (1)$$

where $\rho(x,y)$ represents the spin density distribution in the subject, γ is the magnetogyric ratio, and b and τ are the coefficient and impressing time, respectively, of the quadratic field gradient. Spin density distribution $\rho(x,y)$ can be obtained by taking the inverse Fourier transform of the signal followed by the multiplication of quadratic phase term $\exp[j\gamma b\tau(x^2+y^2)]$ to cancel the scrambled phase.

Equation (1) can be rewritten as the Fresnel transform equation, as shown in Eq.(2), by using the variable substitutions $x'=k_x/2\gamma b\tau$ and $y'=k_y/2\gamma b\tau$.

$$v(x',y')e^{-j\gamma b\tau(x'^2+y'^2)}=\iint\rho(x,y)e^{-j\gamma b\tau\left[(x'-x)^2+(y'-y)^2\right]}dxdy \quad (2)$$

$$v(x',y')e^{-j\gamma b\tau(x'^2+y'^2)}=\rho(x',y')*e^{-j\gamma b\tau\left(x'^2+y'^2\right)} \quad (3)$$

The right-hand of Eq.(2) is known as Fresnel transform equation which is familiar in Optics or sound wave. The parameter $\gamma b\tau$ correspond to the distance parameter in the diffraction wave-front from the object. The Fresnel transform equation is a convolution integral of object function and quadratic phase function, so it can be written as Eq.(3), where * means convolution operation.

1-4244-1484-9/08/$25.00 ©2008 IEEE

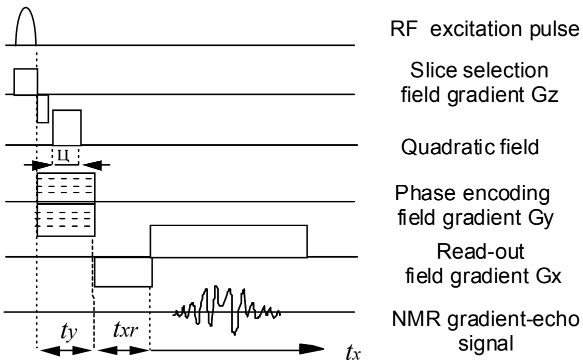

Fig.1 Pulse sequence for Phase-Scrambling Fourier Imaging technique

2.2. Image Reconstruction using Fresnel Transform

Object images can be obtained by inverse Fresnel transform from the signal written as Eq.(2) [7]. Reconstruction involves 1) multiplying a quadratic phase term numerically by the signal in the PSFT to obtain the signal shown in Eq.(1) and 2) solving $\rho(x,y)$ by the inverse filtering technique, as follows:

$$\rho(x',y') = F^{-1}\left[\frac{F\left[v(x',y')e^{-j\gamma b\tau(x'^2+y'^2)}\right]}{F\left[e^{-j\gamma b\tau(x'^2+y'^2)}\right]}\right]$$
$$= \sqrt{\frac{\gamma b\tau}{\pi}}\,e^{j\frac{\pi}{2}}F^{-1}\left[F\left[v(x',y')e^{-j\gamma b\tau(x'^2+y'^2)}\right]\cdot e^{-j\frac{\omega_x^2+\omega_y^2}{4\gamma b\tau}}\right] \qquad (4)$$

The imaging parameter $\gamma b\tau$, which is the coefficient of quadratic phase modulation, is necessary for the image reconstruction as shown in Eq.(4). The spatial resolution of reconstructed images is almost the same as signal step of Fresnel transformed signal,

$$\Delta x' = \frac{\Delta k_x}{2\gamma b\tau} = \frac{\pi}{\gamma b\tau N\Delta x} \qquad (5)$$

3. ANTI-ALIAS IMAGE RECONSTRUCTION

Suppose the parameter $\alpha\gamma b\tau$ is used in place of true $\gamma b\tau$ obtanied experimentally in the reconstruction equations from Eq. (2) to Eq. (4),

$$v_\alpha(x',y') = v(x',y')e^{-j\alpha\gamma b\tau\left(x'^2+y'^2\right)}$$
$$= \iint\left[\rho(x,y)e^{-j\left(\frac{\alpha-1}{\alpha}\right)\gamma b\tau(x^2+y^2)}\right]e^{-j\alpha\gamma b\tau\left\{\left(x'-\frac{x}{\alpha}\right)^2+\left(y'-\frac{y}{\alpha}\right)^2\right\}}dxdy \qquad (6)$$

By substituting the variables as $u=x/\alpha$, $w=y/\alpha$, we obtain the following equation;

$$v_\alpha(x',y') = \iint\left[\alpha^2\rho(\alpha u,\alpha w)e^{-j\gamma b\tau\left(\frac{\alpha-1}{\alpha}\right)\left((\alpha u)^2+(\alpha w)^2\right)}\right] \qquad (7)$$
$$\times e^{-j\alpha\gamma b\tau\left\{(x'-u)^2+(y'-w)^2\right\}}dudw$$

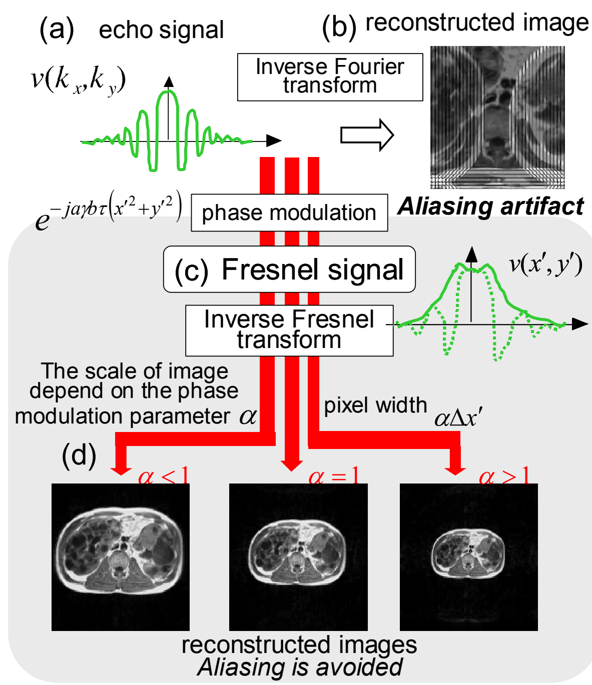

Fig. 2 Schematic of anti-alias image reconstruction : (a) echo signal (b) aliased image by inverse Fourier reconstruction, (c) numerically Fresnel transformed signal given a adequate scaling parameter α, and (d) reconstructed image by inverse Fresnel reconstruction with scaling effect.

$$\rho_\alpha(x',y') = F^{-1}\left[\frac{F\left[v_\alpha(x',y')e^{-j\alpha\gamma b\tau(x'^2+y'^2)}\right]}{F\left[e^{-j\alpha\gamma b\tau(x'^2+y'^2)}\right]}\right]$$
$$= \sqrt{\frac{\gamma b\tau}{\pi}}\,e^{j\frac{\pi}{2}}F^{-1}\left[F\left[v_\alpha(x',y')e^{-j\alpha\gamma b\tau(x'^2+y'^2)}\right]\cdot e^{-j\frac{\omega_x^2+\omega_y^2}{4\gamma b\tau}}\right] \qquad (8)$$
$$= \alpha^2\rho(\alpha x,\alpha y)e^{-j\gamma b\tau\left(\frac{\alpha-1}{\alpha}\right)\left[(\alpha x)^2+(\alpha y)^2\right]}$$

Therefore, the inverse Fresnel transform of Eq.(7) using $\alpha\gamma b\tau$ yields a scaled spin density distribution as follows: The pixel width of the scaled image becomes $\alpha\Delta x'$ according to Eqs.(5) and (8). Note that the Fresnel transformed signal of Eq.(7) remains in the form of the convolution integral equation, even though the true parameter $\gamma b\tau$ is not given in the reconstruction procedure, and so resulting images are free from blurring.

Consider the case in which an aliasing artifact occurs in the Fourier reconstructed image using the PSFT signal written in Eq.(1). Fresnel reconstruction by Eq.(8) then offers an alias-free image by shrinking the image using an adequate scaling parameter α, so as to appear smaller than the field-of-view. Therefore, we can rescale the reconstructed image at the optional scale after data acquisition and thereby avoid the aliasing artifact. Figure 3 shows the scheme of proposed method.

3. EXPERIMENTS

The MRI system used in the experiments generates a static magnetic field of B_0 = 0.0183 T (the resonant frequency is 779 kHz) by the solenoid coil. Since the strength of the main static field of the MRI system is low and the SNR of the NMR signal is small, we improved the SNR of images by averaging the signal over several dozen observations. Figure 3 shows the coil systems that produce quadratic field and static B_0 field. The coil producing $b(x^2+y^2)$ field gradient is one designed for line scan imaging.

The parameters of the experiments were as follows. The repetition time for the pulse sequence was TR =300 msec, and the spatial resolution of Fourier reconstructed image was Δx=Δy=0.05 cm, and the data matrix of the NMR signal was a 64x64 matrix. Since the size of phantom is larger than the field-of-view in the Fourier reconstruction, aliaisng artifact occurs in the Fourier reconstructed image (b).

Figure 4 and 5 show the results of computer simulation and experiments when $\gamma b\tau$=10.0 rad/cm^2. The image obtained by conventional Fourier reconstruction shows serious aliasing artifacts as shown in Fig.4(b) and Fig.5(b). Figures from (c) to (e) show images by Fresnel transform with scaling effect. The scaling effect is not only down-scaling but also up-scaling is possible as shown in (e).

Figure 6 shows a series of scaled images reconstructed using signals having different $\gamma b\tau$ values. The images on the left show the obtained signals and the images on the right show scaled images by the Fresnel reconstruction that are scaled to the same pixel size of $\alpha\Delta x'$ = 0.065 cm by adjusting the scaling parameters. The $\gamma b\tau$ value has a wide range with which reconstructed images are successfully rescaled. It was shown that as $\gamma b\tau$ become small, the surround region of image is blurred or diminished. These results indicated that the parameter $\gamma b\tau$ has a lower limit.

Figure 7 shows the result of scaling using "yuzu" orange. Imaging parameters are $\gamma b\tau$ =6.1 rad/cm^2, the number of data point was 64x64. Fig.7(a) shows the PSFT signal and (b) is Fourier transformed image. Fig.(c) is a Fresnel transformed image with α=0.8, and (d) is Fresnel transformed image using the signal (c). Alias-less image are obtained by the proposed algorithm.

Consider the relation of SNR and scaling parameter α on condition that the same signal data is used in the reconstruction. The signal energy and noise energy never change with the scaling parameter α. When images are scaled by α, the amplitude of object image theoretically becomes α times greater, whereas the standard deviation of the noise does not change in the reconstruction procedure involving FFT and quadratic phase modulation. Therefore, the SNR of the scaled image changes by α times in accordance with scaling coefficient α.

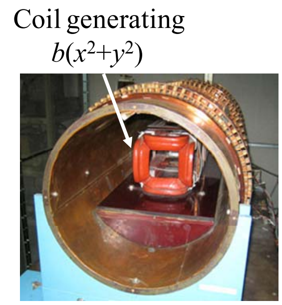

Fig.3 Coil systems producing a quadratic field gradient and static B_0(=0.0183T) field.

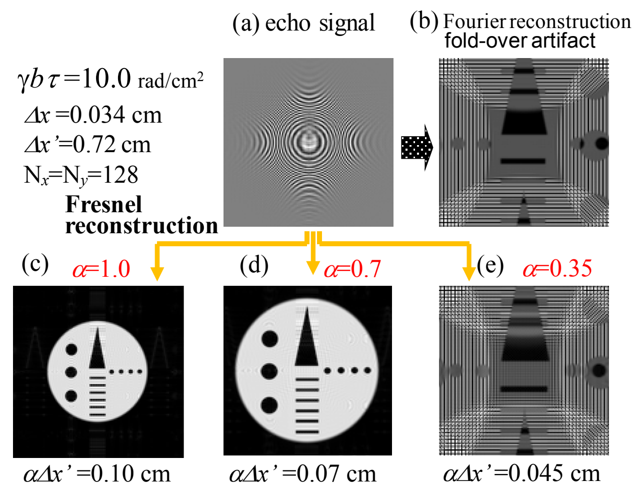

Fig.4 Results of computer simulation in the case $\gamma b\tau$=10.0 rad/cm^2; (a) PSFT echo signal, (b) Fourier transformed image ; aliasing artifact occurs, (c) , (d), (e) scaled images using Fresnel transform reconstruction varying the scaling factor α.

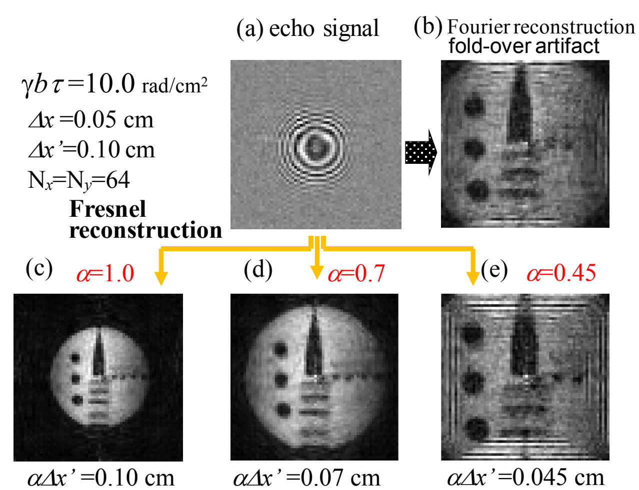

Fig.5 Results of experiments in the case $\gamma b\tau$=10.0 rad/cm^2; (a) PSFT echo signal, (b) Fourier transformed image ; aliasing artifact occurs, (c) , (d), (e) scaled images using Fresnel transform reconstruction varying the scaling factor α.

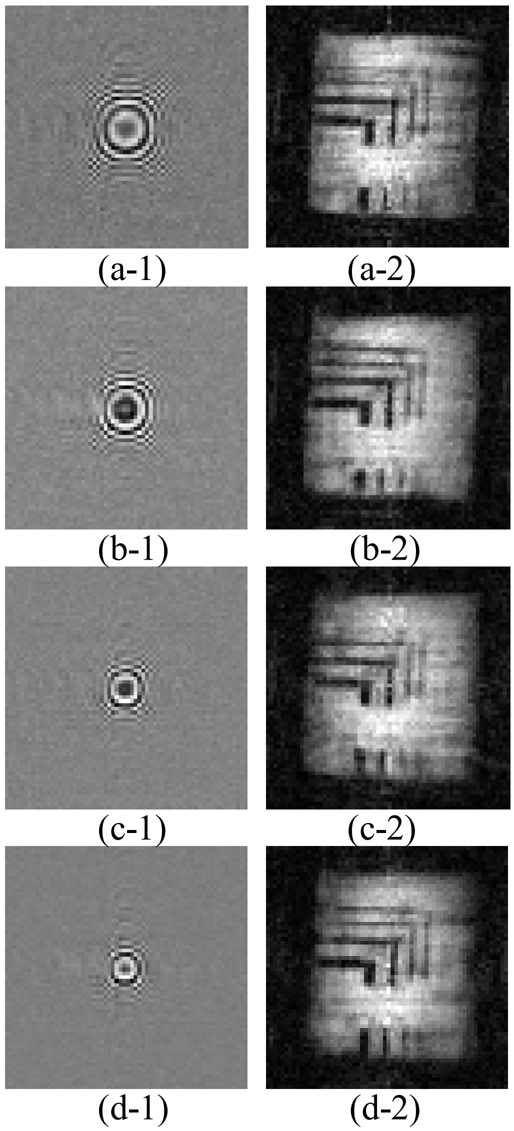

Fig.6 echo signal obtained in each condition; (a-1) phase modulation coefficient $\gamma b\tau$=5.0 rad/cm^2, (b-1) $\gamma b\tau$=4.0 rad/cm^{2,} (c-1) $\gamma b\tau$=3.0 rad/cm^{2,} , (d-1) $\gamma b\tau$=2.5 rad/cm^2, and (a-2), (b-2), (c-2), (d-2) are rescaled images to be the same size in the FOV

4. CONCLUSION

A new anti-alias image reconstruction algorithm that uses the phase-scrambling Fourier imaging technique is presented. This method allows optional scaling in image reconstruction and therefore, images can be resized so as not to exceed the field-of-view of the image. The reconstruction algorithm can be applied to commercial MRI by simply adding a weak quadratic field gradient to the imaging pulse sequences or modulated RF excitation pulses in the presence of *x*- and *y*- field gradients, which will greatly improve the flexibility of image reconstruction.

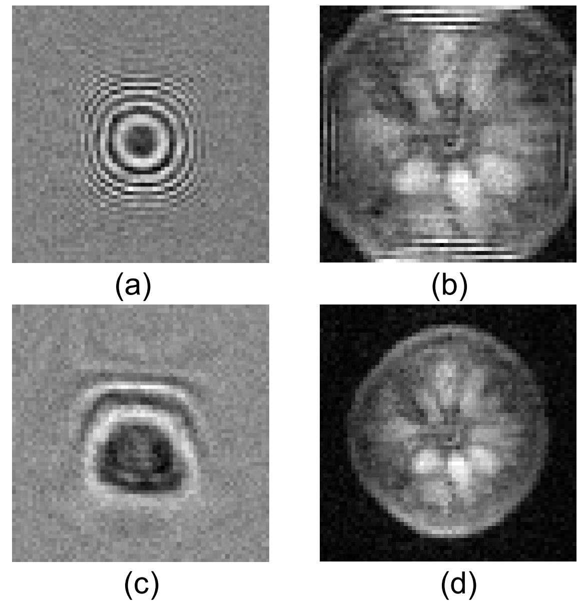

Fig.7 Scaling of "yuzu" orange. (a) PSFT signal, (b) Fourier reconstructed image; resolution 0.055cm, (c) Fresnel transformed signal from signal (a), (d) anti-aliased image using Fresnel transform; resolution 0.11cm

ACKNOWLEDGMENT

This work was supported partly by Grant-in-Aid for Scientific Research(C) (No.19560416) of Japan Society for the Promotion of Science and the Grant in Eminent research selected at Utsunomiya University.

REFERENCES

[1] J Tsao, P Boesiger, KP Pruessmann, "*k-t* BLAST and k-t SENSE: dynamic MRI with high frame rate exploiting spatiotemporal correlations," Magn Reson Med, vol.50, pp.1031-1042, 2003.

[2] DK Sodickson, WJ Manning, "Simultaneous acquisition of spatial harmonics (SMASH): fast imaging with radiofrequency coil arrays," Magn Reson Med, vol.38, pp.591-603, 1997.

[3] KP Pruessmann,M Weigner, MB Scheidegger et al. "SENSE: sensitivity encoding for fast MRI," Magn Reson Med, vol.42, pp.952-962, 1999.

[4] MA Griswold, PM Jakob, RM Heidemann et al. "Generalized autocalibrating partially parallel acquisitions (GRAPPA)," Magn Reson Med, vol.47, pp.1202-1210, 2002.

[5] Maudsley AA, "Dynamic range improvement in NMR imaging using phase scrambling," J Magn Reson, vol.76, pp.287-305, 1988.

[6] Wedeen VJ, Chao YS, Ackerman JL, "Dynamic Range Compression in MRI by Means of a Nonlinear Gradient Pulse," Magn Reson Med, vol.6 , pp.287–295, 1988.

[7] Yamada Y, Tanaka K, Abe Z, "NMR Fresnel transform imaging technique using a quadratic nonlinear field gradient," Rev Sci Instrum, vol.63, pp.5348-5358, 1992.