RHYTHMIC COMPONENT EXTRACTION FOR MULTI-CHANNEL EEG DATA ANALYSIS

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ABSTRACT

A practical method for extracting and enhancing a rhythmic waveform appearing in multi-channel electroencephalogram (EEG) data is proposed. In order to facilitate clinical diagnosis and/or implement so-called brain computer interface (BCI), detecting the rhythmic activity from EEG data recorded in a noisy environment is crucial; however, classical signal processing techniques like linear filtering or the Fourier transform cannot detect such a rhythmic signal if the power of noise is so large. This paper presents a simple but practical method for extracting a rhythmic signal by fully exploiting the multi-variate nature of EEG data. The rhythmic component of interest is estimated as the weighted sum of multi-channel signals, and the optimal weights are then derived so as to maximize the power of the component. After the derivation is illustrated, adaptive weights, which give a new time-frequency analysis, are introduced. Moreover, the application to recently developed empirical mode decomposition (EMD) is presented. Experimental results on real EEG data support the analysis.

Index Terms— Electroencephalogram (EEG), signal extraction, brain computer interface, multi-channel signal processing

1. INTRODUCTION

A robust and efficient electroencephalogram (EEG) signal processing has been highlighted in the areas of neuroscience, medicine, engineering, and so forth [1]. Even though EEG is more simple and convenient to measure and use than other non-invasive, non-hazardous technologies for functional brain mapping, such as magnetoencephalography (MEG) and functional MRI (fMRI), observed signals are so contaminated by noise, especially in engineering and clinical applications, that we need further signal processing to extract physically meaningful information, which may be rhythmic waves such as alpha, theta, and beta waves [1].

Frequency analysis or filtering is an established and classical way to deal with a single channel signal. When we are interested in a specific frequency range, time-invariant band-pass filtering or the Fourier transform (FT) will extract the target frequency component. If the signal is non-stationary, which is very often in EEG, then classical time-frequency analyzers, such as the short-time Fourier transform (STFT) and wavelet transforms (WT) [1, 2], may be applicable to the analysis of data. Another time-frequency analysis is recently developed empirical mode decomposition (EMD) [3], that provides a fully data-dependent time-frequency spectrum called the Hilbert-Huang spectrum (HHS). This method seeks for a collection of linearly independent functions in which the corresponding instantaneous frequency can be well-defined.

What is common among these frequency-based analysis methods is to extract a frequency component or to decompose a signal to a set of frequency components. This implies that even if the input signal is a white Gaussian noise, a sinusoidal waveform may be "extracted," by narrow-band-pass filtering. In other words, from the channel signal highly contaminated by noise, we may observe a rhythmic component, for example, the alpha wave by extracting the range of 8–13 Hz with band-pass filtering or the FT, even though there does not physically exist such a waveform in brain.

In the case of multi-channel signals, we have already had well-established analysis methods for multi-variate data such as principal component analysis (PCA) and independent component analysis (ICA) [1, 4]. The former method is useful to extract a component of which the power is very dominant. The latter is effective to estimate statistically independent components and applicable to blind signal separation (BSS). In particular, the assumption of statistical independence among original sources is often too severe to obtain physically meaningful components inside brain out of observed data [4,5], though it would be useful to exclude signals independent of the brain activity, such as eye blinking and heart beating. For instance, Cao proposed an application of ICA to detecting brain activity from EEG of quasi-brain-death patients recorded in a noisy environment [5]. Even if this method can sometimes extract a component showing physically meaningful rhythm, the assumption of statistical independence does not guarantee the extraction of rhythmic waves. Besides, Washizawa *et al.* developed a method for extracting one global signal highly distorted by noise in each channel [6]. The method seems successful in estimating a periodic component possibly caused by a steady state visually evoked potential (SSVEP). However, if the frequency of interest is known in advance, it is more natural to directly estimate a component with respect to the frequency range.

This paper presents a practical and efficient method for extracting a periodical component from multi-channel EEG data termed *rhythmic component extraction* (RCE). The present work is mainly motivated by time-frequency analysis of highly noisy EEG signals and the detection of brain activity, aiming at BCI and brain-death diagnosis [5]. The extraction is made by combining multi-channel sensor signals with weights that are optimally sought for such that the extracted component maximally contains the power in the frequency range of interest and suppresses that in unnecessary frequencies. The criteria of the optimization is therefore given as the ratio of two power integrals, which is later reduced to a simple matrix eigenvalue problem without any numerical integration. After giving the derivation of the optimal weights, we develop adaptive (time-

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varying) weights that will bring us a novel time-frequency analysis. Moreover, by combining the present method with the EMD, we obtain clearer instantaneous frequency/amplitude analysis [3]. The application of the proposed method to the detection of brain activity from EEG data of quasi-brain-death as well as rhythmic wave detection aiming at BCI is presented to support the analysis.

2. RHYTHMIC COMPONENT EXTRACTION (RCE)

This section establishes the theory and method for extracting the frequency component to be sought for. Extraction is made by weighted averaging and the optimal weights are found out by maximizing a cost function described below. EEG signals often contains a periodic component such as alpha, theta, and beta waves, and mu rhythm [7]. For the proposed analysis, it is assumed that phase difference of the component to be extracted is negligible or pre-aligned.

2.1. Power Ratio Maximization Criteria and Optimal Weights

Let $x_i(t)$ be an observed signal in the ith channel out of M channels, that is, $i = 1, \cdots, M$. The extraction of the desired component is performed by the weighting summation over all the channels. Specifically, the extracted signal is defined as

$$\hat{x}(t) = \sum_{i=1}^{M} w_i x_i(t), \tag{1}$$

where w_i is the weight to be determined. As explained earlier, $\hat{x}(t)$ maximally has the power in the frequency range of interest.

Indeed, we have only a discretized $x_i(t)$ with finite number of samples from an EEG experiment, that is, $x_i[n] = x_i(nT)$, where T is the sampling period and n is the discrete time index, $n = 0, \ldots, N-1$. Let $\hat{X}(e^{j\omega})$ be the discrete-time Fourier transform of $\hat{x}[n]$. If we are interested in the specific frequency range, denoted by $\Omega_1 \subset [0, \pi]$, then the power of $\hat{x}_i[k]$ with respect to this range is described as

$$P_1 = \int_{\Omega_1} |\hat{X}(e^{j\omega})|^2 d\omega. \tag{2}$$

In contrast, we may have the power of frequency range(s) to be maximally suppressed, which we denote by $\Omega_2 \subset [0, \pi]$. Note that it is sufficient to consider positive frequencies because of observed signals being real-valued.

Then a cost function to be maximized may be defined by functional

$$J[\boldsymbol{w}] = \frac{\int_{\Omega_1} |\hat{X}(e^{j\omega})|^2 d\omega}{\int_{\Omega_2} |\hat{X}(e^{j\omega})|^2 d\omega}. \tag{3}$$

It should be noticed that the maximization of the above cost function is accomplished by solving an eigenvalue problem of a finite dimensional matrix, as will be seen hereafter. To show that, we describe the following preliminaries.

Define matrix $\boldsymbol{X} \in \mathbb{R}^{M\times N}$ such that the (m, n)-element of $\boldsymbol{X}$ is given by $[\boldsymbol{X}]_{m,n} = x_m[n]$. Furthermore, define two matrices $\boldsymbol{W}_1, \boldsymbol{W}_2 \in \mathbb{R}^{N\times N}$ as, for $l, k = 0, \ldots, N-1$,

$$[\boldsymbol{W}_1]_{l,k} = \Re \int_{\Omega_1} e^{-j\omega(l-k)} d\omega, \tag{4}$$

and

$$[\boldsymbol{W}_2]_{l,k} = \Re \int_{\Omega_2} e^{-j\omega(l-k)} d\omega, \tag{5}$$

respectively, where $\Re$ is the operator taking the real part. Since it can be verified that $\boldsymbol{W}_2$ is symmetric positive definite, eigenvalue decomposition of $\boldsymbol{X}\boldsymbol{W}_2\boldsymbol{X}^T$ is obtained as

$$\boldsymbol{X}\boldsymbol{W}_2\boldsymbol{X}^T = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}^T, \tag{6}$$

where $\boldsymbol{U}$ is an orthogonal matrix of size $M \times M$ and $\boldsymbol{\Sigma}$ is a diagonal matrix of size $M \times M$ with non-negative elements.

We have the following result.

Theorem 1 *Assume that $\boldsymbol{X}\boldsymbol{W}_2\boldsymbol{X}^T$ has full rank. Functional $J[\boldsymbol{w}]$ is maximized by*

$$\boldsymbol{w} = \boldsymbol{U}\boldsymbol{\Sigma}^{-1/2}\tilde{\boldsymbol{w}}, \tag{7}$$

where $\tilde{\boldsymbol{w}}$ is an eigenvector corresponding to the maximum eigenvalue of $\boldsymbol{\Sigma}^{-1/2}\boldsymbol{U}^T\boldsymbol{X}\boldsymbol{W}_1\boldsymbol{X}^T\boldsymbol{U}\boldsymbol{\Sigma}^{-1/2}$.

(Brief proof) Let $\boldsymbol{x}[n] = [x_1[n], \ldots, x_M[n]]^T$. Then we have

$$\begin{aligned}
P_1 &= \int_{\Omega_1} \left| \sum_{n=0}^{N-1} \hat{x}[n] e^{-j\omega n} \right|^2 d\omega \\
&= \int_{\Omega_1} \left| \sum_{n=0}^{N-1} \boldsymbol{w}^T \boldsymbol{x}[n] e^{-j\omega n} \right|^2 d\omega \\
&= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \boldsymbol{w}^T \boldsymbol{x}[m] \left(\int_{\Omega_1} e^{-j\omega(m-n)} d\omega \right) \boldsymbol{x}^T[n]\boldsymbol{w} \\
&= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \boldsymbol{w}^T \boldsymbol{x}[m] \\
&\quad \times \frac{1}{2} \left(\int_{\Omega_1} e^{-j\omega(m-n)} d\omega + \overline{\int_{\Omega_1} e^{-j\omega(m-n)} d\omega} \right) \boldsymbol{x}^T[n]\boldsymbol{w} \\
&= \boldsymbol{w}^T \boldsymbol{X}\boldsymbol{W}_1\boldsymbol{X}^T \boldsymbol{w},
\end{aligned} \tag{8}$$

where $\bar{\cdot}$ denotes the complex conjugate. This way, we can also write $P_2 = \boldsymbol{w}^T\boldsymbol{X}\boldsymbol{W}_2\boldsymbol{X}^T\boldsymbol{w}$, implying that criterion (3) reads

$$J[\boldsymbol{w}] = \frac{\boldsymbol{w}^T\boldsymbol{X}\boldsymbol{W}_1\boldsymbol{X}^T\boldsymbol{w}}{\boldsymbol{w}^T\boldsymbol{X}\boldsymbol{W}_2\boldsymbol{X}^T\boldsymbol{w}}. \tag{9}$$

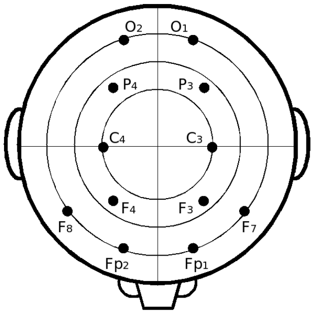
The maximization of the above cost function is equivalent to the maximization of the numerator under the constraint that the denominator is constant. Therefore, the problem is reduced to a general eigenvalue problem. □

2.2. Remarks and Adaptive Algorithm

1. If the range of frequency is continuous, that is, $a \leq \Omega_1 \leq b$, then $\boldsymbol{W}_1$ is explicitly described as

$$[\boldsymbol{W}]_{m,n} = \begin{cases} \dfrac{\sin b(m-n) - \sin a(m-n)}{m-n} & m \neq n, \\ b - a & m = n. \end{cases} \tag{10}$$

2. When Ω_2 spans the whole range of frequency, that is, $\Omega \in [0, \pi]$, then $\boldsymbol{W}_2 = \pi\boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix. Therefore, the denominator of (3) becomes proportional to $\boldsymbol{w}^T\boldsymbol{X}\boldsymbol{X}^T\boldsymbol{w}$, which implies that $\boldsymbol{\Sigma}^{-1/2}\boldsymbol{U}^T$ works as a whitening matrix [4].
3. By windowing a signal under investigation, the maximizer of (3) can provide a time-frequency analysis. Let $\boldsymbol{X}_k \in \mathbb{M} \times \mathbb{N}$ be a matrix consisting of windowed signals with respect to the kth frame. Then $J[\boldsymbol{w}^*(k)]$ associated with the corresponding maximizer, $\boldsymbol{w}^*(k)$ indicate the power of the extracted component.



Ch	Test 1	Test 2
1	Fp1	O2
2	Fp2	O1
3	F3	C4
4	F4	C3
5	F7	F4
6	F8	F3
7		Fp2
8		Fp1
9		P4
10		P3

Fig. 1. Electrode layout

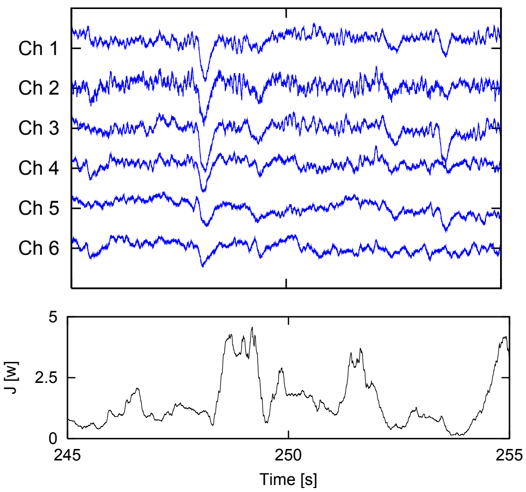


Fig. 2. Recorded quasi-brain-death EEG data (top) and the analyzed power by the proposed method (bottom).

4. The integral in both P_1 and P_2 could be replaced by the sum of the power of the discrete Fourier transform. However, in this case, only discretized frequencies with step $2\pi/N$ are allowed to specify the range.

3. EXPERIMENTS

The proposed RCE is applied to two sets of EEG signals recorded in an noisy (non-shielded) environment in order to show that RCE can successfully extract the rhythmic component of interest. The first set is the one used in [5] and recorded from a quasi-brain-death patient whose symptom was very similar to brain-death case; however, the patient got back consciousness after some treatment. The other is the set of EEG signals recorded from an ordinary person repeating alternatively moving the arms, legs, and nothing while closing eyes. The electrode layout is depicted in Fig. 1.

3.1. Quasi-Brain-Death Patient (Test 1)

EEG data of six channels only in forehead with sampling frequency of 1 kHz are used for the test. Parts of EEG signals for 10 seconds (245–255 s) in all channels are shown in Fig. 2.

First, we applied the proposed RCE to the EEG data for the period of ten seconds (245–255 s), as shown in the top of Fig. 2. To see the variation of the alpha wave component (8–13 Hz), Ω_1 is set to the range corresponding to 8–13 Hz and Ω_2 is defined as $\Omega_2 = [0, \pi] - \Omega_1$. We used the window size, $N = 500$, and tracked the power variation, $J[\boldsymbol{w}^*(k)]$, which is illustrated in the bottom of Fig. 2. In order to confirm that $J[\boldsymbol{w}^*(k)]$ represents the power of the alpha-wave components, we applied RCE to a segment of the signals (245.5–255.5 s). The extracted signal from all the channel is depicted in Fig. 3(a) and its Fourier spectrum is shown in Fig. 3(b), where the theta component (4–7 Hz) is also plotted. It is seen in this figure that the strong peak appears in the alpha wave frequency. The theta has a less strong peak, so it is difficult to judge if the theta component is included in this period. It should be noted that the Fourier spectra of a raw signal and the simply averaged signal over all channels only show the power of their trends.

Another interesting example is the segment with 70–71 s. Figure 3(c) shows the extracted theta and the observed signal in Ch 1. The power spectra of the alpha and theta components are illustrated in Fig. 3(d), which indicates that this segment strongly contains the theta. The alpha component contains the power of the theta rather than that of the alpha. The bottom of Fig. 3(d) slightly shows the power of the theta-wave; however, the proposed method facilitate the interpretation of the recorded data.

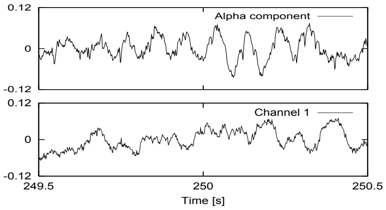
Application to EMD: Empirical mode decomposition (EMD) is a recently developed tool providing a data-driven time-frequency analysis. EMD decomposes the signal to several narrow-band modes called intrinsic mode functions (IMFs). Each IMF allows us to know instantaneous frequency and instantaneous amplitude, which draw a time-frequency spectrogram called the Hilbert-Huang spectrum (HHS). See [3] for more details on EMD. Figure 3(e) and (f) summarizes the effect of applying RCE to the interval of 249.5–250.5 s. It is observed that both the 4th and 5th IMFs indicate stable alpha wave frequencies around at time instance 250s. Moreover, in the top of Fig. 3(f), the instantaneous amplitude of the 4th IMF shows a peak at this time instance in a way similar to the power of the alpha wave component being relatively large in this interval as observed in the bottom of Fig. 2.

3.2. Ordinary Person With Body Action (Test 2)

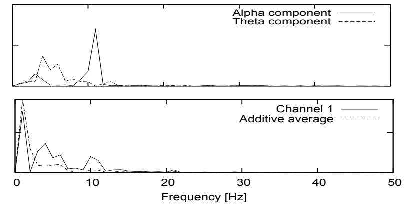
The number of channel was ten, sampling rate was 1 kHz, and the location of each channel is in Fig. 1. The subject of this experiment shook his hands and legs for 5 seconds (5–10 s) of 15 seconds EEG recording. He kept his eyes closed during the EEG recording. The estimated signal using only even number of channels by the optimal weights and the signal of channel 1 are shown in Fig. 4(a) with its Fourier spectra illustrated in Fig. 4(b). We can observe the existence of the alpha-wave component more clearly by the proposed method than averaging. Furthermore, the variations of optimal weights $\boldsymbol{w}^* = \{w_i^*\}$ at Ch 2 ($w_2^*(k)$) and Ch 8 ($w_8^*(k)$) are shown in Fig. 4(c) in order to observe the spatial distribution of the target component. Note that although in the observed data, which are not depicted in the paper due to lack of space, all channels show a similar behavior in amplitude, the weight in Ch 2 is smaller than the one in Ch 8 in the interval of 5–10 s, during moving legs and arms.

4. CONCLUSION

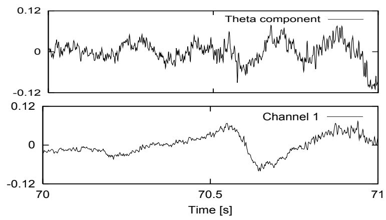
We have proposed the practical method for detecting rhythmic activity observed from multi-channel EEG data, called RCE. RCE successfully estimates a rhythmic signal with respect to the target frequency range from EEG data with the optimal weights that maximize the power in the frequency range. Simulations on real data illustrated



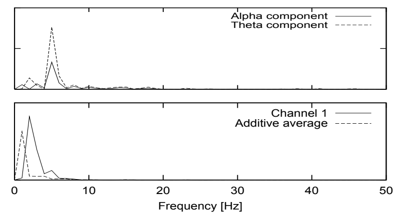
(a) Extracted alpha wave component (top) and raw observed signal of channel 1 (bottom).



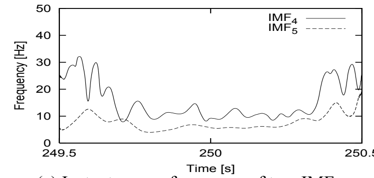
(b) Fourier spectra of extracted alpha and theta wave components (top) and spectra of the signal in ch. 1 and simply averaged signal over all channels (bottom).



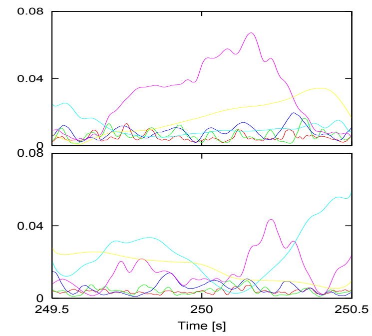
(c) Extracted theta wave component (top) and raw observed signal of channel 1 (bottom).



(d) Fourier spectra of extracted alpha and theta wave components (top) and spectra of the signal in ch. 1 and simply averaged signal over all channels (bottom).

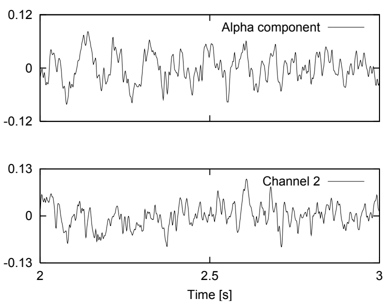


(e) Instantaneous frequency of two IMFs

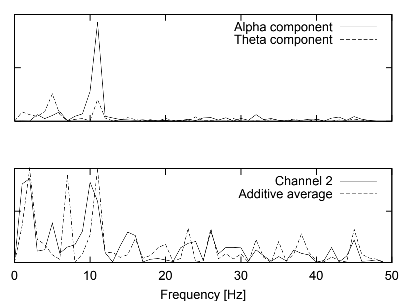


(f) Instantaneous amplitudes of IMFs generated from the alpha wave component by RCE (top) and the signal in ch. 1 (bottom).

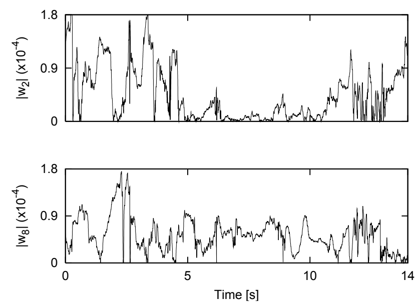
Fig. 3. Results of the analysis to brain-death patient data.



(a) The extracted alpha component (top) and observed signal in Ch 2 (bottom).



(b) Fourier spectra of the extracted alpha and theta components (top) and those of the Ch 2 and simply averaged signal (bottom).



(c) Variations of the optimal weights, w_2^* and w_8^*.

Fig. 4. Results of the analysis to ordinary person EEG data

that the RCE method works well in extracting rhythmic components as well as tracking the power of the component and the spatial distribution. Future research would be concern about automatic detection of active frequency and development of a method considering phase differences.

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