AUXILIARY NOISE POWER SCHEDULING FOR FEEDFORWARD ACTIVE NOISE CONTROL

Alberto Carini and Silvia Malatini

STI - University of Urbino "Carlo Bo"
Piazza della Repubblica, 13 - 61029 Urbino, Italy
E-mails: {carini, malatini}@sti.uniurb.it

ABSTRACT

The paper introduces a self-tuning power scheduling for the auxiliary noise of feedforward active noise control systems with online secondary path modeling. The proposed power scheduling is chosen so that in every operating condition a specific ratio between the powers at the error microphone of the auxiliary noise and of the residual noise is achieved.

Index Terms— Adaptive control, adaptive filters, adaptive signal processing, acoustic noise, noise

1. INTRODUCTION

Active noise control (ANC) systems equipped with the Filtered-x LMS (FX-LMS) adaptation algorithm cannot prescind from the online estimation of the secondary path [1]. Two different approaches can be adopted for the secondary path modeling. A first approach involves the injection of an auxiliary white random noise in the ANC system and it uses a system identification method to model the secondary path [2], [3]. The second approach estimates the secondary path directly from the output of the control filter, without the injection of additional noise [1]. It has been shown in [4] that the first approach is superior for convergence speed of both the control filter and the secondary path modeling filter, for speed of response to modifications in the primary noise and the secondary path, for independence between the primary noise attenuation and the online secondary path identification, and for computational complexity.

Among the ANC systems with secondary path modeling, presently the most performant with broadband input noises are those of [2] and [3]. The ANC of [2] uses three adaptive filters: the first adaptive filter adapts the control filter, the second adapts the secondary path modeling filter, and the third acts as a noise suppressor. The latter is used to remove the residual noise from the error signal of the secondary path modeling filter to improve the convergence performance and the estimation accuracy of the secondary path. Moreover, a cross-update strategy is employed for removing also the disturbance due to the auxiliary noise from the error signals of the control filter and of the noise suppresser. More recently, improved convergence performances were obtained with the ANC structure proposed in [3], where only two adaptive filters are used. One adaptive filter is used for adapting the control filter and one for modeling the secondary path, but in [3] an improved convergence speed of the control filter is obtained by introducing the delay compensation scheme of [5], and by removing the auxiliary noise from the error signal of the control filter.

The use of a scheduling strategy for the auxiliary noise in order to improve the convergence properties of the ANC system was first proposed in [6], where a frequency-domain ANC system was developed. An auxiliary noise power scheduling was introduced in [7] for the time domain ANC system proposed in [2]. In [7] the auxiliary noise power is varied by taking into account both the convergence status of the ANC system and the power of the primary noise. As a result, the ratio between the powers at the error microphone of the auxiliary noise and of the residual noise is approximately kept constant. A larger auxiliary noise power is injected in the early phases of adaptation of the ANC system, while a lower power is injected at steady state. An auxiliary noise power scheduling was also proposed in [8] for the ANC system of [3]. In [8] the power of the auxiliary noise is varied in accordance with the ratio between the power of the secondary path estimation error and the power of the error microphone signal, and it is maintained between a minimum and a maximum power experimentally determined. In this paper a different approach for scheduling the power of the auxiliary noise is proposed. In particular, the power of the auxiliary noise is automatically tuned for guaranteeing in every operating condition the desired ratio between the powers at the error microphone of the auxiliary noise and of the residual noise. Compared with a fixed injected noise, the power scheduling of the auxiliary noise is capable to better meet the conflicting requirements of fast convergence speed of the secondary path modeling filter and of low residual noise in steady state conditions.

In this paper, the proposed power scheduling is derived by referring to the ANC system of [3]. Nevertheless, the proposed power scheduling is also directly applicable to the ANC system of [2].

The paper is organized as follows. Section 2 provides a brief overview of the ANC system of [3], and of the auxiliary noise power scheduling of [7] and [8]. Section 3 introduces the proposed auxiliary noise power scheduling. Section 4 provides simulation results that compare the performances of the ANC system of [3] with the proposed auxiliary noise power scheduling, with the performances of the same system equipped with the noise power scheduling of [8].

Throughout the paper small boldface letters are used to denote vectors, the symbol $*$ denotes the linear convolution, $E[\cdot]$ denotes the mathematical expectation, and $\|\cdot\|$ denotes the Euclidean norm.

2. BACKGROUND THEORY

Figure 1 shows the block diagram of the ANC system with secondary path modeling proposed in [3]. The definition of all quantities introduced in Figure 1, together with the definition of other quantities used in the following of the paper can be found in Table 1.

The active noise control system of [3] exploits the delay compensation scheme of [5] in order to improve the convergence properties of the noise control filter and to avoid the use of a noise sup-

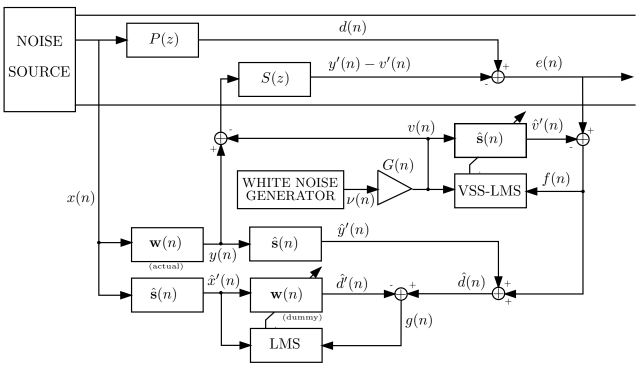

Fig. 1. The ANC system with secondary path modeling of Akhtar, Abe, and Kawamata.

presser in the estimation of the secondary path. The adaptation algorithm for the noise control filter was called in [5] "modified filtered-x algorithm" and it adapts $\mathbf{w}(n)$ with the following rule:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w g(n)\hat{\mathbf{x}}'(n), \qquad (1)$$

where μ_w is a fixed step-size parameter.

The secondary path is estimated from the zero mean white Gaussian auxiliary noise $v(n)$ injected in the secondary path, which is obtained by multiplying a zero mean unit variance white Gaussian noise $\nu(n)$ with a gain $G(n)$. In [3] and [8] the secondary path modeling filter $\hat{\mathbf{s}}(n)$ is adapted with a variable step-size LMS algorithm with the following adaptation rule:

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \mu_s(n) f(n)\mathbf{v}(n), \qquad (2)$$

where $\mu_s(n)$ is the variable step-size parameter. This parameter is varied between a minimum value $\mu_{s_{\min}}$ and a maximum value $\mu_{s_{\max}}$ (with $\mu_{s_{\min}}$ and $\mu_{s_{\max}}$ experimentally determined) on the basis of the ratio $\rho(n)$ between the power of the error signal $f(n)$ and the power of the error microphone signal $e(n)$,

$$\rho(n) = \frac{P_f(n)}{P_e(n)}, \qquad (3)$$

with

$$\begin{aligned} P_f(n) &= \lambda P_f(n-1) + (1-\lambda) f^2(n), &(4)\\ P_e(n) &= \lambda P_e(n-1) + (1-\lambda) e^2(n), &(5)\end{aligned}$$

and λ a forgetting factor close to 1. The variable step-size parameter $\mu_s(n)$ is computed as follows:

$$\mu_s(n) = \rho(n)\mu_{s_{\min}} + (1-\rho(n))\mu_{s_{\max}}. \qquad (6)$$

The choice of this variable step-size parameter was heuristically motivated in [3] with the fact that in the early phases of adaptation of the ANC system (when $y'(n)$ is close to zero) the convergence of the secondary path model is degraded by the large disturbance at the error microphone. Thus, a small step-size $\mu_s(n)$ should be chosen. With the convergence of the active noise control filter the disturbance reduces and a larger step-size can be used in the secondary path model adaptation. It is shown in [3] that $\rho(n) \simeq 1$ in the early phases of the ANC system adaptation, while $\rho(n) \simeq 0$ when the ANC system is converged.

In [3] the auxiliary noise power was kept constant ($G(n) = G$, with G constant) and it was chosen by compromising between the contrasting needs of fast convergence of the secondary path (which benefits from a large auxiliary noise power) and of low steady state residual noise (which requires a low auxiliary noise power). A scheduling strategy for the auxiliary noise was later introduced in [8]. This scheduling strategy provides a large auxiliary noise power in the early phases of adaptation of the ANC system, and a much lower power at steady state. In particular, the auxiliary noise power is varied between a maximum value $\sigma^2_{v_{\max}}$ and a minimum value $\sigma^2_{v_{\min}}$ (with $\sigma^2_{v_{\max}}$ and $\sigma^2_{v_{\min}}$ experimentally determined) by varying the gain $G(n)$ with the following rule:

$$G(n) = \sqrt{\rho(n)\sigma^2_{v_{\max}} + (1-\rho(n))\sigma^2_{v_{\min}}}, \qquad (7)$$

where $\rho(n)$ is defined in Equation (3).

Another scheduling strategy was introduced in [7]. In this scheduling strategy the ratio between the powers at the error microphone of the auxiliary noise and of the residual noise is approximately kept constant. In particular, for the ANC structure of Figure 1, the power scheduling of [7] is:

$$G(n) = \begin{cases} c\sqrt{P_f(n)} & \text{if } P_f(n) < P_x(n) \\ c\sqrt{P_x(n)} & \text{if } P_f(n) \geq P_x(n) \end{cases} \qquad (8)$$

with c a properly tuned positive constant, $P_f(n)$ given by (4) and

$$P_x(n) = \lambda P_x(n-1) + (1-\lambda)x^2(n). \qquad (9)$$

3. PROPOSED AUXILIARY NOISE POWER SCHEDULING

As discussed in [7], in order to ensure a secondary path modeling with stable accuracy, it is desirable that the ratio between the power of the residual noise $d(n) - y'(n)$ and the power of auxiliary noise at the error microphone $v'(n)$ is approximately constant no matter how $x(n)$ is varied,

$$\frac{E[(d(n) - y'(n))^2]}{E[(v'(n))^2]} = R = \text{const.} \qquad (10)$$

By exploiting the fast convergence properties of the secondary path modeling filter in the ANC system of Figure 1, we can find an accurate estimate of the auxiliary noise gain $G(n)$ that guarantees the desired value of the ratio R. Indeed, since $v'(n)$ is uncorrelated with $d(n)$ and $y'(n)$, it is

$$E[(e(n))^2] = E[(d(n) - y'(n))^2] + E[(v'(n))^2], \qquad (11)$$

In the hypothesis that $G(n)$, $\nu(n)$, and $s(n)$ are independent, and $G(n)$ is slowly varying, it is

$$E[(v'(n))^2] = G^2(n)\|\mathbf{s}\|^2 E[(\nu(n))^2] = G^2(n)\|\mathbf{s}\|^2, \qquad (12)$$

with $\mathbf{s}$ the vector that collects the samples $s(n)$. By approximating $\|\mathbf{s}\|^2$ with $\|\hat{\mathbf{s}}(n)\|^2$ and taking into account Equations (11) and (12) we find that a good estimate of the gain $G(n)$, which ensures the ratio R between the power of the residual noise and the power of auxiliary noise at the error microphone, is:

$$G(n) = \sqrt{\frac{P_e(n)}{(R+1)P_s(n)}} \qquad (13)$$

where $P_e(n)$ is an estimate of the power of $e(n)$,

$$P_e(n) = \lambda P_e(n-1) + (1-\lambda)e^2(n), \qquad (14)$$

and $P_s(n)$ is an exponentially smoothed estimate of $\|\hat{\mathbf{s}}(n)\|^2$,

$$P_s(n) = \lambda P_s(n-1) + (1-\lambda)\hat{\mathbf{s}}^T(n)\hat{\mathbf{s}}(n). \qquad (15)$$

Table 1. Quantities used for the ANC system description.

Quantity	Description
$P(z)$	transfer function of the primary path;
$p(n)$	impulse response of the primary path;
$S(z)$	transfer function of the secondary path;
$s(n)$	impulse response of the secondary path;
$\hat{\mathbf{s}}(n) = [s_0(n), s_1(n), \ldots, s_{M-1}(n)]^T$	coefficient vector of the secondary path modeling filter, an FIR filter of memory length M;
$\mathbf{w}(n) = [w_0(n), w_1(n), \ldots, w_{N-1}(n)]^T$	coefficient vector of the noise control filter, an FIR filter of memory length N;
$x(n)$	reference signal;
$\mathbf{x}_N(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T$	data vector with the last N samples of $x(n)$;
$y(n) = \mathbf{w}^T(n)\mathbf{x}_N(n)$	output of the *actual* noise control filter;
$\nu(n)$	internally generated zero mean, unit variance, white Gaussian noise;
$G(n)$	amplification factor of the white Gaussian noise;
$v(n) = G(n)\nu(n)$	auxiliary noise injected in the system;
$d(n) = p(n) * x(n)$	primary disturbance signal;
$y'(n) = s(n) * y(n)$	canceling signal;
$v'(n) = s(n) * v(n)$	modeling signal;
$e(n) = d(n) - y'(n) + v'(n)$	error microphone signal;
$\mathbf{v}(n) = [v(n), v(n-1), \ldots, v(n-M+1)]^T$	data vector with the last M samples of $v(n)$;
$\hat{v}'(n) = \hat{\mathbf{s}}^T(n)\mathbf{v}(n)$	output of the adaptive secondary path modeling filter;
$f(n) = e(n) - \hat{v}'(n)$	estimation error of the secondary path;
$\mathbf{y}(n) = [y(n), y(n-1), \ldots, y(n-M+1)]^T$	data vector with the last M samples of $y(n)$;
$\hat{y}'(n) = \hat{\mathbf{s}}^T(n)\mathbf{y}(n)$	internal estimate of the canceling signal;
$\hat{d}(n) = f(n) + y'(n)$	internal estimate of the primary disturbance signal;
$\mathbf{x}_M(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$	data vector with the last M samples of $x(n)$;
$\hat{x}'(n) = \hat{\mathbf{s}}^T(n)\mathbf{x}_M(n)$	reference signal filtered with $\hat{\mathbf{s}}(n)$;
$\hat{\mathbf{x}}'(n) = [\hat{x}'(n), \hat{x}'(n-1), \ldots, \hat{x}'(n-N+1)]^T$	data vector with the last N samples of $\hat{x}'(n)$;
$\hat{d}'(n) = \mathbf{w}^T(n)\hat{\mathbf{x}}'(n)$	output of the *dummy* adaptive noise control filter;
$g(n) = \hat{d}(n) - \hat{d}'(n)$	estimation error of the noise control filter.

It is interesting to note that the expression of $G(n)$ in (13) directly depends on the chosen ratio R and that, in contrast with the other power scheduling techniques proposed in the literature, no parameter tuning is needed to guarantee a specific value of this ratio. In particular, with stationary input signals and by properly tuning the parameter c, the power scheduling in (8) can obtain the same ratio R between the power of $d(n) - y'(n)$ and the power of $v'(n)$ guaranteed by the proposed power scheduling. In these conditions, the two power scheduling strategies of (8) and (13) provide the same convergence and noise control performances of the active noise controller. Nevertheless, when the acoustic paths or the primary noise change, in order to guarantee the chosen power ratio R in (8) the parameter c must be re-tuned. On the contrary, the proposed power scheduling is a self-tuning algorithm that ensures in every condition the same ratio between the power of $d(n) - y'(n)$ and the power of $v'(n)$.

4. EXPERIMENTAL RESULTS

In this section we provide some experimental results that compare the performance of the ANC system of Figure 1 equipped with the proposed auxiliary noise power scheduling with the performance of the same ANC system equipped with the auxiliary noise power scheduling of [8], i.e., of Equation (7).

We consider the same experimental conditions of the second experiment of [3]. The sampling frequency is 2 kHz. The primary path $P(z)$ and the secondary path $S(z)$ are FIR filters with memory length 48 and 16, respectively. They have been obtained by truncating the impulse response of the models provided in the companion disk of [1]. The reference signal $x(n)$ is a multi-tonal signal with frequencies 100 Hz, 200 Hz, 300 Hz and 400 Hz and variance 2.0. The signal is corrupted with a zero-mean white Gaussian noise till a 30 dB SNR. The control filter and the secondary path modeling filter have memory lengths $N = 32$ and $M = 16$, respectively.

In our experimental set-up we assume that the secondary path cannot be modeled off-line by switching off the primary noise source, but it must be modeled online with the noise source $x(n)$ active. Thus, we consider two phases of operation of the ANC system. In the first phase we keep inactive the control filter and we adapt the secondary path modeling filter to obtain a first estimate of the secondary path. In the second phase, we operate the ANC system by adapting both the secondary path modeling filter and the control filter. We have tuned the duration of the first phase to the minimum duration that guarantees a stable operation of the ANC system.

The performance comparison of the ANC systems is done on the basis of different performance measures: the power of the error microphone signal ($E[e^2(n)]$), the power of the auxiliary noise at the error microphone ($E[(v'(n))^2]$), the relative modeling error of the secondary path, defined as $\Delta S(n) = 10\log_{10}\left[\frac{\|\mathbf{s}-\hat{\mathbf{s}}(n)\|^2}{\|\mathbf{s}\|^2}\right]$, and the relative modeling error of the control filter, defined as $\Delta W(n) = 10\log_{10}\left[\frac{\|\mathbf{w}_o-\mathbf{w}(n)\|^2}{\|\mathbf{w}_o\|^2}\right]$, with $\mathbf{w}_o$ the MMS optimal control filter [1], which has been a priori determined.

For the power scheduling of Equation (13) we have set the desired ratio $R = 1$. Thus, in every condition we enforce the power of $v'(n)$, to be equal to the power of $d(n) - y'(n)$. On the contrary, for the power scheduling of Equation (7) we have chosen $\sigma^2_{v_{\max}} = 4$ and $\sigma^2_{v_{\min}} = 0$. With this choice, during the initialization phase the auxiliary noise power is almost equal to that obtained with the power

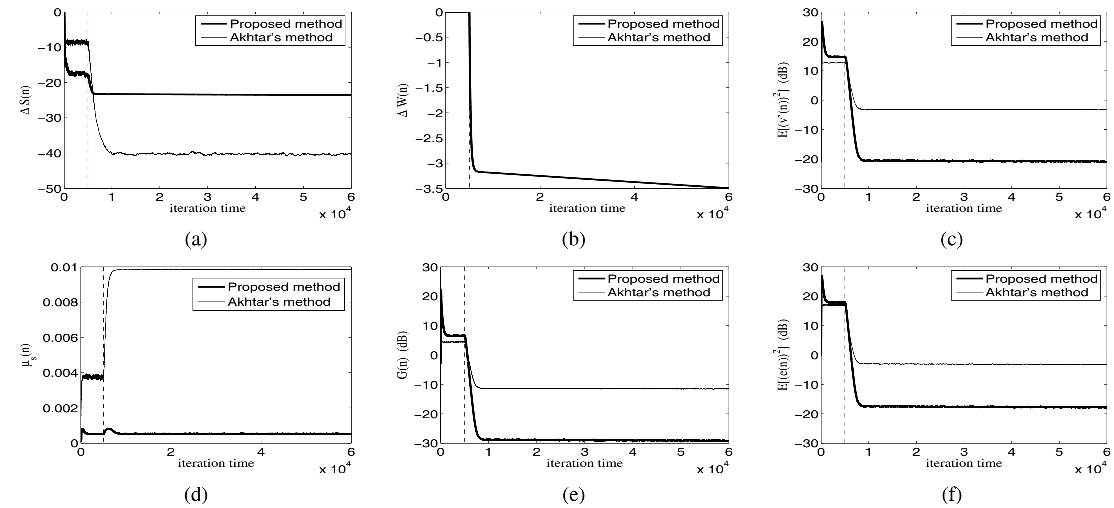

Fig. 2. Performance comparison of the ANC system equipped with the auxiliary noise power scheduling of (13) (proposed method), and the ANC system equipped with the auxiliary noise power scheduling of (7) (Akhtar's method).

scheduling of Equation (13). Moreover, since $\sigma_{v_{\min}} = 0$, the power of the auxiliary noise at steady state is set to the minimum achievable power for that choice of $\sigma^2_{v_{\max}}$. Indeed, in (7) $\rho(n)$ is never zero at steady state and $\sigma^2_{v_{\max}}$ influences the steady-state value of the auxiliary noise power.

As for the other parameters of the adaptation algorithm, we have always set $\lambda = 0.99$ and $\mu_w = 5 \cdot 10^{-5}$. $\mu_{s_{\min}}$ and $\mu_{s_{\max}}$ were set to 10^{-3} and 10^{-2}, respectively, with the power scheduling of Equation (7). On the contrary, in order to compensate for the different input and output noise conditions they were set to to 10^{-4} and 10^{-3}, respectively, with the power scheduling of Equation (13).

Figure 2 provides the performance comparison of the two systems. In this figure, plot (a) diagrams the system distance $\Delta S(n)$, plot (b) the system distance $\Delta W(n)$, plot (c) the evolution of the power of the auxiliary noise at the error microphone $v'(n)$, plot (d) the evolution of the step-size $\mu_s(n)$, plot (e) the evolution of $G(n)$, plot (f) the evolution of the power of the error microphone signal $e(n)$. All plots have been obtained with ensemble averages over 100 runs of the system. From there figures we notice a strong performance improvement obtained with the proposed auxiliary noise power scheduling in the power of error microphone signal. At steady state with the proposed auxiliary noise power scheduling we improve the power of $e(n)$ by more than 15 dB compared to the power scheduling of [8]. The lower accuracy in the estimation of the secondary path modeling filter with the proposed power scheduling is caused by the lower power of the auxiliary noise compared with that of [8]. Nevertheless, the estimation accuracy of $s(n)$ is still very good and it does not affect the estimation of the optimal controller which is the same for the two algorithms as can be seen from plot (b). The initial peaks on the power of $v'(n)$ observed at the beginning of the simulation in plots (c) with the proposed power scheduling are due to the low estimation accuracy of the secondary path in the early phases of adaptation. They can be easily removed by limiting the maximum power of the auxiliary noise, as for example done in [7].

5. CONCLUSION

In this paper we have introduced a novel self-tuning power scheduling for the auxiliary noise of feedforward ANC systems with secondary path modeling. The power scheduling controls the auxiliary noise gain such that the ratio at the error microphone between the power of the auxiliary noise and the power of the residual noise keeps a desired value. In this way, the power scheduling provides a large auxiliary noise power in the early phases of adaptation, which facilitates the convergence of the algorithm. On the contrary, a lower auxiliary noise power is injected at steady state and, thus, a low noise power is obtained at the error microphone after the convergence of the ANC system. The proposed power scheduling requires only the choice of the desired ratio between the powers at the error microphone of the auxiliary noise and of the residual noise. On the contrary, all other solutions proposed in the literature [7], [8], require the accurate tuning of one or two parameters.

6. REFERENCES

[1] S. M. Kuo and D. R. Morgan, *Active noise control Systems – Algorithms and DSP implementation*. New York: Wiley, 1996.

[2] M. Zhang, H. Lan, and W. Ser, "Cross-updated active noise control system with online secondary path modeling," *IEEE Trans. Speech and Audio Processing*, vol. 9, no. 5, pp. 598–602, July 2001.

[3] M. T. Akhtar, M. Abe, and M. Kawamata, "A new variable step size LMS algorithm-based method for improved online secondary path modeling in active noise control systems," *IEEE Trans. Audio, Speech and Language Processing*, vol. 12, no. 2, pp. 720–726, Mar. 2006.

[4] C. Bao, P. Sas, and H. V. Brussel, "Comparison of two online identification algorithms for active noise control," in *Proc. Recent Advances in Active Control of Sound Vibration*, 1993, pp. 38–51.

[5] E. Bjarnason, "Active noise cancellation using a modified form of the filtered-x LMS algorithms," in *Proc. 6th European Signal Processing Conference*, Brussels, Belgium, Aug. 1992, pp. 1053–1056.

[6] D. Cornelissen and P. C. Sommen, "New online secondary path estimation in a multipoint filtered-x algorithm for acoustic noise canceling," in *Proc. IEEE ProRIS 99*, 1999, pp. 97–103.

[7] M. Zhang, H. Lan, and W. Ser, "A robust online secondary path modeling method with auxiliary noise power scheduling strategy and norm constraint manipulation," *IEEE Trans. Speech and Audio Processing*, vol. 11, no. 1, pp. 45–53, Jan. 2003.

[8] M. T. Akhtar, M. Abe, and M. Kawamata, "Noise power scheduling in active noise control systems with online secondary path modeling," *IEICE Electronics Express*, vol. 4, no. 2, pp. 66–71, 2007.