DISTRIBUTED TOTALLY ASYNCHRONOUS ITERATIVE WATERFILLING FOR WIDEBAND INTERFERENCE CHANNEL WITH TIME/FREQUENCY OFFSET

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ABSTRACT

This paper considers the competitive maximization of information rates in the Gaussian frequency-selective interference channel, subject to global power and spectral mask constraints. We focus on the practical case in which the transmission by the different users contains time and frequency synchronization offsets. We propose a unified framework based on a distributed algorithm called asynchronous iterative waterfilling algorithm. In this algorithm, the users update their power spectral density in a completely distributed and asynchronous way: some users may update their power allocation more frequently than others and they may even use outdated measurements of the received interference. Moreover the users are not required to know time and frequency offsets. Our main contribution is to provide a unified set of convergence conditions for the whole class of algorithms obtained from the asynchronous iterative waterfilling algorithm.

Index Terms— Gaussian frequency-selective interference channel, game theory, Nash equilibrium, totally asynchronous algorithm, iterative waterfilling algorithm.

1. INTRODUCTION

The *sequential* Iterative Water-Filling Algorithm (IWFA) proposed by Yu et al. [1] is by now a popular low complexity distributed algorithm to compute the Nash equilibrium point of the power allocation game in a Gaussian frequency-selective interference channel. The algorithm is based on a distributed sequential updating where, at each iteration, the users choose, one after the other, their power allocation to maximize their own information rate, treating the interference generated from the other users as additive colored noise.

However, despite its appealing properties (i.e., its low complexity and its distributed nature), the sequential IWFA may suffer from slow convergence if the number of users in the network is large because of the sequential updating strategy. In addition, the algorithm requires some kind of central scheduling to determine the order in which users are updated.

To overcome the drawback of slow speed of convergence, the *simultaneous* IWFA was proposed in [3], where at each iteration, all the users update their power allocations simultaneously, rather than sequentially. This reduces the convergence time considerably, specially when the number of users is large. Furthermore, differently from [1, 2], the algorithm takes explicitly into account spectral masks constraints. However, the simultaneous IWFA still requires some form of synchronism as all the users need to be simultaneously updated.

In a real network with many users, the kind of synchronism requirements of the sequential and simultaneous IWFAs may not be feasible. To overcome this limitation, in [4], a unified framework based on the so-called *asynchronous* IWFA, that falls within the class of totally asynchronous schemes of [5] was developed. In this more general algorithm, all users still update their power allocations according to the waterfilling solution, but the updates can be performed in a *totally asynchronous* way (in the sense of [5]). This feature makes the asynchronous IWFA appealing for all practical scenarios where strong constraints on synchronization cannot be met.

The asynchronous aspect of the algorithm proposed in [4] concerned the updating schedule. However, in a real implementation, lack of synchronization among uncoordinated users arises also as a consequence of mismatch between the oscillators of different transmitters, propagation delays, Doppler effects, etc. Whenever this happens, a frequency-coupling among the users arises. In this case, the game-theoretic approach proposed in [1, 2, 3, 4] is not adequate anymore, since it ignores the presence of frequency-coupling in the expression of the rates of the users, and thus the resulting NEs can lead to poor performance.

In this paper, we generalize results of [1, 2, 3, 4] and formulate the competitive maximization of information rates taking explicitly into account the effect of time and/or frequency offsets in the system. We fully characterize the game in terms of existence and uniqueness of NE and make the asynchronous IWFA suitable for implementation in the presence of frequency coupling among the users. Interestingly, the presence of time/frequency misalignments does no affect the main features of asynchronous IWFA [4]. In fact, all users are still allowed to update their own strategies whenever they want, and they may even use an *outdated* measurement of the interference caused from the others. Moreover, they do not need to know the time/frequency offsets to choose their optimal transmission strategy.

Our main contribution is to provide a unified set of convergence conditions that are valid for all the algorithms that can be obtained from the asynchronous IWFA as special cases.

2. SYSTEM MODEL

We consider a Gaussian frequency-selective interference channel composed by multiple links. Aiming at finding distributed algorithms, we focus on transmission techniques where no interference cancellation is performed and multiuser interference (MUI) is treated as additive colored noise from each receiver. To deal easily with the frequency-selectivity of the channel, we adopt a multicarrier approach without loss of optimality (since it is a capacity-lossless structure for sufficiently large block length).

In large scale distributed systems, where no cooperation among different users is allowed, the assumption of perfect synchronization in time and/or frequency among the transmissions of all the links, as made in [1, 2, 3, 4], may not be satisfied, because of large propagation delays, timing errors, and/or transmit-receive oscillators' mismatch. Whenever this happens, multiuser Inter-Carrier Interference (ICI) arises, since the signal transmitted by each source over one carrier interferes with the other links not only at the same carrier, but also at neighboring frequencies.

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As every link results in a different (unknown) time/frequency shift from the others, the loss of the orthogonality among the carriers cannot be recovered by trying to compensate time/frequency offsets with a proper tuning of each local oscillator, as in single-user systems (e.g., [6]). The correction made with respect to one user would in fact misalign other already aligned users. As an alternative, one could perform some ICI cancellation technique (e.g., [7] and references therein), but this would require exchange of signaling among the links, with a consequent traffic overhead. As our interest is in totally distributed algorithms, we do not consider multiuser ICI cancellation, and ICI is treated as additive noise at the receivers, which leads to carrier-coupling in the (information) rate of each link.

Under this system model, we consider the competitive maximization of the information rate on each link, subject to global power and spectral mask constraints. To this end, we first derive a unified expression for the ICI, in the cases of both time and frequency offsets.

2.1. Time offsets

The propagation delays and timing errors induce at each receiver a misalignment among the intended (OFDM) block and those transmitted from the interfering links [6, 8]. If the propagation delays are larger than guard time, ICI occurs and orthogonality among the carriers is lost. More specifically, given the multiuser ICI as additive noise, the SINR $\mathsf{sinr}_q(k)$ on the k-th carrier for the q-th link can be approximated for large N as [6, 8][1]:

$$\mathsf{sinr}_q(k) = \frac{|H_{qq}(k)|^2 p_q(k)}{\sigma^2_{w_q}(k) + \sum_{r\neq q}\sum_{k'}\eta_{rq}(k-k')\,|H_{rq}(k')|^2 p_r(k')}, \tag{1}$$

where $H_{rq}(k) \triangleq \sqrt{P_r/d_{rq}^{\gamma}}\,\bar{H}_{rq}(k)$, with $\bar{H}_{rq}(k)$ denoting the normalized frequency-response of the channel between source r and destination q; d_{rq} denotes the distance between source r and destination q, and γ is the path loss exponent; P_q is the transmit power of user q and $p_q(k)$ is the normalized (by P_q) power allocated by the q-th user over the k-th subcarrier, subject to the spectral mask constraints $p_q(k) \leq p_q^{\max}(k), \forall k$,[2] and the power constraint $(1/N)\sum_{k=0}^{N-1} p_q(k) \leq 1$; $\eta_{rq}(k)$ is the ICI function defined as

$$\eta_{rq}(k) \triangleq \begin{cases} \dfrac{2}{N^2}\dfrac{\sin^2\left(\frac{\pi}{N}k\nu_{rq}\right)}{\sin^2\left(\frac{\pi}{N}k\right)}, & \text{if } k \neq 0 \\ \dfrac{\nu_{rq}^2 + (N-\nu_{rq})^2}{N^2}, & \text{otherwise,} \end{cases} \tag{2}$$

and ν_{rq} denotes the (unknown) time offset at receiver q between the block transmitted from user q and the block transmitted from user r.

2.2. Frequency offsets

Frequency offsets are introduced by differences in oscillator's frequencies in the transmitters and receivers of different links or by Doppler shift. The frequency offset leads to a loss of orthogonality among the carriers, as we show next.

Assuming perfect time temporization (within the cyclic prefix) among the blocks, and denoting by Δf_{rq} the carrier frequency offset between transmitter r and receiver q, the sequence received from the q-th link, after discarding the cyclic prefix and performing the FFT, is (dropping the dependence on time index) [9]

$$\mathbf{y}_q = \mathbf{H}_{qq}\mathbf{s}_q + \sum_{r\neq q}\mathbf{W}^H\mathbf{D}\left(\Delta f_{rq}\right)\mathbf{W}\mathbf{H}_{rq}\mathbf{s}_r + \mathbf{w}_q, \tag{3}$$

where $\mathbf{H}_{rq}$ is the $N\times N$ diagonal matrix, defined as $[\mathbf{H}_{rq}]_{kk} \triangleq \bar{H}_{rq}(k)/\sqrt{d_{rq}^{\gamma}}$; $\mathbf{s}_q$ is the vector of N symbols transmitted from user q; $\mathbf{W}$ is the $N\times N$ IFFT matrix with $[\mathbf{W}]_{nm} = \exp\left(j2\pi nm/N\right)/\sqrt{N}$; and $\mathbf{D}\left(\Delta f_{rq}\right)$ is a $N\times N$ diagonal matrix, with diagonal entries $[\mathbf{D}\left(\Delta f_{rq}\right)]_{kk} \triangleq \exp\left(j2\pi k\Delta f_{rq}\right)$, with $k = 0,\ldots,N-1$.

Exploring the structure of (3), the SINR on the k-th carrier for the q-th link can be written as in (1), where the ICI function $\eta_{rq}(k)$ is now defined as

$$\eta_{rq}(k) \triangleq \frac{1}{N^2}\frac{\sin^2\left(\pi\left(k - N\Delta f_{rq}\right)\right)}{\sin^2\left(\frac{\pi}{N}\left(k - N\Delta f_{rq}\right)\right)}. \tag{4}$$

3. PROBLEM FORMULATION AS A GAME

Taking ICI into account, we provide now a unified game theoretic formulation for the *joint* maximization of information rates of the links, in both cases of time and frequency offsets. Specifically, we consider a strategic non-cooperative game, in which the players are the links and the payoff functions are the information rates on the links (given MUI as additive noise): Each player competes rationally against the others by choosing the signaling (i.e. its strategy) that maximizes its own rate, given constraints on the transmit power and spectral masks. A Nash Equilibrium (NE) of the game is reached when each user, given the strategy profile of the others, does not get any rate increase by changing its own strategy.

It is straightforward to see that all the NEs of the game are obtained if each user transmits using Gaussian signaling, with a proper Power Spectral Density (PSD) [3, 9]. Hence, the maximum achievable rate for the q-th user is given by

$$R_q = \frac{1}{N}\sum_{k=0}^{N-1}\log\left(1 + \mathsf{sinr}_q(k)\right), \tag{5}$$

where $\mathsf{sinr}_q(k)$ is given in (1) and the ICI function $\eta_{rq}(k)$ is defined in (2) for time offsets, and in (4) for frequency offsets[3].

In summary, the structure of the game is the following:

$$\mathscr{G} = \left\{\Omega, \{\mathscr{P}_q\}_{q\in\Omega}, \{R_q\}_{q\in\Omega}\right\}, \tag{6}$$

where $\Omega \triangleq \{1,2,\ldots,Q\}$ denotes the set of the active links, $\mathscr{P}_q$ is the set of admissible power allocation (pure) strategies, across the N available sub-carriers, for the q-th player, defined as

$$\mathscr{P}_q \triangleq \left\{\mathbf{p}_q \in \mathbb{R}^N : \frac{1}{N}\sum_{k=0}^{N-1} p_q(k) = 1,\ 0 \leq p_q(k) \leq p_q^{\max}(k),\ \forall k\right\}, \tag{7}$$

and R_q is the payoff function of the q-th player, defined in (5).

All the NEs of the game $\mathscr{G}$ in (6) are reached using pure strategies, given by the following simultaneous waterfilling power allocation [9]

$$\mathbf{p}_q^{\star} = \mathsf{WF}_q\left(\mathbf{p}_1^{\star},\ldots,\mathbf{p}_{q-1}^{\star},\mathbf{p}_{q+1}^{\star},\ldots,\mathbf{p}_Q^{\star}\right), \quad \forall q\in\Omega, \tag{8}$$

[1] We have considered in (1) only the ICI due to propagation delays. In the presence of both timing errors and propagation delays, an additional term of ICI has to be considered in writing the SINR [6]. Since this term is similar to the one already written in (1), for the sake of notation we will not consider it in the following.

[2] In order to avoid the trivial solution $p_q(k) = p_q^{\max}(k), \forall k, q$, we assume, for each q, $\sum_k p_q^{\max}(k) > N$.

[3] In the presence of both time and frequency offsets, an additional term of ICI has to be considered in writing the SINR [6]. Since this term is similar to the one already written in (1), for the sake of notation we will not consider it in the following.

with $\mathsf{WF}_q(\cdot)$ defined as

$$[\mathsf{WF}_q(\mathbf{p}_{-q})]_k \triangleq [\mu_q - \mathsf{insr}_{q,k}(\mathbf{p}_{-q})]_0^{p_q^{\max}(k)}, \quad k=0,\ldots,N-1, \tag{9}$$

where $[x]_a^b$ denotes the Euclidean projection of x onto the interval $[a,b]$ and

$$\mathsf{insr}_{q,k}(\mathbf{p}_{-q}) \triangleq \frac{\sigma_{w_q}^2(k) + \sum_{r\neq q}\sum_{k'}\eta_{rq}(k-k')\left|H_{rq}(k')\right|^2 p_r(k')}{\left|H_{qq}(k)\right|^2}. \tag{10}$$

The water-level μ_q in (9) is chosen to satisfy the power constraint $(1/N)\sum_{k=0}^{N-1} p_q^\star(k) = 1$.

The existence of at least one NE for the game $\mathscr{G}$ in (6) is guaranteed by the following proposition that comes directly from standard results of game theory.

Proposition 1 ([9]) *The game $\mathscr{G}$ in (6) always admits at least one NE in pure-strategies, for any set of time/frequency offsets, channel realizations, power and spectral mask constraints.*

Once proved that a NE always exists, the problem of how to reach such an equilibrium arises. We address this issue in the next section. By direct product of our derivations, we also provide sufficient conditions for the uniqueness of the equilibrium.

4. ASYNCHRONOUS ITERATIVE WATERFILLING IN THE PRESENCE OF TIME/FREQUENCY OFFSETS

To compute the NE points of the game $\mathscr{G}$ in (6), we propose a distributed iterative waterfilling procedure, called asynchronous Iterative WaterFilling Algorithm, which is an instance of the totally asynchronous scheme of [5]. We show in the following that, whatever the asynchronous mechanism is, such a procedure converges to a stable NE of the game $\mathscr{G}$ in (6), under mild conditions on the multiuser interference.

In order to provide a formal description of the asynchronous IWFA, we need some preliminary definitions, as we introduce next. We assume, without loss of generality, that the set of times at which one or more users update their strategies is the discrete set $T = \mathbb{N}_+ = \{0,1,2,\ldots\}$. Let $\mathbf{p}_q^{(n)}$ denote the power allocation of user q at the n-th iteration, and let $T_q \subseteq T$ denote the set of times n at which $\mathbf{p}_q^{(n)}$ is updated (thus, at time $n \notin T_q$, $\mathbf{p}_q^{(n)}$ is left unchanged). Let $\tau_r^q(n)$ denote the most recent time at which the interference from user r is perceived by user q at the n-th iteration (observe that $\tau_r^q(n)$ satisfies $0 \leq \tau_r^q(n) \leq n$). Hence, if user q updates its power allocation at the n-th iteration, then it waterfills, according to (9), the interference level caused by

$$\mathbf{p}_{-q}^{(\tau^q(n))} \triangleq \left(\mathbf{p}_1^{(\tau_1^q(n))},\ldots,\mathbf{p}_{q-1}^{(\tau_{q-1}^q(n))},\mathbf{p}_{q+1}^{(\tau_{q+1}^q(n))},\ldots,\mathbf{p}_Q^{(\tau_Q^q(n))}\right). \tag{11}$$

The overall system is said to be totally asynchronous if the following weak assumptions are satisfied for each q [5]: $A1)$ $0 \leq \tau_r^q(n) \leq n$; $A2)$ $\lim_{k\to\infty}\tau_r^q(n_k) = +\infty$; and $A3)$ $|T_q| = \infty$; where $\{n_k\}$ is a sequence of elements in T_q that tends to infinity. Assumption A1)-A3) are standard in asynchronous convergence theory [5], and they are fulfilled in any practical implementation.

Using the above notation, the asynchronous IWFA is described in Algorithm 1.

Algorithm 1: Asynchronous Iterative Waterfilling Algorithm

Set $\mathbf{p}_q^{(0)}$ = any feasible power allocation;
`for` $n = 0$: Number_of_iterations,

$\forall q \in \Omega$:

$$\mathbf{p}_q^{(n+1)} = \begin{cases} \alpha\mathbf{p}_q^{(n)} + (1-\alpha)\mathsf{WF}_q\left(\mathbf{p}_{-q}^{(\tau^q(n))}\right), & \text{if } n \in T_q, \\ \mathbf{p}_q^{(n)}, & \text{otherwise;} \end{cases} \tag{12}$$

`end`

The factor $\alpha \in [0,1)$ in Algorithm 1 can be interpreted as a forgetting factor: the larger α is, the longer the memory of the algorithm is. The choice of α depends on the channel stationarity and on possible channel fluctuations or estimation errors.

Remark 1. Since the asynchronous IWFA is based on the waterfilling solution (9), it can be implemented in a distributed way, where each user, to maximize its own rate, only needs to locally measure the PSD of the interference-plus-noise (see (1)) and waterfill over this level. More importantly, it does not require knowledge of the unknown time/frequency offsets by each link. Moreover, according to the asynchronous scheme, the users may update their strategies using a potentially outdated version of the PSD of the interference and, furthermore, some users are allowed to update their power allocation more often than others, without affecting the convergence of the algorithm. These features strongly relax the constraints required on the synchronization of the updates of the users needed in the sequential IWFA [1, 2] and simultaneous IWFAs [3].

The convergence of the algorithm is guaranteed under the following conditions.

Theorem 1 ([9]) *The asynchronous IWFA converges to the unique NE of the game $\mathscr{G}$ in (6), if the following condition is satisfied*

$$\rho(\mathbf{H}) < 1, \tag{C1}$$

where $\rho(\mathbf{H})$ denotes the spectral radius of the matrix $\mathbf{H}$, defined as

$$[\mathbf{H}]_{qr} = \begin{cases} \|\boldsymbol{\Upsilon}_{rq}\mathbf{H}_{rq}\|_2, & \text{if } r \neq q, \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

with

$$[\boldsymbol{\Upsilon}_{rq}]_{kk'} = \begin{cases} \eta_{rq}(k-k'), & \text{if } k \in \mathcal{D}_q \text{ and } k' \in \mathcal{D}_r, \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

and $\mathbf{H}_{rq}$ is a diagonal matrix, whose diagonal entries are:

$$[\mathbf{H}_{rq}]_{kk} = \begin{cases} \dfrac{|\bar{H}_{rq}(k)|^2}{|\bar{H}_{qq}(k)|^2}\dfrac{d_{qq}^\gamma}{d_{rq}^\gamma}\dfrac{P_r}{P_q}, & \text{if } k \in \mathcal{D}_r, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

The set $\mathcal{D}_q$ is defined as $\mathcal{D}_q \triangleq \{k \in \{0,\ldots,N-1\} : \exists\, \mathbf{p}_{-q} \in \mathscr{P}_{-q}$ such that $[\mathsf{WF}_q(\mathbf{p}_{-q})]_k \neq 0\}$, with $\mathsf{WF}_q(\cdot)$ given in (9).

Corollary 1 *A sufficient condition for (C1) is given by one of the following*

$$\frac{1}{w_q}\sum_{r=1,r\neq q}\|\boldsymbol{\Upsilon}_{rq}\|_2 \max_{k\in\mathcal{D}_r}\left\{\frac{|\bar{H}_{rq}(k)|^2}{|\bar{H}_{qq}(k)|^2}\right\}\frac{d_{qq}^\gamma}{d_{rq}^\gamma}\frac{P_r}{P_q}w_r < 1,\ \forall q \in \Omega, \tag{C2}$$

$$\frac{1}{w_r}\sum_{q=1,q\neq r}\|\boldsymbol{\Upsilon}_{rq}\|_2 \max_{k\in\mathcal{D}_r}\left\{\frac{|\bar{H}_{rq}(k)|^2}{|\bar{H}_{qq}(k)|^2}\right\}\frac{d_{qq}^\gamma}{d_{rq}^\gamma}\frac{P_r}{P_q}w_q < 1,\ \forall r \in \Omega, \tag{C3}$$

where $\boldsymbol{\Upsilon}_{rq}$ is defined in (14) and $\mathbf{w} \triangleq [w_1,\ldots,w_Q]^T$ is any positive vector.

Remark 2. The set $\mathcal{D}_q$ defined in Theorem 1 represents the set $\{0,\dots,N-1\}$ (possibly) deprived of the carrier indices that user q would never use as the best response set to any strategies used by the other users, for the given set of transmit power and propagation channels. Observe that one can always choose $\mathcal{D}_q = \{0,\dots,N-1\}$. However, less stringent conditions are obtained by removing unnecessary subcarriers, which are never used. A simple algorithm to estimate the set $\mathcal{D}_q$ was given in [9].

Remark 3. In the presence of ICI, the convergence of the asynchronous IWFA and the uniqueness of NE are affected by both the MUI and the coupling due to the ICI. As expected, the convergence is ensured if the links are sufficiently far apart from each other. In fact, from (C1) (or (C2)-(C3)) we infer that there exists a minimum distance beyond which the convergence of the asynchronous IWFA (and the uniqueness of NE) is guaranteed, corresponding to the maximum level of interference and ICI that may be tolerated by each receiver. But, the most interesting result coming from (C1) (or (C2)-(C3)) is that the convergence of the asynchronous IWFA is robust against the worst normalized channels $|H_{rq}(k)|^2/|H_{qq}(k)|^2$; in fact, the subchannels corresponding to the highest ratios $|H_{rq}(k)|^2 /|H_{qq}(k)|^2$ (and, in particular, those where $|H_{qq}(k)|^2$ is vanishing) do not necessarily affect the convergence of the asynchronous IWFA, as their carrier indices may not belong to the set $\mathcal{D}_q$.

Observe that conditions (C1) and (C2)-(C3) generalize those obtained in [3, 4] in the absence of ICI. In fact, in the case of no ICI (i.e., $\eta_{rq}(k) = \delta(k)$, $\forall r \neq q$), the asynchronous IWFA coincides with the algorithm proposed in [4], whose convergence is guaranteed under conditions (C1) (or (C2)-(C3)), where $\|\mathbf{\Upsilon}_{rq}\mathbf{H}_{rq}\|_2$ is replaced by $\max_{k\in\mathcal{D}_r\cap\mathcal{D}_q}\{|\bar{H}_{rq}(k)|^2/\,|\bar{H}_{qq}(k)|^2\}(d_{qq}^{\gamma}\,P_r)/\left(d_{rq}^{\gamma}P_q\right)$.

Remark 4. The asynchronous IWFA given in Algorithm 1 represents a general framework to solve the rate maximization game $\mathscr{G}$ (6), in the presence of time/frequency offsets. In fact, it contains as special cases a plethora of algorithms, each one obtained by a possible choice of the scheduling of the users in the updating procedure (i.e. the parameters $\{\tau_r^q(n)\}$ and $\{T_q\}$). The important result proved in Theorem 1 is that, since the convergence conditions of the asynchronous IWFA do not depend on the particular scheduling used in the updates of the PSDs, all the algorithms, either synchronous or asynchronous, resulting as special cases of the asynchronous IWFA are guaranteed to reach the unique NE of the game, under the *same* set of convergence conditions. Clearly, the sequential [1, 2] and simultaneous [3] updates previously considered in the literature (in the absence of ICI) are particular cases of our asynchronous IWFA. Our new result also shows that the convergence for these two algorithms is robust to situations where some users may fail to follow the sequential or simultaneous scheduling of updates.

5. NUMERICAL RESULTS

As an example, we plot in Figure 1 the optimal power spectral density at the NE for a system with two users in the case of high multiuser interference (due to small ratios d_{rq}/d_{qq}) and ICI due to frequency offsets. We compare the optimal power allocation obtained in the presence of ICI (subplot b)) with the one obtained in the case of perfect time/frequency synchronization (subplot a)). As expected, in the case of high interference, the power allocation corresponds to transmission over non-overlapping bands, emulating OFDMA strategy. In fact, if the interference is the dominant factor of degradation in the transmission, the only possibility to maximize the rate of all the links simultaneously (i.e. according to the NE optimality) is to totally remove it. Interestingly, in the presence of frequency offsets the simultaneous waterfilling solution still provides an orthogonal transmission, but to limit the degradation induced by frequency-coupling a guard interval among the bands occupied from different users is naturally introduced.

In conclusion, in this paper we have proposed a unified framework based on the totally asynchronous IWFA that solves the rate maximization game in frequency-selective Gaussian interference channels, in the presence of time/frequency misalignments. Our main contributions have been to fully characterize the game in terms of existence and uniqueness of NE and to provide the conditions ensuring the convergence of the algorithm to the unique NE.

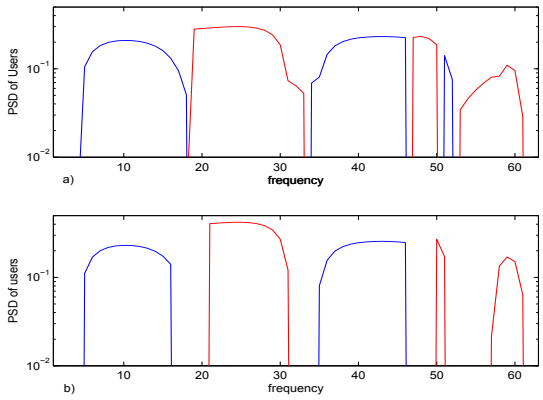

Fig. 1. Optimal PSD at NE in the absence of ICI (subplot a)) and in the presence of ICI (subplot b)) due to frequency offset $\Delta f_{rq} = 0.5/N$; $\gamma = 2.5$, $d_{12} = d_{21}$, $d_{11} = d_{22} = 1$, $P_1 = P_2$, $P_1/\sigma_w^2 = 3$ dB, $P_1/(\sigma_w^2 d_{12}^{\gamma}) = 1$ dB.

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