# A TIME-VARYING PHASE COHERENCE MEASURE FOR QUANTIFYING FUNCTIONAL INTEGRATION IN THE BRAIN

*Selin Aviyente, Westley S. Evans*

Department of Electrical and Computer Engineering
Michigan State University
East Lansing, MI 48824

*Edward M. Bernat, Scott Sponheim*

Department of Psychology
University of Minnesota
Minneapolis, MN 55455

## ABSTRACT

The functional integration between the different parts of the brain is usually quantified through a measure of coherence. Most of the existing measures define coherence based on the spectral energy distribution of the signals rather than the phase, and therefore cannot be reliably used as measures of neural synchrony. Moreover, the most common methods for quantifying coherence are formulated in the frequency domain and thus, do not take into account the time-varying nature of brain activity. Recently, coherence measures have been extended to account for the energy and the phase relationships between the given signals and the time-varying nature of the signals using the wavelet transform. In this paper, we extend this idea by introducing a new time-varying phase coherence measure based on Cohen's class of time-frequency distributions. This new measure is applied to both synthesized signals and electroencephalogram (EEG) data to show the effectiveness of the proposed measure in estimating phase changes and in quantifying the neural synchrony in the brain.

***Index Terms***— Time-frequency analysis, Electroencephalography, Phase synchronization

## 1. INTRODUCTION

Cognitive acts require the integration of numerous functional areas widely distributed over the brain and in constant interaction with each other. This dynamic interaction is often characterized by the phase relationships between the activities of two neuronal populations, termed as phase synchronization. The most common measures used to quantify phase synchrony is time domain cross-correlation and spectral coherence. These measures have limitations for two reasons. First, they assume stationarity of the underlying signals whereas most real life signals, including EEGs, are not. Second, coherence is a measure of spectral covariance and does not separate the effects of amplitude and phase from each other. Therefore, there is a need for time-frequency based coherence measures [1] that separate the phase component of coherence from the amplitude component.

In order to address these limitations, two different approaches for quantifying phase synchrony have been proposed. The first approach employs the Hilbert transform of the signal to get an analytic form of the signal and estimates instantaneous phase directly from its analytic form [2]. In order to be able to estimate the instantaneous phase of a signal from its analytic form one has to make sure that it is a narrowband signal. Since most real life EEG signals are not narrowband, this is not a very realistic assumption. For this reason, the Hilbert transform based phase synchrony measure first bandpass filters the signal around a frequency of interest and then uses Hilbert transform to get the instantaneous phase. This is an indirect way of obtaining the frequency dependent phase estimates and is not exact. The second approach, on the other hand, computes a time-varying complex energy spectrum using either the continuous wavelet transform (CWT) with a complex Morlet wavelet [3] or the short-time Fourier transform (STFT) [4]. It has been observed that the two approaches are similar in their results with the time-varying spectrum based methods giving sharper phase synchrony estimates over time and frequency, especially at the low frequency range [2]. Although the wavelet and STFT based phase coherence estimates address the issue of non-stationarity, they suffer from limited time-frequency resolution due to the limited number of available scales in the case of wavelets and the tradeoff between the window length and resolution in the case of STFT. Moreover, these estimates are biased since the phase coherence is estimated from a windowed signal and not the signal itself. For these reasons, there is a need for high time-frequency resolution phase distributions for quantifying phase coherence.

In this paper, we introduce a new measure of time-varying phase estimation and phase coherence based on Cohen's class of distributions. Section 2 gives the background on Rihaczek distribution, a complex energy distribution belonging to Cohen's class, and defines measures of time-varying phase spectrum and phase coherence. Section 3 presents the results for both synthesized signals and EEG data collected during two different experiments. Finally, Section 4 gives the conclusions and suggests future work.

## 2. RIHACZEK DISTRIBUTION

Cohen's class of distributions are bilinear time-frequency distributions (TFDs) that are expressed as [1] [5]:

$$C(t,f) = \int\int\int \phi(\theta,\tau) s(u+\frac{\tau}{2}) s^*(u-\frac{\tau}{2}) e^{j(\theta u - \theta t - 2\pi\tau f)} du\, d\theta\, d\tau \quad (1)$$

where the function $\phi(\theta,\tau)$ is the kernel function and $s$ is the signal. The kernel completely determines the properties of its corresponding TFD. Most of the members of Cohen's class are real valued energy distributions such as the spectrogram and the Wigner distribution. These distributions are successful at describing the energy of the signal over time and frequency, simultaneously. However, they do not carry any information about the phase of the signal. For this reason, they cannot be directly used for describing the phase information in an individual signal and estimating the phase coherence between two signals.

Rihaczek introduced the complex energy distribution and gave a plausibility argument based on physical grounds [6]. For a signal, $x(t)$, Rihaczek distribution is expressed as

$$C(t,\omega) = x(t) X^*(\omega) e^{-j\omega t} \quad (2)$$

and measures the complex energy of a signal around time $t$ and frequency $\omega$. The complex energy density function provides a fuller appreciation of the properties of phase-modulated signals that is not available with other time-frequency distributions. Rihaczek distribution is a bilinear, time and frequency shift covariant, complex-valued time-frequency distribution belonging to Cohen's class. This distribution satisfies the marginals and preserves energy. Rihaczek distribution provides both a time-varying energy spectrum as well as a phase spectrum, and thus is useful for estimating the phase coherence between any two signals.

### 2.1. Reduced Interference Rihaczek Distribution (RID-Rihaczek)

One of the disadvantages of Rihaczek distribution is the existence of cross-terms for multicomponent signals. For any signal, $x(t) = x_1(t) + x_2(t)$, the Rihaczek distribution is:

$$\begin{aligned} C(t,\omega) &= x_1(t)X_1^*(\omega)e^{-j\omega t} + x_2(t)X_2^*(\omega)e^{-j\omega t} \\ &+ x_1(t)X_2^*(\omega)e^{-j\omega t} + x_2(t)X_1^*(\omega)e^{-j\omega t} \end{aligned} \quad (3)$$

where the last two terms in the above expression are the cross-terms. Unlike the cross-terms in Wigner distribution, these cross-terms are located at the same time locations and occupy the same frequency bands as the original signals.

In order to get rid of these cross-terms, we propose to apply a kernel function such as the Choi-Williams (CW) kernel with $\phi(\theta,\tau) = \exp(-\frac{(\theta\tau)^2}{\sigma})$ to filter the cross-terms in the ambiguity domain. Using the ambiguity domain definition of Cohen's class of time-frequency distributions, $C(t,\omega) = \int\int \phi(\theta,\tau) A(\theta,\tau) e^{-j(\theta t + \tau\omega)} d\tau d\theta$, we define the reduced interference Rihaczek distribution as:

$$C(t,\omega) = \int\int \underbrace{\exp\left(-\frac{(\theta\tau)^2}{\sigma}\right)}_{\text{CW kernel}} \underbrace{\exp(j\theta\tau)}_{\text{Rihaczek kernel}} A(\theta,\tau) e^{-j(\theta t + \tau\omega)} d\tau d\theta \quad (4)$$

where $A(\theta,\tau)$ is the ambiguity function of the signal and the kernel function is the product of the Choi-Williams kernel and the Rihaczek kernel. This new distribution still satisfies the time and the frequency marginals and preserves the energy since $\phi(0,\tau) = \phi(\theta,0) = 1$ [5]. The value of $\sigma$ can be adjusted to achieve a desired trade-off between resolution and the amount of cross-terms retained. Fig. 1 illustrates the original and the reduced interference Rihaczek distributions for the sum of two Gabor logons.

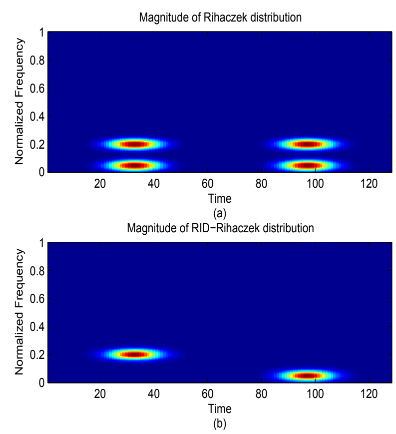


**Fig. 1**. a) Magnitude of Rihaczek distribution and b) Magnitude of Reduced interference Rihaczek Distribution

### 2.2. Time-Varying Phase Spectrum

The time-varying phase estimate based on the Rihaczek distribution can be defined as

$$\begin{aligned} \Phi(t,\omega) &= \arg\left[\frac{C(t,\omega)}{|C(t,\omega)|}\right], \\ &= \arg\left[e^{j\phi(t)} e^{-j\theta(\omega)} e^{-j\omega t}\right], \\ &= \phi(t) - \theta(\omega) - \omega t \end{aligned} \quad (5)$$

where $\phi(t)$ and $\Phi(\omega)$ refer to the phase in the time and the frequency domains, respectively.

Once the time-varying phase spectrum is defined, the phase

[1]All integrals are from $-\infty$ to $\infty$ unless otherwise stated.

between two signals, $x_1(t)$ and $x_2(t)$ can be computed as:

$$\begin{aligned}\Phi_{12}(t,\omega) &= \arg\left[\frac{C_1(t,\omega)}{|C_1(t,\omega)|}\frac{C_2^*(t,\omega)}{|C_2(t,\omega)|}\right], \\ &= (\phi_1(t)-\phi_2(t))-(\theta_1(\omega)-\theta_2(\omega)), \quad (6)\end{aligned}$$

where $\phi_1(t)$ and $\phi_2(t)$ correspond to the phase of the time domain signals, whereas $\theta_1(\omega)$ and $\theta_2(\omega)$ correspond to the phase of the Fourier transform of the two signals.

It can be shown that for a real-valued signal, the phase between a signal $x_1(t)$ and its shifted version $x_1(t-t_0)$ is given by

$$\begin{aligned}\Phi_{12}(t,\omega) &= \arg\left[\frac{x_1(t)X_1^*(\omega)e^{-j\omega t}}{|x_1(t)||X_1(\omega)|}\frac{x_1^*(t-t_0)X_1(\omega)e^{-j\omega t_0}e^{j\omega t}}{|x_1(t-t_0)||X_1(\omega)|}\right] \\ &= \arg\left[\frac{x_1(t)}{|x_1(t)|}\frac{x_1^*(t-t_0)e^{-j\omega t_0}}{|x_1(t-t_0)|}\right] \\ &= -\omega t_0 \quad (7)\end{aligned}$$

which is a linear function of frequency as expected [2].

### 2.3. Phase Coherence

In most applications, the time-varying phase spectrum is not particularly useful for measuring the synchrony between the signals. In order to further quantify the synchrony between signals, we define a measure of phase coherence based on the time-varying phase spectrum estimate introduced in the previous section. Similar to the definition given in [3], we define a phase coherence measure based on Rihaczek distribution:

$$PC(t,\omega)=\left|\frac{1}{\delta}\int_{t-\delta/2}^{t+\delta/2}\exp(j\Phi_{12}(\tau,\omega))d\tau\right| \quad (8)$$

where $\delta$ is the length of the time window used for smoothing the phase difference estimates. In previous work based on the wavelet transform, $\delta$ is chosen as a function of frequency since the time-bandwidth product is not a constant for the wavelet transform. However, in the proposed approach, the phase coherence is based on the time-frequency distribution which has a constant time-bandwidth product over the whole time-frequency plane. Therefore, $\delta$ will be a constant determined based on the signal to achieve a reasonable trade-off between resolution and accuracy of the phase coherence estimates.

Phase coherence is always between 0 and 1, and can be used to detect neural synchrony between different parts of the brain. When the phase difference between the two signals is constant over time, the value of phase coherence is equal to 1.

## 3. RESULTS

In this section, we will first test the validity of the proposed measure on a synthesized signal and then apply it to EEG signals collected during different experiments.

[2] $\Phi_{12}(t,\omega)=-\omega t_0$ with modulus of $\pi$.

*Example 1: Time-Varying Phase Tracking*: In this example, we consider two complex exponential signals with a time-varying phase difference, $x_1(t)=\exp(j\omega_1 t)$ and $x_2(t)=\exp(j\omega_1(t-at^2))$, where the phase difference is a second order polynomial as a function of time. Classical measures of phase coherence based on the Fourier transform will not be able to detect this time-varying change in the phase. In this example, the Rihaczek distribution of the two complex exponentials are computed and the phase difference between them is computed using equation 6. We compare the theoretical phase difference which is a second order polynomial and the estimated one at the frequency of interest, $\omega_1$. Fig. 2 illustrates the two signals and shows that the proposed method is successful at estimating the time-varying phase difference with high accuracy.

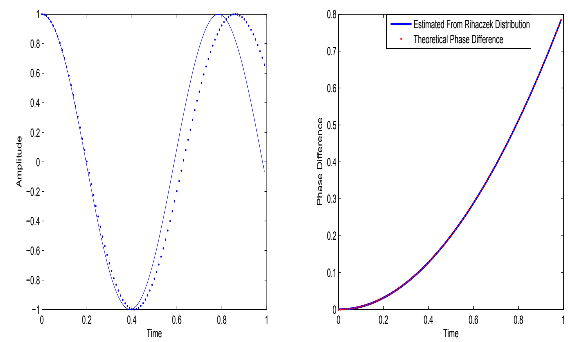


**Fig. 2**. Comparison of the theoretical and estimated time-varying phase difference

*Example 2: Gamma Band Synchrony in Schizophrenic Patients*: Cognitive acts require the integration of numerous functional areas widely distributed over the brain. In recent years, there has been evidence that such large-scale integration is mediated by neuronal groups that oscillate in the gamma range (30-80 Hz). In earlier studies, it has been found that schizophrenic patients exhibit deficits in gamma band neural synchrony compared to normal subjects [7].

In this study, we examined the high-frequency (gamma) electrophysiological abnormalities by studying one schizophrenic and one non-psychiatric control subjects performing a continuous performance task (CPT). Measures of gamma energy indicate that schizophrenia patients exhibit reduced responses to target stimuli relative to non-psychiatric control subjects. The current study extends this analysis by utilizing the proposed phase coherence measure to evaluate whether gamma activity for controls represented synchrony between frontal and parietal areas during target perception, and then whether this synchrony is attenuated for schizophrenia patients. The phase coherence was computed over a window of maximal gamma energy for target stimuli, 200-600 ms after the stimulus, and 30-55 Hz over all trials. The average phase synchrony for the time and frequency region was computed over all trials.

The average phase coherence over 100 trials is 0.8206 for the control subject, whereas the average coherence is 0.6640 for the schizophrenic subject. The difference between the phase coherence values over trials was found to be significant by the t-test at $\alpha = 0.001$ significance level. Fig. 3 shows the phase coherence over time and frequency in the P300 time range and $\gamma$ frequency range. It can be seen that for control subjects the coherence is higher especially at the upper gamma frequencies.

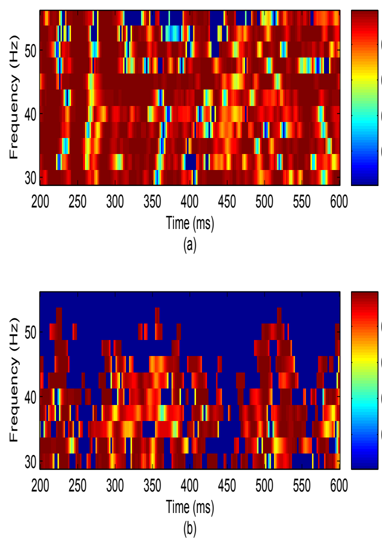


**Fig. 3**. a) Phase coherence over time and frequency for the control subject, b) Phase coherence over time and frequency for the schizophrenic subject

## 4. CONCLUSIONS

In this paper, we have introduced a new time-varying measure of phase coherence for quantifying the large-scale neural synchrony in the brain. The proposed measure is based on a complex-valued time-frequency distribution introduced by Rihaczek. The time-varying phase and phase coherence measures are defined based on this complex distribution. The effectiveness of the proposed measure in quantifying time-varying phase synchrony is verified both through simulated and real EEG data. Results based on the analysis of EEG signals recorded from schizophrenic patients during a sustained attention task revealed the significant differences in the amount of integration in the brain for the schizophrenic and control subjects. The proposed method differs from the existing wavelet-based phase coherence measures in a couple of aspects. First, the time and frequency resolution in the proposed method is constant over the whole time-frequency plane as opposed to the wavelet coherence measure. Preliminary analysis shows that this property of the Rihaczek-based coherence measures results in a higher resolution estimate of the phase coherence. Second, there is no bias due to windowing unlike the wavelet coherence measure. Future work will focus on the statistical analysis of the proposed measure as well as the application to other EEG studies.

## 5. REFERENCES


[1] S. Haykin, R. J. Racine, Y. Xu, and A. Chapman, "Monitoring neuronal oscillations and signal transmission between cortical regions using time-frequency analysis of electroencephalographic activity," *Proceedings of the IEEE*, vol. 84, no. 9, pp. 1295–1301, 1996.

[2] M. Le Van Quyen, J. Foucher, J-P. Lachaux, E. Rodriguez, A. Lutz, J. Martinerie, and F. J. Varela, "Comparison of Hilbert transform and wavelet methods for the analysis of neuronal synchrony," *Journal of Neuroscience Methods*, vol. 111, pp. 83–98, 2001.

[3] J-P. Lachaux, A. Lutz, D. Rudrauf, D. Cosmelli, M. Le Van Quyen, J. Martinerie, and F. Varela, "Estimating the time course of coherence between single-trial brain signals: an introduction to wavelet coherence," *Neurophysiol. Clin.*, vol. 32, no. 1-3, pp. 1–18, 2002.

[4] D. Li and R. Jung, "Quantifying coevolution of nonstationary biomedical signals using time-varying phase spectra," *Annals of Biomedical Engineering*, vol. 28, pp. 1101–1115, 2000.

[5] L. Cohen, *Time-Frequency Analysis*, Prentice Hall, New Jersey, 1995.

[6] A. W. Rihaczek, "Signal energy distribution in time and frequency," *IEEE Trans. on Info. Theory*, vol. 14, no. 3, pp. 369–374, 1968.

[7] K. M. Spencer, P. G. Nestor, R. Perlmutter, M. A. Niznikiewicz, M. C. Klump, M. Frumin, M. E. Shenton, and R. W. McCarley, "Neural synchrony indexes disordered perception and cognition in schizophrenia," *Proc. of the National Academy of Sciences*, vol. 101, no. 49, pp. 17288–17293, 2004.