ON THE PERFORMANCE OF UNIFORM THRESHOLD QUANTIZATION FOR A SUM OF INDEPENDENT MEMORYLESS LAPLACIAN SOURCES

Vadim Sheinin and Da-ke He

IBM T. J. Watson Research
Emails: vadims@us.ibm.com, dakehe@us.ibm.com

ABSTRACT

The performance of uniform threshold quantization subject to an entropy constraint is studied for a sum of two and three independent zero-mean memoryless Laplacian sources. Both symmetric and asymmetric quantizers are considered, and approximate parametric expressions for the operational rate-distortion function $R(D)$ are obtained for all rates. In particular, the low rate regime (rates below 1 bit per sample) is considered and simpler expressions for $R(D)$ are derived. It is envisioned that these expressions will facilitate rate estimation in practical Wyner-Ziv coding with low complexity encoder.

Index Terms: Laplacian sources, rate-distortion, quantization, Wyner-Ziv coding.

1. INTRODUCTION

In this paper we describe a scalar quantizer Q defined on the real line by two sequences of real numbers: $\{x_k\}_{k=-\infty}^{\infty}$ and $\{y_k\}_{k=-\infty}^{\infty}$. To quantize a real number x, the quantizer Q simply finds an index $k^* = Q(x)$ such that $x_{k^*} < x \leq x_{k^*+1}$, and reconstructs x as y_{k^*}. For this reason, $\{x_k\}$ are called threshold levels, and $\{y_k\}$ are called reconstruction levels. In a special case when there exists a constant $\Delta > 0$ such that $x_{k+1} - x_k = \Delta$ for all k, the quantizer Q is called a uniform threshold quantizer. If in addition $x_k = k\Delta$, Q is called a uniform threshold symmetric quantizer.

If Q is applied to a random variable X, its performance can be analyzed by the rate

$$H_Q \stackrel{\Delta}{=} H(Q(X)), \tag{1}$$

where $H(Q(X))$ denotes the entropy of the random variable $Q(X)$, and the average distortion D_Q incurred in the quantization process. In most practical applications, the distortion of interest is the mean square error, i.e.,

$$D_Q \stackrel{\Delta}{=} \mathbf{E}(X - Q^{-1}(Q(X)))^2, \tag{2}$$

where the inverse mapping $Q^{-1}(k) \stackrel{\Delta}{=} y_k$ for any integer k, and $\mathbf{E}$ stands for standard expectation. Throughout this paper D_Q refers to mean square error unless specified otherwise.

The operational rate-distortion performance of uniform threshold quantization for a single source has been studied extensively in the literature of traditional lossy source coding. However except for some very special cases, the research has been mostly relying on numerical approaches [1]. One of the few exceptions is the performance analysis of uniform threshold quantization for memoryless Laplacian sources [2], [3], where analytical results were derived. Specifically, for memoryless Laplacian sources, uniform threshold quantizers were shown in [2] to satisfy the necessary conditions for optimality. In the same paper, Berger also provided parametric equations to analytically calculate H_Q and D_Q of a uniform threshold *symmetric* quantizer Q for a zero-mean Laplacian source. Parametric expressions for H_Q and D_Q when Q is a uniform threshold quantizer but not symmetric were derived in [3].

In this paper, we aim at deriving parametric expressions of H_Q and D_Q for a sum of independent memoryless Laplacian sources. Formally, let N, N', and N'' be Laplacian random variables independent each of other. For convenience, throughout this paper we assume that N, N', and N'' are all with zero mean. Let $X = N + N'$ or $X = N + N' + N''$. We are interested in the operational rate-distortion performance of uniform threshold quantization, both symmetric and asymmetric, for the source $\{X_i\}_{i=1}^{\infty}$, where each X_i, $i \geq 1$, is an independent copy of X. Clearly, the $\{X_i\}_{i=1}^{\infty}$ can be regarded as a sum of independent memoryless Laplacian sources.

Our interest in analyzing the performance of uniform threshold quantization for the sum of independent memoryless Laplacian sources is motivated by, apart from pure theoretical curiosity, the following observations. Recently we have seen surging interest in lossy source coding with side information available only to the decoder, also called Wyner-Ziv coding for brevity [4], One motivating factor of such interest is the prospect that one can build compression systems using Wyner-Ziv coding with low-complexity encoders that are ideally suited for applications like distributed source coding or asymmetric video compression [6]. To make Wyner-Ziv coding practical, a critical problem is how to efficiently estimate the achievable compression rates on the encoder side as the encoder does not have access to the side information. In order to address this problem, we see that on the one hand, in these applications of Wyner-Ziv coding, uniform threshold quantization is often a natural choice due to its simplicity and low computational complexity; and, on the other hand, empirical evidence shows that the noise in the channel between the source and the side information in these applications demonstrates behavior between Laplacian and Gaussian, and is often better modeled by a sum of independent memoryless Laplacian sources. Clearly when the encoder complexity is concerned, numerical approaches to estimating the achievable rates cannot be afforded. In view of these, we see that it is necessary to analyze the operational rate-distortion performance of uniform threshold quantization for a sum of independent memoryless Laplacian sources.

Let Q denote a uniform threshold quantizer. In this paper, we will derive parametric expressions of H_Q and D_Q for a sum of two or three independent memoryless Laplacian sources, and for both symmetric and asymmetric Q. Simplified approximations will be provided for the low-rate regime, which can be utilized to address the above rate estimation problem in Wyner-Ziv coding. Throughout this paper, we use the low-rate regime to denote the region of rates below 1 bit per sample. It should be noted that despite the fact that the low-rate regime is practically more important than the high-rate regime in video and audio compression, it is less investigated analytically than the high-rate regime in the literature partly due to technical difficulty. Finally we note that our analysis in this paper is made possible by and is an interesting demonstration of the famous Euler-Maclaurin summation formula in infinite series theory [5].

2. PERFORMANCE FOR THE SUM OF TWO LAPLACIAN SOURCES: SYMMETRIC CASE

In this section, we investigate the performance of symmetric uniform threshold quantization for the sum of two or three independent memoryless Laplacian sources with zero mean.

Consider the one-dimensional quantization of a discrete-time memoryless zero-mean stationary process (source) $\{X_i\}_{i=1}^{\infty}$ with marginal pdf $f(x)$ and variance $\sigma_X^2 = \mathbf{E}[X_1^2]$. When Q is clear from the context, we shall drop the subscript Q in H_Q and D_Q defined in (1) and (2), respectively. From (1) and (2) we see that an optimum quantizer Q minimizes

$$D = \sum_k \int_{x_k}^{x_{k+1}} f(x)(y_k - x)^2 dx. \tag{3}$$

subject to the constraint

$$H = -\sum_k p_k \log_2 p_k, \tag{4}$$

where

$$p_k = \int_{x_k}^{x_{k+1}} f(x) dx. \tag{5}$$

Note that quantizer reconstruction levels are assumed to be entropy coded at a rate R which is arbitrary close to the entropy H_Q. Using the technique from [2], we have the following expression for the y_k,

$$y_k = \frac{1}{p_k} \int_{x_k}^{x_{k+1}} x f(x) dx, \tag{6}$$

which minimizes the mean square error distortion for $X_1 \in [x_k, x_{k+1}]$.

The performance of uniform threshold quantization for an memoryless Laplacian source was analyzed in [3], where parametric expressions for D and H above were obtained. In the following, we consider a source which is a sum of two independent memoryless Laplacian sources with zero-mean. We are interested in getting analytical expressions for both H and D, especially in the low-rate regime for the reason stated in the introduction section.

Let $\{X_i\}_{i=1}^{\infty}$ denote the source as the sum of two independent memoryless Laplacian sources with zero mean and variance $2/\lambda^2$. Let Q be a symmetric uniform threshold quantizer with threshold levels $\{x_k\}$ and reconstruction levels $\{y_k\}$, where $x_k = k\Delta$. In the above Δ is a positive consant. We now compute D and H according to (3) and (4) for the source $\{X_i\}_{i=1}^{\infty}$. Note that due to page limit, some derivation steps are omitted.

Let $f(x)$ be the probability density function of X_1. It is easy to verify that

$$f(x) = \frac{\lambda}{4} e^{-\lambda|x|} \left(1 + \lambda|x|\right), \tag{7}$$

and that the variance of X_1 is given by

$$\sigma_X^2 = \frac{4}{\lambda^2}. \tag{8}$$

First, we calculate p_k from (4).

$$\begin{aligned} p_k &= \int_{x_k}^{x_{k+1}} \frac{\lambda}{4} e^{-\lambda|x|} \left(1 + \lambda|x|\right) dx \\ &= \frac{1}{4} e^{-x\,a} \left(x\,b + c\right) \end{aligned} \tag{9}$$

for $k \geq 0$. In the above $a \triangleq \lambda\Delta$, $b \triangleq \left(1 - e^{-a}\right) a$, and $c \triangleq 2 - 2\,e^{-a} - ae^{-a}$. From symmetry of $f(x)$ it is clear that p_k will have the same expression as (9) for $k < 0$.

Second, we derive an expression for y_k. It follows from (6) and (9) that

$$y_k = \frac{1}{\lambda} \frac{k^2 a^2 A + k[3aA + 2a^2] + 3 + a^2 + 3A}{kaA + 2A + a}, \tag{10}$$

where $A \triangleq 1 - e^a$.

Using the symmetry of $f(x)$ again, we get from (3) that

$$\begin{aligned} D &= \sum_{k \in I} \int_{x_k}^{x_{k+1}} f(x)(y_k - x)^2 dx \\ &= 2\sum_{k=0}^{\infty} \Big[\int_{k\Delta}^{(k+1)\Delta} f(x) y_k^2 dx - \int_{k\Delta}^{(k+1)\Delta} f(x) 2 y_k dx + \\ &\quad \int_{k\Delta}^{(k+1)\Delta} f(x) x^2 dx \Big]. \end{aligned}$$

In the above, the last summand on the right hand side is the variance of X_1 given by (8). Taking into account (9) we have

$$D = \frac{4}{\lambda^2} - 2\sum_{k=0}^{\infty} {y_k}^2 p_k. \tag{11}$$

In view of (11), (10), and (9), we see that to perform the summation in (11) on the index k we cannot get a closed form expression. Nonetheless, we now propose to use a famous result from infinite series theory, namely, Euler-Maclaurin summation formula [5], which states the partial sum of the first n terms in the series $\{f(k)\}_{k=1}^{\infty}$ can be convert to an integral as follows.

$$\begin{aligned} & f_1 + f_2 + ... + f_n \\ = & \int_1^n f(x) dx + \frac{1}{2}[f_n + f_1] + \frac{B_2}{2!}[f_n^{'} - f_1^{'}] + \frac{B_4}{4!}[f_n^{'''} - \\ & f_1^{'''}] + ... + \frac{B_{2k}}{2k!}[f_n^{(2k-1)} - f_1^{(2k-1)}] + R_k, \end{aligned} \tag{12}$$

where B_k are Bernoulli numbers, and R_k is an error term. It can be shown that $|R_k| < \frac{4}{(2\pi)^k} |f^{(2k)}(1)|$ when the sum is infinite and $f^{(2k)}(\infty) = 0$.

Finally, using (12) we can approximate D and R to any given precision. In fact, by keeping only first derivative, we find that the error term is in the order of 10^{-2}. When more precision is required, more terms shall be taken into account in (12). The final expressions for D and R are quite lengthy. In later sections we will see that in the low-rate regime these formulas become more manageable. For the sake of the completeness we show the expressions for D and R here for any rate.

$$\begin{aligned} D = & \big[4 - 0.5(4e^{3a} - 4 + 2e^{2a}a^2 - 12e^{2a} + a^3 e^{2a} - \\ & 2e^a a^2 + e^a a^3 + 12e^a)\big] / \big[(e^a - 1)(1 - 2e^a + e^{2a})\big] - \\ & 2(1 + 4e^{2a}a^2 - 2e^{3a}a^2 + e^{2a}a^4 + 6e^{2a} - 2e^a a^2 \\ & -4e^{3a} + e^{4a} - 4e^a)\big[0.25\frac{e^{ca/b} Ei(a + ca/b)}{b} + \end{aligned}$$

$$\frac{1}{8}\frac{e^{-a}}{b+c} + \frac{833}{40000}\frac{ae^{-a}}{b+c} + \frac{833}{40000}\frac{be^{-a}}{(b+c)^2} + \frac{1}{4c}]/(-1+e^a)^2, \quad (13)$$

where $a \triangleq \lambda\Delta$, $b \triangleq e^a a(e^a - 1)$, $c \triangleq e^a(2e^a - a - 2)$, and $Ei(x)$ denotes an exponential integral, i.e., $Ei(t) \triangleq \int_1^\infty \frac{e^{-tx}}{x}dx$.

$$\begin{aligned} H = & -0.5\frac{(ac+b+ab)\,e^{-a}\ln(b+c)}{\ln(2)\,a^2} - \\ & 0.5be^{\frac{ac}{b}} Ei\left(1, a+\frac{ac}{b}\right)(\ln(2))^{-1}a^{-2} \\ & -0.5\frac{e^{-a}b}{\ln(2)\,a^2} - 0.25\frac{e^{-a}(b+c)\ln(b+c)}{\ln(2)} - \\ & \frac{1}{24}\frac{ae^{-a}(b+c)\ln(b+c)}{\ln(2)} + \frac{1}{24}\frac{e^{-a}b\ln(b+c)}{\ln(2)} + \\ & \frac{1}{24}\frac{e^{-a}b}{\ln(2)} + \frac{e^a(b-c+ce^a)}{(-1+e^a)^2} + \\ & 0.7213\,\frac{a(e^a)^2\left((b+c)e^{2a} + e^a(b-c)\right)}{(-1+e^a)^3 e^{2a}} - \\ & 0.5\frac{c\ln(c)}{\ln(2)}, \end{aligned} \quad (14)$$

where $b \triangleq \left(1-e^{-a}\right)a$ and $c \triangleq 2 - 2\,e^{-a} - ae^{-a}$.

Table (1) compares the values D for a fixed H obtained by numerical methods, and the ones obtained by parametric expressions (13) and (14). Note that in the symmetric case, the rate is always greater than or equal to 1 bit per sample. Throughout this paper, SNR denotes the signal to noise ratio, i.e., $SNR = 10\log_{10}\frac{\sigma_X^2}{D}$.

H bits per sample	1.01	2	3
a	8.64	2.13	1.02
Analytical SNR, dB	3.75	10.64	16.7
Numerical SNR, dB	3.73	10.66	16.73

Table 1. Comparison of analytical and numerical SNRs for the sum of two Laplacian sources

3. PERFORMANCE FOR THE SUM OF THREE LAPLACIAN SOURCES: SYMMETRIC CASE

In this section, we investigate the performance of symmetric uniform threshold quantization for the sum of three independent memoryless Laplacian sources with zero mean. Let $\{X_i\}_{i=1}^\infty$ denote the source as the sum of three independent memoryless Laplacian sources with zero mean and variance $2/\lambda^2$. Let $f(x)$ be the probability density function of X_1. It is easy to verify that

$$f(x) = \frac{\lambda}{16}e^{-\lambda|x|}(3+3\lambda|x|+\lambda^2x^2), \quad (15)$$

and the variance of X_1 is given by $\sigma_X^2 = \frac{6}{\lambda^2}$. Along the line in the previous section, we now compute D and H according to (3) and (4) for the source $\{X_i\}_{i=1}^\infty$.

First, we calculate p_k from (4).

$$p_k = \int_{x_k}^{x_{k+1}} f(x)dx = \frac{1}{16}e^{-ak}(bk^2+ck+d), \quad (16)$$

where $a \triangleq \lambda\Delta$, $b \triangleq a^2(1-e^{-a})$, $c \triangleq -2a^2e^{-a} + 5a(1-e^{-a})$, and $d \triangleq 8(1-e^{-a}) - e^{-a}(a^2+5a)$. From the symmetry of $f(x)$ it is clear that p_k will have the same expression as (9) for $k < 0$.

Second, we derive an expression for y_k according to (6) and (16).

$$\begin{aligned} y_k = & \frac{1}{\lambda}\Big[k^3a^3A + k^2a^2(-3a+6A) + \\ & ka(-12a - 3a^2 + 15A) + 15A - 6a^2 - a^3 - 15a\Big]/ \\ & \Big[k^2a^2A + ka(5A-2a) - 5a - a^2 + 8A\Big], \end{aligned} \quad (17)$$

where $A \triangleq e^a - 1$.

Finally, for the sum of three Laplacian sources, we can use Matlab symbolic toolbox to obtain final expressions for D and H, which as expected turn out to be very lengthy, and will not be provided in this paper. In the low bit rate regime of interest, however, the expressions are much shorter, and will be provided in the next section. In the following, we compare analytical and numerical results. As in the case of two sources, these result agree to each other nicely.

H bits per sample	1.01	2	3
a	10.05	2.63	1.26
Analytical SNR, dB	3.96	10.6	16.63
Numerical SNR, dB	3.95	10.6	16.58

Table 2. Comparison of analytical and numerical SNRs for the sum of two Laplacian sources

4. PERFORMANCE FOR THE SUM OF TWO OR THREE LAPLACIAN SOURCES: ASYMMETRIC CASE

In this section, we investigate the performance of asymmetric uniform threshold quantization for the sum of two or three independent memoryless Laplacian sources with zero mean, respectively.

Our sources are the same as in Section 2 and Section 3, respectively. The difference is that the uniform threshold quantizer Q is now asymmetric, i.e., the threshold levels satisfy $x_k = \Delta k + \frac{\Delta}{2}$, where k is an integer.

Consider now the source in Section 2. To calculate p_k, we consider three different cases: $k \geq 0$, $k < -1$ and $k = -1$. For the case of $k \geq 0$ we have

$$p_k = \int_{x_k}^{x_{k+1}} f(x)dx = \frac{1}{8}e^{-ak}(bk+c), \quad (18)$$

where $a \triangleq \lambda\Delta$, $b \triangleq 2ae^{-a/2}(1-e^{-a})$, and $c \triangleq (4+a)e^{-a/2} - (4+3a)e^{-3a/2}$. Similarly one can derive the formula for the case of $k < -1$. For p_{-1} we have $p_{-1} = 1 - e^{-a/2}(1+a/4)$. It follows immediately from the symmetry of $f(x)$ that $y_{-1} = 0$. In view of this, and (11), we see that

$$D = \frac{4}{\lambda^2} - S, \quad (19)$$

where

$$S = \sum_{k=0}^{\infty} {y_k}^2 p_k + \sum_{k=-\infty}^{-2} {y_k}^2 p_k. \quad (20)$$

Examining carefully ${y_k}^2 p_k$, we recognize that it can be split into two part: the exponential function of ak multiplied by a function of a, and the exponential function of ak multiplied by a ratio of polynomials of k. Evaluating the infinite sum $\sum_{k\in I} {y_k}^2 p_k$ for rates smaller than 1 bit per sample shows that with relative error by far smaller than 10^{-2}, which is adequate for most practical purposes, we can keep in the expression for D only the first part. Performing the similar evaluation for R simplifies expressions as well. It can be shown, after taking into account (18-20), that in this case we have the following expressions for D and R.

$$H = -2(V_1 + V_2 + T_0) - q\log_2(q),$$

where $V_1 \triangleq -3/8\ (4+a)\, e^{-1/2\,a}$,
$V_2 \triangleq -.1803 \frac{a\left((b+c)e^{2\,a} - e^{a}(-b+c)\right)}{(-1+e^{a})^3}$, $T_0 = 1/8\, c\log_2 c$,
$q \triangleq 1 - e^{-1/2\,a}\,(1 + 1/4\,a)$, $b \triangleq 2\left(e^{-1/2\,a} - e^{-3/2\,a}\right) a$,
and $c \triangleq (4+a)\, e^{-1/2\,a} - (4+3\,a)\, e^{-3/2\,a}$.

$$\lambda^2 D = 4 - 2S_1,$$

and $SNR = 10\log_{10}\frac{4}{D\lambda^2}$, where $S1 = AC/B$. In the above,

$$\begin{aligned} A &\triangleq -208 - 208\,a + 64\,e^{a}a^3 - e^{4\,a}a^4 + 4\,e^{2\,a}a^4 - \\ &\quad 6\,e^{3\,a}a^4 - 144\,e^{4\,a}a - 60\,e^{4\,a}a^2 - 12\,a^3e^{4\,a} - \\ &\quad 496\,e^{2\,a}a^2 - 1056\,e^{2\,a}a + 38\,e^{a}a^4 + 264\,a^2e^{3\,a} - \\ &\quad 3\,a^4 + 640\,ae^{3\,a} - 24\,e^{2\,a}a^3 - 148\,a^2 - 928\,e^{2\,a} + \\ &\quad 592\,e^{3\,a} - 28\,a^3 - 144\,e^{4\,a} + 672\,e^{a} + 768\,e^{a}a + \\ &\quad 440\,e^{a}a^2 + 16\,e^{-a}, \\ C &\triangleq \left(e^{-1/2\,a}\right)\frac{-1}{32}, \text{ and} \\ B &\triangleq (-4 - 3\,a + 4\,e^{a} + e^{a}a)\,(e^{a} - 1)^3 . \end{aligned}$$

Table (3) below compares the values D for a fixed H obtained by numerical methods, and the ones obtained by parametric expressions

H bits per sample	1	0.9	0.7	0.5	0.3	0.1
a	4.6	4.98	5.85	6.97	8.57	11.7
Analytical SNR, dB	5.16	4.65	3.66	2.67	1.68	0.64
Numerical SNR, dB	5.16	4.66	3.65	2.66	1.68	0.63

Table 3. Comparison of analytical and numerical SNRs for the sum of two Laplacian sources in the asymmetric case

We then consider the sum of three independent memoryless Laplacian sources with zero mean in Section 3. The following parametric expressions of H and D are obtained in the low-bit rate regime.

$$H = -2(V_1 + V_2 + T_0) - q\log_2(q),$$

where $V_1 \triangleq \frac{3}{32}\,\frac{e^{a}\left(b - c + ce^{a} + be^{a} + d - 2\,de^{a} + de^{2\,a}\right)}{(-1+e^{a})\left(-1+2\,e^{a} - e^{2\,a}\right)}$,
$V_2 \triangleq 0.0225\,\frac{ae^{a}\left(b - c + 4\,be^{a} + d - 2\,de^{a} + de^{2\,a} + ce^{2\,a} + be^{2\,a}\right)}{(-1+e^{a})^2\left(-1+2\,e^{a} - e^{2\,a}\right)}$,
$T_0 \triangleq \frac{1}{64}\,d\log_2 d$, $q \triangleq 1 - e^{-1/2\,a}\left(1 + \frac{5}{16}\,a + 1/32\,a^2\right)$,
$b \triangleq 4\,a^2e^{-1/2\,a}\left(1 - e^{-a}\right)$,
$c \triangleq 20\,e^{-1/2\,a}a\left(1 - e^{-a}\right) + 4\,a^2e^{-1/2\,a}\left(1 - 3\,e^{-a}\right)$, and $d \triangleq 10\,e^{-1/2\,a}a\left(1 - 3\,e^{-a}\right) + a^2e^{-1/2\,a}\left(1 - 9\,e^{-a}\right) + 32\,e^{-1/2\,a}\left(1 - e^{-a}\right)$.

$$D = \lambda^2 D = 6 - 2S_1,$$

and $SNR = 10\log_{10}\frac{6}{D\lambda^2}$, where $S_1 \triangleq A/B$. In the above,

$$\begin{aligned} A &\triangleq 92\,e^{4\,a}a^2 + 312\,e^{4\,a}a + 14\,e^{4\,a}a^3 + e^{4\,a}a^4 + 416\,e^{4\,a} + \\ &\quad 32\,e^{3\,a}a^4 - 304\,e^{3\,a}a^2 - 1248\,e^{3\,a}a + 64\,a^3e^{3\,a} - \\ &\quad 1664\,e^{3\,a} + 2496\,e^{2\,a} + 1872\,ae^{2\,a} + 94\,a^4e^{2\,a} + \\ &\quad 424\,e^{2\,a}a^2 + 4\,e^{2\,a}a^3 - 304\,e^{a}a^2 - 1248\,e^{a}a - \\ &\quad 96\,e^{a}a^3 - 1664\,e^{a} + 92\,a^2 + 312\,a + 14\,a^3 + a^4 + 416 \\ B &\triangleq 256\,\left(e^{a} - 1\right)\left(e^{3\,a} - 3\,e^{2\,a} + 3\,e^{a} - 1\right)e^{1/2\,a}. \end{aligned}$$

Table (4) compares the values D for a fixed H obtained by numerical methods, and the ones obtained by parametric expressions

H bits per sample	1	0.9	0.7	0.5	0.3	0.1
a	5.75	6.2	7.23	8.52	10.34	13.8
Analytical SNR, dB	4.91	4.42	3.44	2.49	1.54	0.58
Numerical SNR, dB	4.96	4.44	3.45	2.49	1.55	0.57

Table 4. Comparison of analytical and numerical SNRs for the sum of three Laplacian sources in the asymmetric case

5. CONCLUSIONS

In this paper we have derived parametric expressions of H_Q and D_Q for a sum of two or three independent memoryless Laplacian sources, and for both symmetric and asymmetric optimum uniform threshold quantizers Q. Simplified approximations have been provided for the practically important low-rate regime. These approximations may be utilized to provide an efficient solution to the rate estimation problem in practical Wyner-Ziv coding. A key approximation technique that makes our analysis poissbile is the famous Euler-Maclaurin summation formula.

6. REFERENCES

[1] N. Farvardin and J. Modestino, "Optimum quantizer performance for a class of non-gaussian memoryless sources," *IEEE Trans. Inform. Theory*, vol. IT-30, pp.485–497, May 1984.

[2] T. Berger, "Optimum quantizers and permutation codes," *IEEE Trans. Inform. Theory*, vol. IT-18, pp.759–765, Nov. 1972.

[3] V. Sheinin, A. Jagmohan, and D.-k. He, "Uniform Threshold Scalar Quantizer Performance in Wyner-Ziv Coding with Memoryless, Additive Laplacian Correlation Channel," *to appear in the Proc. ICASSP*, 2006.

[4] A. D. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," *IEEE Transactions on Information Theory*, vol. 22, pp. 1–10, Jan. 1976.

[5] K.Knopp, *Theory and Application of Infinite Series* NY, NY: Dover Publications, 1990.

[6] B. Girod, A. Aaron, S. Rane, and D. Robollo-Monedero, "Distributed source coding," *Proc. of the IEEE*, vol. 93, pp. 71–83, 2005.