ADAPTIVE RATE FILTERING FOR A SIGNAL DRIVEN SAMPLING SCHEME

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ABSTRACT

This work is a contribution to enhance the signal processing chain required in mobile systems. The system must be low power as it is powered by a battery. Thus a signal driven sampling scheme based on level crossing is employed, adapting the sampling rate and so the system activity by following the input signal variations. In order to filter the non-uniformly sampled signal obtained at the output of this sampling scheme a new adaptive rate FIR filtering approach is devised. The idea is to combine the features of both uniform and non-uniform signal processing tools to achieve a smart online filtering process. The computational complexity of the proposed approach is deduced and compared to one of the classical FIR filtering approach. It promises a significant gain of the computational efficiency and hence of the processing power.

Key Words: *Level-crossing sampling, Asynchronous design, Activity selection, Adaptive rate filtering.*

1. CONTEXT OF THE STUDY

This work is part of a large project aimed to enhance the signal processing chain required in mobile systems. The motivation is to reduce their power consumption, electromagnetic emission and processing noise by smartly reorganizing their associated signal processing theory and architectures. The idea is to combine the signal's event driven processing with asynchronous design in order to reduce the system's dynamic activity. Most of the systems are processing signals with interesting statistical properties, but Nyquist signal processing architectures do not take advantage of such properties. These systems are highly constrained due to the Shannon theory especially in the case of signals such as electrocardiograms, speech, seismic signals etc which are almost always constant and may vary sporadically only during brief moments [3]. This condition causes a large number of samples without any relevant information, a useless increase of system activity so a useless increase of power consumption. In order to avoid this problem a novel approach is adopted, the idea is to realize a signal driven sampling scheme of the analog input signal based on its amplitude variations. This sampling scheme drastically reduces the activity of the post processing, analysis or communication chain because it only captures the relevant information. It is based on "level-crossing" that provides a non-uniform time repartition of the samples. In this context an AADC (Asynchronous Analog to Digital Converter) [2] based on LCSS (Level Crossing Sampling Scheme) [1] has been designed by the CIS group of the TIMA Laboratory. Algorithms for processing [3] and analysis [8] & [9] of the non-uniformly spaced out in time sampled data obtained at the output of AADC have also been developed. The aim of this work is to combine the features of both non-uniform and uniform signal processing tools in order to develop an efficient FIR filtering approach. The idea is to adapt the filter order according to the variations of sampling rate.

2. LCSS (LEVEL CROSSING SAMPLING SCHEME)

In [4], authors have shown that ADC based upon LCSS has a reduced activity and thus allows power saving and noise reduction compared to Nyquist ADCs.
An M-bit resolution AADC has $2^M - 1$ quantization levels which are disposed according to the input signal amplitude dynamic. A sample is captured only when the analog signal *x(t)* crosses one of these predefined levels. The samples are not uniformly spaced in time because they depend on the signal variations as it is clear from Figure 1.

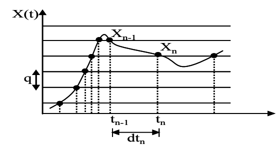

Figure 1: Level-crossing sampling scheme

In [5], Beutler showed that the reconstruction of an original continuous signal is possible, if the average sampling frequency $\overline{F}$ of the non-uniformly sampled signal is greater than twice of the signal bandwidth f_{max}. This condition can be expressed mathematically by $\overline{F} > 2f_{\max}$. According to [2], in the case of LCSS, the number of samples is directly influenced by the resolution of the AADC. For a M-bit resolution AADC, the average sampling frequency of a signal can be calculated by exploiting its statistical characteristics. Then an appropriate value of M can be chosen in order to respect the Beutler's criterion.

3. PROPOSED FILTERING APPROACH

The block diagram of the proposed filtering approach is shown in Figure 2.

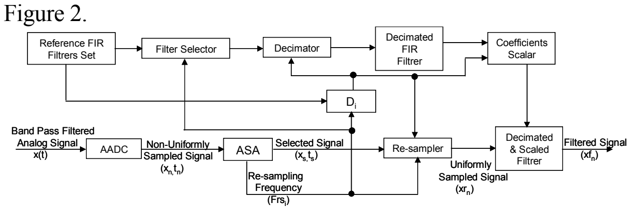

Figure 2: Block diagram of the proposed filtering approach

3.1. AADC + ASA

For a non-uniformly sampled signal obtained at the output of an AADC, the sampling instants (according to [1]) are defined by the Equation 1.

$$t_n = t_{n-1} + dt_n . \quad (1)$$

In Equation 1, t_n is the current sampling instant, t_{n-1} is the previous one and dt_n is the time delay between current and previous sampling instant, as shown in Figure 1.

Let δ be the processing delay of AADC for one sample point. For proper signal capturing the incoming signal must satisfy the "tracking condition" [2] given by Equation 2. In Equation 2, q is the quantum of AADC and is defined by Equation 3.

(2) $$\frac{dx(t)}{dt} \leq \frac{q}{\delta}.$$ (3) $$q = \frac{\Delta x(t)}{2^M - 1}.$$

In Equation 3, $\Delta x(t)$ represents the amplitude dynamic of the band pass filtered signal, $x(t)$ and M represents the resolution of AADC. As AADC has a finite sampling frequency so in order to respect the Beutler's criteria [5] and tracking condition [2] we have employed a band pass filter with pass band $F_{min} \sim F_{max}$, at the input of AADC.

The interesting (active) part of non-uniformly sampled signal is selected and windowed by employing ASA (activity selection algorithm). This algorithm has been implemented by employing the values of dt_n (Equation 1) the complete procedure of activity selection has explained in [8]. ASA displays interesting features with LCSS which are not available in the classical case. It correlates the length of selected window with the signal activity. In addition, it also provides an efficient reduction of the phenomenon of spectral leakage in case of transient signals (signals which start and finish at zero). This is done by avoiding the signal truncation problem occurs in the classical case with the use of a simple and efficient algorithm instead of a smoothening window function (used in classical scheme) [8].

3.2. Adaptive Rate Sampling

In case of AADC the sampling is triggered when the input signal crosses one of the pre-specified threshold levels defined in the amplitude domain. As a result, the temporal density of the sampling operation in the level crossing scheme adapts with time by following the input signal variations. The ASA is used to select and window this non-uniformly sampled data. It is possible to use the non-uniformly sampled sequence directly to do the digital processing on it. However In the studied case it is required to uniformly re-sample the selected data obtained at the output of ASA. So there will be an additional error due to this transformation. Nevertheless, prior to this transformation, one can take advantage of the inherent over-sampling of the relevant signal parts in the system [3]. The interesting signal parts are locally over-sampled in time while keeping the global (average) sampling rate lesser than the classical one. This idea is well suited for the low activity sporadic signals. This over-sampling improves the accuracy of signal acquisition and post interpolation processes. The NNR (nearest neighbour re-sampling) interpolator is employed for data re-sampling. The reasons of inclination towards NNR interpolation are discussed in [8] & [10]. The re-sampling frequency (Frs_i) of each selected window obtained at the output of ASA can be specific depending upon the window length (in seconds) and slope of the signal activity lying within this window [8]. The value of Frs_i for the i^{th} selected window can be calculated by using the following equations.

(4) $$Ts_i = tmax_i - tmin_i\ ,$$

(5) $$Frs_i = N_i / Ts_i.$$

In Equation 4, $tmax_i$ and $tmin_i$ are the final and the initial times of the i^{th} selected window; these parameters describe the window length Ts_i in seconds. In Equation 5, N_i is the number of samples lying in the i^{th} selected window, it depends upon the slope of the signal activity lying within this window. Frs_i is the re-sampling frequency of the i^{th} selected window [8].

3.3. Adaptive Rate Filtering

The motivation behind the proposed filtering approach is to achieve smart sampling (only relevant information to process) along with adaptive and optimal filter order (minimum number of operations per output sample), without performing the complex filter design (calculation) algorithm during online computation. This leads to maximise the computational gain and power efficiency while performing the proposed adaptive rate FIR filtering.

This filtering approach is a smart alternative of the multirate filtering, achieves computational efficiency which is not attainable with a time invariant FIR filter operates at a fixed sampling rate [6] & [7].

As in case of FIR filters with fixed frequency domain parameters the filter order changes as a function of the operational sampling rate. For high sampling rate the order is high and vice versa. In case of proposed approach the sampling rate and the filter order both are adapted by following the incoming signal variation. The computational efficiency is achieved by using the simple (low order) adaptive rate filters operate at reduced sampling rates instead of a unique complex (high order) filter operates at a high sampling rate.

The idea is to offline design an appropriate set of reference filters for a specific application by exploiting the statistical characteristics of incoming signals. In this case the reference filters are designed by taking into account the worst case (highest sampling rate). Here the worst case points towards the Bernstien's inequality [2], given by Expression 6. In Expression 6, $\Delta x(t)$ is the amplitude dynamics of the $x(t)$. The term on left hand side is the slope of the signal and f_{max} is the bandwidth of $x(t)$. Thus for a known resolution of AADC the highest sampling rate F_{max} occurs for a sinusoid of frequency f_{max} and amplitude $\Delta x(t)$ and can be calculated by employing the Equation 7. In Equation 7, M represents the resolution of AADC.

(6) $$\frac{dx(t)}{dt} \leq 2.\pi.\Delta x(t).f_{\max}.$$ (7) $$F_{\max} = 2.f_{\max}.(2^M - 1).$$

A set of reference FIR filters with appropriate specifications can be designed for a set of reference frequencies $Fref$, consists of F_{max} and its dividers. Then during online processing an appropriate reference filter can be chosen for the i[th] selected window from this pre-calculated set. This choice is made on the basis of $Fref$ and the effective value of Frs_i. Let us introduce here the index notation c in order to make the distinction between the chosen reference filter and the complete set of reference filters. The reference filter whose corresponding value of $Fref_c$ is closest and greater or equal to Frs_i is chosen. Afterwards the chosen reference filter's impulse response can be adjusted (decimated) as a function of Frs_i for the i^{th} selected window. The decimation factor D_i can be specific for each selected window depending upon the value of Frs_i.

Various methods can be adapted to deal with the fractional values of D_i and to keep the sampling rate of decimated filter coherent with the re-sampling rate of the data lying in the i^{th} selected window. The employed method is depicted in Figure 3.

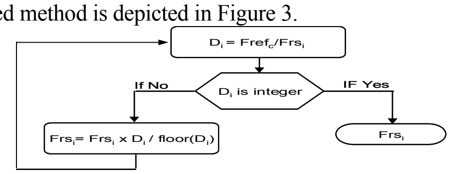

Figure 3: Flowchart of method to avoid fractional value of D_i.

From Figure 3 it is clear that the values of D_i and Frs_i are correlated. First the D_i is calculated by using the $\mathrm{Frs_i}$ and then a decision is made on the basis of D_i, weather an adjustment of Frs_i is required or not. If D_i is an integer keep the same value of Frs_i. If D_i is not an integer, make an increment in the value of Frs_i depending upon the fractional part of D_i and then D_i is recalculated for this new Frs_i. This fulfils the both above stated goals of keeping the value of D_i as a whole number and the sampling rate of decimated filter coherent with the Frs_i. Moreover it further improves the accuracy of the interpolation process when re-sampling the data lying in the i^{th} selected window.

A simple decimation leads to a reduction of filter's energy which will lead to an attenuated version of the filtered signal. As D_i is a

good approximate of the ratio of energy of the original (chosen reference filter) and energy of the decimated filter (decimated for the i^{th} selected window). So this effect of decimation is compensated by scaling (weighting) the coefficients of the decimated filter with the value of D_i as a scaling factor.
The process of obtaining the decimated and scaled filter for the i^{th} selected window from the chosen reference filter is shown mathematically by Equations 8 and 9.

(8) $hdi_j = hc_{D_i k}$. (9) $hwi_j = hdi_j \times D_i$.

According to Equation 8 the decimated filter's impulse response hdi_j for the i^{th} selected window is obtained by picking every D_i^{th} coefficient from the impulse response hc_k of the chosen reference filter. Here k and j represents the indexes of the impulse responses of the chosen reference filter and of the decimated filter respectively. If the length of hc_k is A_c then the length of hdi_j will be $P_i = A_c/D_i$. The process of scaling the hdi_j in order to obtain the decimated and scaled impulse response hwi_j for the i^{th} selected window is clear from Equation 9.

4. ILLUSTRATIVE EXAMPLE

In order to illustrate the proposed filtering approach an input signal summarised in Table 1 is employed. Its total duration is 20 seconds and consists of three active parts. From Table 1 it is clear that the signal is band limited up to 1 kHz. This signal is sampled by employing a 3-bit resolution AADC. So in this case the highest sampling rate F_{max}, calculated by using Equation 7 is 14 kHz.

Active Part	Signal Components	Length (sec)
First	0.6.sin(2.pi.10.t) + 0.3.sin(2.pi.300.t)	1.0
Second	0.45.sin(2.pi.20.t) + 0.45.sin(2.pi.150.t)	1.0
Third	0.65.sin(2.pi.5.t) + 0.25.sin(2.pi.1000.t)	0.5

Table 1: Summary of input signal active parts

The input signal and the selected signal obtained at the output of ASA are shown on the Figure 4.

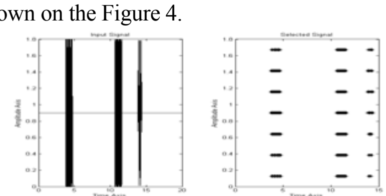

Figure 4: Input signal (left) and the selected signal obtained at the output of ASA (right).

From Table 1 it is clear that each active part of the signal has a low and high frequency component. In order to separate the low frequency component of each activity from higher one two low pass reference FIR filters are implemented as a standard Parks-McClellan algorithm. The filters parameters are given in Table 2.

Cut-off Freq	Transition Band (Hz)	Pass Band Ripples	Stop Band Ripples	Fref (k Hz)	A
30 (Hz)	30~100	-25 (dB)	-80 (dB)	14	506
30 (Hz)	30~100	-25 (dB)	-80 (dB)	7	253

Table 2: Summary of reference filters parameters

In Table 2 *Fref* represents the set of reference frequencies for which the reference filters are designed. Where as *A* represents the set of orders of the designed reference filters.
The parameters of each selected window along with the filter's order (P_i), adapted for each selected window are summarized in Table 3. Here the chosen reference filter in case of each selected window is $h2_k$ i.e. one designed for the 7 kHz sampling rate. Table 3 exhibits the interesting features of the proposed filtering approach. The values of *Ni* represent the sampling rate adaptation by following the input signal slope. It is achieved due to the smart features of AADC based on LCSS. It is also clear from *Ni* that the interesting (active) parts of signal are over-sampled locally like any harmonic signal [3]. The values of T_i in Table 3 exhibit the dynamic feature of ASA which is to correlate the window length with the signal activity lying in the window. On the other hand in the classical case during the windowing process we are not able to select only the active part of the signal. Moreover the window length remains static and is not correlated with the signal activity in the window. For this studied example a 1 second window length would lead to 20, 1-second windows for the whole signal duration (20 sec) in classical case. It follows that the system has to process more than the relevant part of the information. The parameter P_i represents the adaptation of filter's order according to the values of Frs_i it is another advantage of the proposed approach achieved due to the appealing feature of ASA. Whereas in the classical case the filter remains time invariant so has to be designed for the worst case. As the input signal is band limited to 1 kHz. So, if the sampling rate is chosen equal to 2.5 kHz in order to satisfy Shannon's criteria then for the same filter parameters (Table 2) Parks-McClellan algorithm design requires a 90^{th} order FIR filter. As in standard case the signal regardless of its activity is sampled at a fixed sampling rate (2.5 kHz) so a fixed order filter (H = 90) has to be employed for the whole signal length, cause the extra system activity.

Selected Window	T_i (Sec)	N_i (Samples)	Frs_i (Hz)	D(i)	(P_i)
First	0.9995	1400	1400	5	51
Second	0.9994	1166	1166	6	43
Third	0.4995	1750	3500	2	127

Table 3: Summary of the parameters of the selected windows

Spectrum of the signal lying in 2nd window obtained after filtering by the chosen reference filter $h2_k$ (with sampling rate of 7 kHz) and by the decimated and scaled filter $hw2_j$ (with sampling rate of 1166 Hz) are shown in Figure 5.

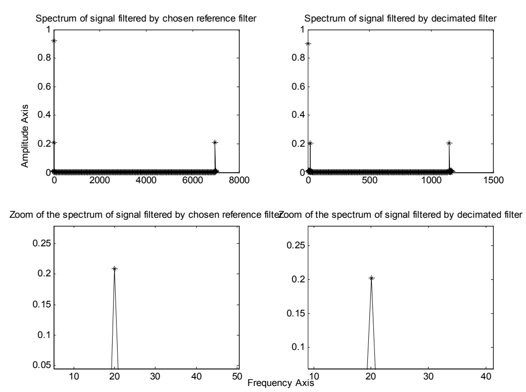

Figure 5: Spectrum of signal filtered by chosen reference filter and by decimated and scaled filter (top-left) and (top-right) respectively. Zoom of the spectrum of signal filtered by chosen reference filter and by decimated and scaled filter (bottom-left) and (bottom-right) respectively.

From Figure 5 it is clear that the results obtained by the decimated and scaled filter are of comparable quality with the results obtained by the chosen reference filter. It is obvious that by decimating the chosen reference filter we are loosing its quality in terms of desired filter response. This loose of quality can be used as an upper bound on decimation factor (D_i) , the maximum value of D_i for which the decimated and scaled filter provides response with an acceptable level of accuracy.

5. ALGORITHM EFFICIENCY

This section compares the computational complexity of the proposed filtering approach with standard one. The complexity evaluation is made by considering the number of online operations executed to perform the algorithm. For simplicity, it is assumed that each operation like addition, multiplication and division has equal complexity.

It is known that in classical case a time invariant, fixed order filter is employed for a specific application. As for a H order FIR filter, H multiplications and H accumulations are computed for each output sample so the operations count per output sample is $2.H$. The total computational complexity C_1 for N_u (total number of uniform) samples can be calculated by employing Equation 10.

Where as in the proposed approach the filter order is not fixed and adapts for each selected window according to the value of Frs_i. In comparison to classical case this approach requires extra operations. Let q be the length of the set of reference frequencies $Fref$. Then the choice of reference filter for the i^{th} selected window requires q comparisons between the values of $Fref$ and Frs_i in worst case. The calculation of D_i requires six operations in worst case, three divisions, one multiplication, one comparison and one floor operation as is clear from Figure 3. These operations repeat once for each selected window. In order to make the complexity comparison we are taking in to account the operations count for the worst case. The decimator has a negligible complexity as compare to the operations like addition or multiplication. This is the reason that the complexity of the decimator used to decimate the chosen reference filter for the i^{th} selected window is not taken into the consideration here. Coefficients scalar (weightner) performs P_i multiplications here P_i represents the order of decimated filter for the i^{th} selected window. Another step of data re-sampling is required before filtering. In this case we are employing NNR interpolator to re-sample the data. The NNR interpolator requires only a comparison operation so perform N_i comparisons here N_i represents the total number of samples lying in the i^{th} selected window. The combined computational complexity C_2 of the proposed filtering approach is given by Equation 11. The processes of designing the set of reference filters is performed offline so is not included in the online algorithm complexity calculation.

(10) $$C_1 = 2.H.N_u \,.$$ (11) $$C_2 = L(q+6) + \sum_{i=1}^{L} P_i + N_i + 2.P_i.N_i \,.$$

In Equation 11 i = 1,2,3,…..,L represents the index of selected window. The computational gain of the proposed filtering approach over classical one can be calculated by employing Equation 12.

(12) $$G = \frac{C_1}{C_2} = \frac{2.H.N_u}{L(q+6) + \sum_{i=1}^{L} P_i + N_i + 2.P_i.N_i} \,.$$

The computational gain of the proposed adaptive rate filtering approach over time invariant classical one has been calculated for the results of illustration example (Table 3) for different time spans by employing Equation 12. It gives 4.4 times gain for the second selected window (1 sec) and a gain of 13 times for the whole signal span (20 sec) respectively. This gain has achieved by signal driven sampling (only the relevant number of samples to process) along with adaptive rate filtering (relevant filter order; relevant number of operations per output sample).

6. CONCLUSIONS

A new adaptive rate filtering approach has been proposed. This technique is well suited for the signals which remain constant most of the time and vary sporadically as electro-cardiograms, speech, seismic signals, etc. A set of reference filters is designed offline by taking into account the signal statistics and application requirements. A complete methodology of firstly choosing the most appropriate reference filter from the pre-calculated set and then decimating and scaling it during the online computation is demonstrated. It shows how the order of chosen reference filter is smartly adapted for each selected window by following the value of Frs_i. The computational complexity of the proposed adaptive rate filtering approach is deduced and compared with the time invariant classical one by using the results of an illustrative example. The result shows 13 times computational gain over classical one which shows that the proposed filtering technique leads to a significant reduction of the total number of operations. This reduction in number of operations is achieved by combining the adaptive rate sampling (only process the relevant information) along with adaptive filter order (reduce the number of operations per output sample).

The decimation of reference filters reduces the quality of decimated filters as compared to the reference ones. The maximum value of decimation factor can be decided by offline calculations for which the decimated and scaled filter gives response with acceptable level of accuracy. Moreover in case of applications where high quality filtering is required, an appropriate filter can be calculated directly online for each selected window at the cost of increased computational load.

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