COMPENSATION OF AN ESTIMATION DELAY IN SELF-OPTIMIZING ADAPTIVE NOTCH FILTERS

Maciej Niedźwiecki and Piotr Kaczmarek

Faculty of Electronics, Telecommunications and Computer Science, Department of Automatic Control
Gdańsk University of Technology, ul. Narutowicza 11/12, Gdańsk, Poland
maciekn@eti.pg.gda.pl,   k.piotr@autograf.pl

ABSTRACT

It is shown that estimation accuracy of adaptive notch filters (ANFs) can be increased by combining two techniques that were previously used separately: automatic gain adjustment and frequency debiasing. To achieve this goal one has to solve a nontrivial problem of determining estimation delay introduced by a variable-gain ANF filter.

Index Terms— adaptive filters, frequency estimation

1. INTRODUCTION

Consider the problem of extraction or elimination of nostationary complex sinusoidal signals (cisoids) $s_i(t), i = 1, \ldots, k$ from noisy measurements $y(t)$

$$y(t) = \sum_{l=1}^{k} s_i(t) + v(t)$$

$$s_i(t) = a_i(t)e^{j \sum_{s=1}^{t} \omega_i(s)}, \quad i = 1, \ldots, k \qquad (1)$$

We will assume that the complex-valued amplitudes $a_i(t)$ and real-valued instantaneous frequencies $\omega_i(t) \in [-\pi, \pi]$ are slowly varying quantities, and that the measurement noise $v(t)$ is circular white.

For a single noisy cisoid ($k = 1$) the normalized steady state version of the ANF algorithm presented in [1] can be written down in the form

$$
\begin{aligned}
\varepsilon(t) &= y(t) - e^{j\widehat{\omega}(t)}\widehat{s}(t-1) \\
\widehat{s}(t) &= e^{j\widehat{\omega}(t)}\widehat{s}(t-1) + \mu_o\varepsilon(t) \\
g(t) &= \mathrm{Im}\left[\frac{\varepsilon^*(t)e^{j\widehat{\omega}(t)}}{\widehat{s}^*(t-1)}\right] \\
\widehat{\omega}(t+1) &= \widehat{\omega}(t) - \gamma_o g(t)
\end{aligned}
\qquad (2)
$$

The ANF algorithm (2) can be controlled by means of adjusting two user-dependent coefficients: the adaptation gain μ_o, $0 < \mu_o \ll 1$, which decides upon the speed of amplitude tracking, and another adaptation gain γ_o, $0 < \gamma_o \ll \mu_o$,

which decides upon the speed of frequency tracking. A thorough analysis of tracking properties of this algorithm (including the proof of its statistical efficiency under certain frequency variation scenarios) was presented in [1].

Using the technique described in [2], the multiple-frequency version of (2) can be easily obtained by combining several single-frequency algorithms into appropriately designed parallel-form or cascade-form structures.

A typical way of increasing tracking capabilities of adaptive notch filters is by means of automatic gain/bandwidth tuning [3], [4], [5] (see [5] for an interesting overview of different approaches to this problem). Irrespective of tuning principles, all solutions mentioned above have the same main feature - they try to balance the estimation bias and the estimation variance. In order to achieve this, they increase adaptation gains when signal parameters change faster, and decrease adaptation gains when signal parameters slow down. The solution proposed recently in [6] is based on recursive prediction error (RPE) minimization. The structure of the self-optimizing ANF algorithm derived in [6] is identical with (2) except that the constant gains μ_o and γ_o are replaced with time-varying gains $\mu(t)$ and $\gamma(t)$ adjusted automatically by an external gain tuning loop.

An entirely different technique of increasing estimation accuracy of ANF filters was proposed in [7]. It was shown that frequency biases, which arise in ANF algorithms, can be significantly reduced by incorporating in the adaptive loop an appropriately chosen decision delay. Such delay is acceptable in many practical applications. The proposed solution is a cascade of two filters. The "pilot" ANF filter, given by (2), provides preliminary (biased) frequency estimates. The estimates yielded by the pilot algorithm are fed into the following "frequency-guided" ANF filter

$$
\begin{aligned}
\bar{\varepsilon}(t - l_o) &= y(t - l_o) - e^{j\widehat{\omega}(t)}\bar{s}(t - l_o - 1) \\
\bar{s}(t - l_o) &= e^{j\widehat{\omega}(t)}\bar{s}(t - l_o - 1) + \mu_o\bar{\varepsilon}(t - l_o) \\
l_o &= \mathrm{int}[\mu_o/\gamma_o]
\end{aligned}
\qquad (3)
$$

which does not estimate signal frequency on its own, and which operates on a delayed data sequence; $\mathrm{int}[x]$ denotes an integer number that is closest to x.

Frequency debiasing improves tracking performance of ANF algorithms and increases their robustness to the choice of design parameters.

We will show that if the two techniques mentioned above – automatic gain tuning and frequency debiasing – are used jointly, the estimation accuracy of ANF filters can be further increased.

2. EVALUATION OF AN ESTIMATION DELAY

2.1. Constant gain case

Tracking properties of (2) were analyzed in [1] in the special case of a constant-modulus cisoid ($|s(t)| = a$, $\forall t$) and constant adaptation gains. Using the approximating linear filter (ALF) technique, developed by Tichavský and Händel [8], it was shown there that the steady state relationship between the mean path of frequency estimates $\bar{\omega}(t) = \mathrm{E}[\widehat{\omega}(t)|\omega(s), s \leq t]$ and the true frequency trajectory $\omega(t)$ can be approximately described by the following linear equation

$$\bar{\omega}(t) \cong F(q^{-1})\omega(t) \qquad (4)$$

$$F(q^{-1}) = \frac{(1 - \delta_o)q^{-1}}{1 - (\lambda_o + \delta_o)q^{-1} + \lambda_o q^{-2}}$$

where q^{-1} denotes the backward shift operator and $\lambda_o = 1 - \mu_o$, $\delta_o = 1 - \gamma_o$.

Since, for small values of μ_o and γ_o, $F(q^{-1})$ is a lowpass filter, it introduces the lag effect: the mean trajectory of frequency estimates yielded by the ANF filter (2) can be regarded a delayed version of the true trajectory. As it was shown in [7], a pretty good approximation of this delay can be obtained using the formula $l_o = \mathrm{int}[\tau_o]$, where τ_o denotes the so-called nominal (low-frequency) delay of the filter $F(e^{-j\xi}) = A(\xi)e^{j\phi(\xi)}$

$$\tau_o = -\lim_{\xi \mapsto 0} \frac{d\phi(\xi)}{d\xi} = -\lim_{\xi \mapsto 0} \frac{\phi(\xi)}{\xi} = \frac{\mu_o}{\gamma_o} \qquad (5)$$

where ξ denotes the standard Fourier-domain frequency variable.

2.2. Variable gain case

When the adaptation gains are time-varying the estimation delay τ is also a time-dependent quantity. Quite obviously, it cannot be evaluated using the steady state formula (5), based on frequency domain concepts. In order to derive the time-varying counterpart of (5), note that equation (4) can be rewritten in the form

$$\bar{\omega}(t) \cong \sum_{i=1}^{\infty} f(i)\omega(t - i) \qquad (6)$$

where $f(i) = \mathcal{Z}^{-1}[F(z^{-1})]$ is the impulse response of the filter $F(q^{-1})$.

Denote by $w(t) = \omega(t) - \omega(t - 1)$ the one-step frequency change and consider the situation where the instantaneous frequency varies linearly with time (linear chirp signal), that is $\omega(t) = \alpha t$ or equivalently $w(t) \equiv \alpha$, $\forall t$. Then, according to (6), it holds that $\bar{\omega}(t) \cong \alpha t - \alpha \sum_{i=1}^{\infty} if(i) = \omega(t - \tau_o)$ where

$$\tau_o = \sum_{i=1}^{\infty} if(i) \qquad (7)$$

Using elementary properties of the $\mathcal{Z}$-transform one can show equivalence of (5) and (7). Hence, the nominal delay, originally defined in the frequency domain, can be also justified using the time-domain concepts – τ_o can be regarded a mean (steady state) delay of the estimated chirped frequency. Using such interpretation the concept of an estimation delay can be easily extended to the time-varying case, leading to the following definition

$$\tau(t) = \left[\frac{\omega(t) - \bar{\omega}(t)}{\alpha}\right]_{\omega(t) = \alpha t} = [\omega(t) - \bar{\omega}(t)]_{w(t) \equiv 1} \qquad (8)$$

Suppose that the true signal evolves according to

$$s(t) = (1 + \Delta r(t))e^{j\omega(t)}s(t - 1)$$

where the quantity $\Delta r(t)$ is real-valued and denotes small relative changes in the magnitude of the cisoid. Note that, unlike the constant-modulus model adopted in [1], such description admits both amplitude and frequency changes. Furthermore, suppose that both adaptation gains are kept at constant levels μ_o and γ_o until the instant t_o, when the gain-tuning mechanism is switched on. We will assume that t_o is large enough to guarantee that the ANF algorithm reaches its steady state behavior. The following proposition summarizes the main result of the paper

Proposition

The estimation delay $\tau(t)$ introduced by the ANF algorithm (2) equipped with time-varying adaptation gains can be evaluated recursively using the following equations

$$\begin{aligned} \eta(t) &= [1 - \mu(t)]\eta(t - 1) - [1 - \mu(t)]\tau(t) \\ \tau(t + 1) &= [1 - \gamma(t)]\tau(t) + \gamma(t)\eta(t - 1) + 1 \end{aligned} \qquad (9)$$

$$t \geq t_o$$

with initial conditions set to $\mu(t_o) = \mu_o$, $\gamma(t_o) = \gamma_o$, $\tau(t_o) = \mu_o/\gamma_o$ and $\eta(t_o) = (\mu_o - 1)/\gamma_o$.

Derivation: See Appendix.

3. IMPROVED ANF ALGORITHM

As argued in [6] setting $\gamma = \mu^2$ may be a good way of reducing the number of design degrees of freedom of an ANF algorithm from two (μ, γ) to one (μ). Let

$$[x]_a^b = \begin{cases} a & \text{if} & x < a \\ x & \text{if} & a \leq x \leq b \\ b & \text{if} & x > b \end{cases}$$

The debised version of the self-optimizing ANF filter proposed in [6] can be summarized as follows

pilot filter:

$$\varepsilon(t) = y(t) - e^{j\widehat{\omega}(t)}\widehat{s}(t-1)$$

$$\zeta(t) = -e^{j\widehat{\omega}(t)}[j\chi(t)\widehat{s}(t-1) + \psi(t-1)]$$

$$\psi(t) = \varepsilon(t) - [1 - \mu(t-1)]\zeta(t)$$

$$\rho(t) = \mathrm{Im}\left\{ \frac{e^{j\widehat{\omega}(t)}}{\widehat{s}^*(t-1)} \left[\zeta^*(t) + j\varepsilon^*(t)\chi(t) \right. \right.$$
$$\left. \left. - \frac{\varepsilon^*(t)\psi(t-1)}{\widehat{s}^*(t-1)} \right] \right\}$$

$$r(t) = \beta r(t-1) + |\zeta(t)|^2$$

$$\mu(t) = \left[\mu(t-1) - \frac{\mathrm{Re}[\varepsilon(t)\zeta^*(t)]}{r(t)} \right]_{\mu_{\min}}^{\mu_{\max}}$$

$$\widehat{s}(t) = e^{j\widehat{\omega}(t)}\widehat{s}(t-1) + \mu(t)\varepsilon(t)$$

$$g(t) = \mathrm{Im}\left[\frac{\varepsilon^*(t)e^{j\widehat{\omega}(t)}}{\widehat{s}^*(t-1)} \right]$$

$$\widehat{\omega}(t+1) = \widehat{\omega}(t) - \mu^2(t)g(t)$$

$$\chi(t+1) = \chi(t) - \mu(t)[2g(t) + \mu(t)\rho(t)] \qquad (10)$$

computation of estimation delay:

$$\eta(t) = [1 - \mu(t)]\eta(t-1) - [1 - \mu(t)]\tau(t)$$

$$\tau(t+1) = [1 - \mu^2(t)]\tau(t) + \mu^2(t)\eta(t-1) + 1$$

$$l(t+1) = [\mathrm{int}[\tau(t+1)]]_{l_{\min}}^{l_{\max}} \qquad (11)$$

frequency-guided filter:

$$k = t - l_{\max}$$

$$\bar{\omega}(k) = \widehat{\omega}(i), \text{ where } i \in [k,t] \text{ obeys } i - l(i) = k$$

$$\bar{\varepsilon}(k) = y(k) - e^{j\bar{\omega}(k)}\bar{s}(k-1)$$

$$\bar{s}(k) = e^{j\bar{\omega}(k)}\bar{s}(k-1) + \mu(k)\bar{\varepsilon}(k) \qquad (12)$$

where $0 < \beta < 1$ in (10) is the forgetting constant which decides upon the estimation memory of the RPE estimator of μ. Note that the algorithm is equipped with two "safety valves". First, when the calculated value of μ exceeds its upper limit, it is truncated to $\mu_{\max}$; similarly, μ is set to $\mu_{\min}$ whenever the calculated value becomes too close to zero. Second, the computed value of $l(t)$ is constrained to the range $[l_{\min}, l_{\max}]$, where $l_{\min} = \mathrm{int}[1/\mu_{\max}]$ and $l_{\max} = \mathrm{int}[1/\mu_{\min}]$ are the steady state estimation delays corresponding to the adopted values of $\mu_{\max}$ and $\mu_{\min}$, respectively.
The initial conditions should be set to $\{\mu(t) = \mu_o, \eta(t) = (\mu_o - 1)/\mu_o^2, \tau(t) = 1/\mu_o, \forall\ t \leq t_o\}$.
The inverse frequency mapping problem $\bar{\omega}(k) \mapsto \widehat{\omega}(i)$, solved in the second line of (12), may occasionally get underdetermined, which happens when no value of i exists such that
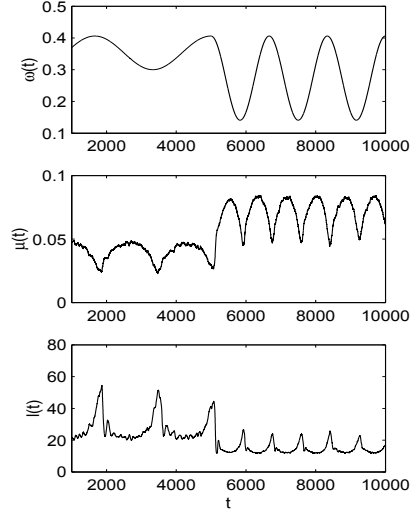

Fig. 1. True frequency changes (upper plot) and the mean values of $\mu(t)$ (middle plot) and $l(t)$ (lower plot) yielded by the proposed ANF filter.

$i - l(i) = k$, or overdetermined, which takes place when there are several values of i such that $i - l(i) = k$. The first difficulty can be overcome using linear interpolation, and the second one – by means of averaging.

4. COMPUTER SIMULATIONS

The simulated signal consisted of a single noisy cisoid $y(t) = ae^{j\sum_{s=1}^{t}\omega(s)} + v(t)$ with a constant amplitude $a = 1$ and a time-varying frequency. Four noise levels were considered ($\sigma_v^2 = 1, 1/\sqrt{10}, 0.1$ and 0.01) to check estimation efficiency of the compared ANF filters under different SNR conditions (SNR=0dB, 5dB, 10dB and 20dB, respectively). The forgetting constant β was set to 0.99 and the limiting values of μ were equal to $\mu_{\min} = 0.005$ and $\mu_{\max} = 0.2$, respectively.
Figure 1 shows evolution of the true instantaneous frequency and evolution of the mean values of the adaptation gain $\mu(t)$ and estimation delay $l(t)$ yielded by the algorithm (10)-(12). All averages were computed from the results of 50 simulation runs, corresponding to different realizations of $\{v(t)\}$ (SNR=5dB). Frequency estimates observed in a typical simulation experiment are displayed in Figure 2. Note how debiasing improves estimation accuracy of the pilot filter.
Table I shows comparison of the average mean-squared signal reconstruction errors yielded by the self-optimizing pilot ANF filter (10), by the debiased frequency-guided filter (12) and by four variants of the fixed-gain ANF filters. All results were averaged over time and 50 different realizations of the measurement noise. Note that debiasing improves results yielded by the pilot filter by approximately 27%, and that both self-optimizing algorithms work better than any of the fixed-gain algorithms.
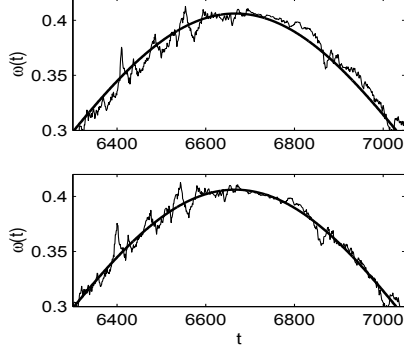
Fig. 2. True frequency changes (thick lines) and their estimates (thin lines) obtained using the pilot ANF filter ($\widehat{\omega}(t)$, upper plot) and its debiased version ($\bar{\omega}(t)$, lower plot).

$$\times 10^{-1}$$

μ	0 dB	5 dB	10 dB	20 dB
pilot	0.471	0.182	0.070	0.011
debiased	0.343	0.132	0.051	0.008
0.02	2.455	2.359	2.327	2.312
0.04	0.559	0.354	0.288	0.261
0.06	0.504	0.195	0.097	0.056
0.08	0.621	0.207	0.076	0.022

Table 1. Average values of the signal reconstruction errors observed for the pilot ANF filter, for the debiased frequency-guided ANF filter and for constant-gain ANF filters with four values of μ.

Appendix

Let $\Delta\widehat{\omega}(t) = \widehat{\omega}(t) - \omega(t)$ and $\Delta\widehat{s}(t) = \widehat{s}(t) - s(t)$. Our derivation of (9) will be based on the ALF approximation, which means that we will examine dependence of $\Delta\widehat{\omega}(t)$ and $\Delta\widehat{s}(t)$ on $v(t)$, $w(t)$ and $\Delta r(t)$, neglecting higher than first-order terms of all quantities listed above (including all cross-terms).

Using the approximation $e^{j\Delta\widehat{\omega}(t)} \cong 1 + j\Delta\widehat{\omega}(t)$, which holds for small frequency estimation errors, and neglecting all higher-order terms mentioned above, one arrives at

$$\Delta\widehat{s}(t) \cong \lambda(t)e^{j\omega(t)}\Delta\widehat{s}(t-1) + j\lambda(t)s(t)\Delta\widehat{\omega}(t)$$

$$-\lambda(t)\beta(t)\Delta r(t) + \mu(t)v(t)$$

where $\lambda(t) = 1 - \mu(t)$.
Let $\Delta\psi(t) = \text{Im}[\Delta\widehat{s}(t)/s(t)]$. After dividing both sides of the last equation by $s(t)$, and taking imaginary parts, one obtains

$$\Delta\psi(t) \cong \lambda(t)\Delta\psi(t-1) + \lambda(t)\Delta\widehat{\omega}(t) + \mu(t)z(t) \quad (13)$$

where $z(t) = \text{Im}[v(t)/s(t)]$.

A similar technique can be used to cope with the frequency update in (2), leading (after elementary but tedious calculations) to

$$g(t) \cong \Delta\widehat{\omega}(t) + \Delta\psi(t-1) - z(t)$$

and consequently to

$$\begin{aligned}\Delta\widehat{\omega}(t+1) &\cong \delta(t)\Delta\widehat{\omega}(t) - \gamma(t)\Delta\psi(t-1) \\ &+ \gamma(t)z(t) - w(t+1)\end{aligned} \quad (14)$$

Note that neither (13) nor (14) depends on $\Delta r(t)$.
According to (8) it holds that $\tau(t) = -\text{E}[\Delta\widehat{\omega}(t)|w(t) \equiv 1]$. Let $\eta(t) = \text{E}[\Delta\psi(t)|w(t) \equiv 1]$. Taking expectations of both sides of (13) and (14), and noting that the process $\{z(t)\}$ is zero-mean, one arrives at (9). The initial conditions correspond to the steady state solution of (9) under $\mu(t) \equiv \mu_o$ and $\gamma(t) \equiv \gamma_o$.

5. REFERENCES

[1] M. Niedźwiecki and P. Kaczmarek, "Tracking analysis of a generalized adaptive notch filter," *IEEE Trans. on Signal Processing*, vol. 54, pp. 304–314, 2006.

[2] M. Niedźwiecki and P. Kaczmarek, "Generalized adaptive notch filters," in *Proc. 2004 IEEE Int. Conf. on Acoustics, Speech and Signal Proc.*, Montreal, Canada, 2004, pp. II-657–II-660.

[3] B. Farhang-Boroujeny, "An IIR adaptive line enhancer with controlled bandwidth," *IEEE Trans. on Signal Processing*, vol. 45, pp. 477–481, 1997.

[4] V. DeBrunner and S. Torres, "Multiple fully adaptive notch filter design based on allpass sections," *IEEE Trans. on Signal Processing*, vol. 48, pp. 550–552, 2000.

[5] Y.-C. Lim , Y.-X. Zou and N. Zheng, "A piloted adaptive notch filter," *IEEE Trans. on Signal Processing*, vol. 53, pp. 1310–1323, 2005.

[6] Niedźwiecki, M. and P. Kaczmarek, "Self-optimizing adaptive notch filters – comparison of three optimization strategies," in *Proc. 14th European Signal Processing Conference*, Florence, Italy, 2006, pp. Fri.4.1: 1–5.

[7] M. Niedźwiecki and A. Sobociński, "A simple way of increasing estimation accuracy of generalized adaptive notch filters," *IEEE Signal Processing Letters*, vol. 14, no. 3, 2007.

[8] P. Tichavský and P. Händel, "Two algorithms for adaptive retrieval of slowly time-varying multiple cisoids in noise," *IEEE Trans. on Signal Processing*, vol. 43, pp. 1116–1127, 1995.