WAVELET-PACKET-BASED ADAPTIVE ALGORITHM FOR SPARSE IMPULSE RESPONSE IDENTIFICATION

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ABSTRACT

This paper proposes a wavelet-packet-based (WPB) algorithm for efficient identification of sparse impulse responses with arbitrary frequency spectra. The discrete wavelet packet transform (DWPT) is adaptively tailored to the energy distribution of the unknown system's response spectrum. The new algorithm leads to a reduced number of active coefficients and to a reduced computational complexity, when compared to competing wavelet-based algorithms. Simulation results illustrate the applicability of the proposed algorithm.

Index Terms— Adaptive systems, echo cancellation, sparse impulse response.

1. INTRODUCTION

One of the main challenges in adaptive filtering is to deal with a large number of coefficients, which causes a high computational complexity and a low convergence speed. For sparse impulse responses, however, the significant coefficients of the response to be identified (those that are larger than a specified threshold) can be detected prior to adaptation. Then, only a small number of coefficients must be adapted, reducing the computational complexity and increasing the speed of convergence of the adaptive algorithm.

Some works have shown improvements in the convergence speed of adaptive filtering algorithms when implemented in the wavelet transform (WT) domain, as compared to the implementations in the time domain [1,2]. In these works, the WT has been used only to reduce the time correlation of the input signal. The important wavelet temporal hierarchy was not utilized. Other works aim at the reduction of the number of coefficients that are effectively adapted [3–5]. From these, [4] and [5] make use of the wavelet temporal hierarchy. In [4], two short adaptive filters are used, one in the partial Haar domain to estimate the impulse response bulk delay and one in time domain to adapt over a time window centered about the delay previously estimated. As the bulk delay is estimated from the location of the peak of the impulse response, this approach is effective when the unknown sparse response has only one nonzero region. The approach in [5] uses the coefficients of a selected scale of the wavelet transform as a set of control coefficients which are always adapted. The control coefficients are compared with a threshold to determine the active ones. Then, the significant coefficients of the remaining scales are determined to be those corresponding to the significant coefficients in the control scale according to the temporal hierarchy of the wavelet transform. This approach works with the entire impulse response. Thus, it can be used to identify sparse responses with more than one active region. One limitation of the technique proposed in [5] is that it is designed for sparse responses which are rich in frequency content. This is mainly because of the need for the choice of a fixed control scale. As a consequence, the technique may lead to poor results for some impulses responses. Examples can be found, for instance, among the typical responses included in [6].

Wavelets divide the frequency band in subbands that enhance the low frequency components of a signal. Wavelet packets generalize the wavelet theory, allowing efficient frequency-domain representations of signal with any spectral content. Thus, wavelet packet expansions can be tailored to the frequency spectrum of the signal [7,8].

This paper proposes a wavelet-packet-based solution to the identification of sparse impulse responses. The derivation of the new technique had the following objectives: 1) To enable the identification of sparse systems with impulse responses having any profile of energy distribution in frequency; 2) To reduce the number of adaptive coefficients to levels close to the actual number of nonzero samples of the response to be identified; 3) To keep the computational complexity at levels compatible with the competing algorithms. The resulting algorithm incorporates the design of the wavelet packet structure in the adaptive process. The choice of subbands for the structure are based on estimates of the energy content of the unknown response in each candidate subband. The frequency bands emphasized by the structure are then tailored to the spectrum of the response to be identified. The adaptive design strategy leads to a reduced number of active coefficients, as thus to a low computational complexity. Simulation results illustrate the effectiveness of the proposed technique.

The paper is organized as follows. Section 2 presents the proposed strategy for the on-line definition of the DWPT structure and describes the adaptation process. Section 3 compares the computational complexity of the proposed technique with that of [5]. Section 4 presents simulation results as Section 5 concludes the paper.

2. ADAPTATION STRATEGY

The proposed adaptation strategy is composed of two phases. Phase 1 includes the construction of the DWPT, a first reduction in the number of active coefficients and adaptation of these coefficients. Phase 2 is dedicated to the activation or deactivation of coefficients and to the adaptation of the active coefficients for the DWPT defined in Phase 1.

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2.1. Phase 1: Construction of the DWPT

This section describes the construction of the DWPT, which is schematically depicted in Fig. 1 for a three-level transform. In this figure $\tilde{y}(n)$ (output of the adaptation system) is the estimate of the desired signal $y(n)$, which is the output of the unknown system, and $e(n)$ is the error between the desired signal and its estimate $\tilde{y}(n)$.

Given an $N-1$ input vector $\boldsymbol{x}(n)$ with $N = 2^M$, the first step in Phase 1 is to transform $\boldsymbol{x}(n)$ using a partial DWPT with just one level. This is done by two filters $\boldsymbol{H}_L$ (low pass) and $\boldsymbol{H}_H$ (high pass). The two $N/2-1$ transformed vectors are denoted $\boldsymbol{z}_{H_1}$ and $\boldsymbol{z}_{L_1}$, where H and L stand for high and low frequency bands, respectively, and the subscript 1 indexes the DWPT level.

The next step is called the first adaptation interval (AI_1), and is itself comprised of three steps. In the first step, two $N/2$-long adaptive filters with weight vectors $\boldsymbol{w}_{H_1}$ and $\boldsymbol{w}_{L_1}$ are adapted for some iterations. This generates a set of rough estimates of the optimum weights. In the second step, the weights in $\boldsymbol{w}_{H_1}$ and $\boldsymbol{w}_{L_1}$ are compared against a threshold. Those larger than the threshold are considered to be active weights. The third step is a new adaptation cycle with several iterations in which only the active weights just determined are adapted. These active weights now form vectors $\tilde{\boldsymbol{w}}_{H_1}$ and $\tilde{\boldsymbol{w}}_{L_1}$, which are excited by the pruned signal vectors $\tilde{\boldsymbol{z}}_{H_1}$ and $\tilde{\boldsymbol{z}}_{L_1}$. This concludes the adaptation interval AI_1, which is realized only once for the entire identification process.

The third step of Phase 1 generates the next level of the DWPT, in which only one of the last set of subbands in again subdivided. In order to decide which subband will be subdivided, the portions of the sparse response's energy in each subband are estimated from the weights of $\tilde{\boldsymbol{w}}_{H_1}$ and $\tilde{\boldsymbol{w}}_{L_1}$. The band with more energy is subdivided. The adaptive filter weights corresponding to the lower energy subband are migrated into a new adaptive vector $\tilde{\boldsymbol{w}}_a$, which is excited by signal vector $\tilde{\boldsymbol{z}}_a$ with the corresponding signal samples. Weight vector $\tilde{\boldsymbol{w}}_a$ will always be adapted from this point on. The higher energy subband is subdivided by a new set of filters $\boldsymbol{H}_L$ and $\boldsymbol{H}_H$. This leads to new transformed signal vectors $\tilde{\boldsymbol{z}}_{H_2}$ and $\tilde{\boldsymbol{z}}_{L_2}$ at the output of the second level of the DWPT and corresponding weight vectors $\tilde{\boldsymbol{w}}_{H_2}$ and $\tilde{\boldsymbol{w}}_{L_2}$. In this third step no adaptive weights are deactivated. After a new adaptation interval (AI), a new decision is made about which subband will be split, and the process described in this paragraph is repeated for the new level of the DWPT until the complete DWPT structure is constructed. Fig. 1 illustrates a possible structure for $M = 3$ ($N = 8$). This concludes Phase 1.

2.2. Phase 2: Adaptation

In this phase, the adaptive weights defined by the DWPT constructed in Phase 1 are iteratively adapted and activated/deactivated according to the temporal hierarchy of the DWPT. Fig. 2 illustrates this temporal hierarchy for a 5-level DWPT ($M = 5$). The horizontal direction shows the distribution in time. The vertical direction shows the 5 levels ($m = 1, \dots, 5$). Each rectangle corresponds to the region of greater influence of a transformed coefficient. As an illustration of the temporal hierarchy, the dark rectangles in Fig. 2 belong to the set of coefficients in the same temporal hierarchy of element (3,2) (marked with *).

In this phase, the WPB algorithm uses the DWPT to process the input signal $\boldsymbol{x}$ to determine the elements of the reduced transformed vector $\tilde{\boldsymbol{z}}_a$ that will excite the adaptive filter with weight vector $\tilde{\boldsymbol{w}}_a$.

Phase 2 proceeds iteratively as follows. At the beginning of the phase or whenever the level m is equal to 1, the non-significant coefficients (smaller than the threshold) in level 1 are deactivated. Then, the coefficients of level $M-1$ in the same temporal hierarchy of the

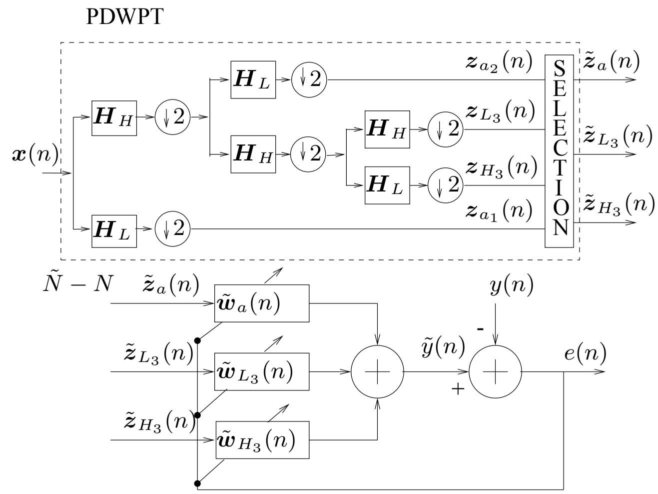

Fig. 1. Block diagram with 3 levels.

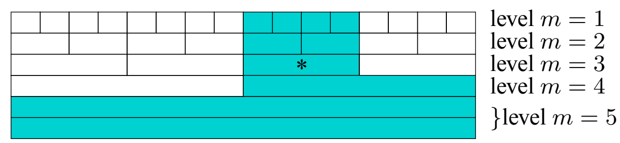

Fig. 2. Hierarchic structure of the DWPT-Haar.

active coefficients of level M are activated. For other levels $m \neq 1$, the small adaptive weights of level m are deactivated and the adaptive weights of level $m-1$ in the same temporal hierarchy of the active weights of level m are activated. The levels m are continuously cycled in regressive order ($M-1, \dots, 1$) starting at $m = 1$. For each value of m, the adaptive weight activation/deactivation at levels m and $m-1$ is followed by an adaptation interval for adaptive weights updating.

2.3. Adaptive Filtering

The algorithm WPB can be applied with any adaptive algorithm. Here we follow [5] and present the adaptation strategy for the normalized least mean squares (NLMS) algorithm. Fig. 3 shows a typical block diagram of the adaptive system for Phase 1 of the WPB algorithm.

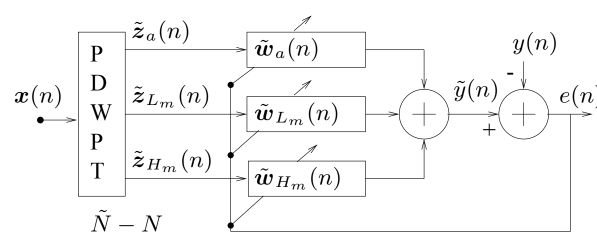

Fig. 3. Block diagram of the 1st phase of WPB.

Considering the already defined variables, the output of the adaptive system is given by $\tilde{y}(n) = \tilde{\boldsymbol{z}}^T(n)\tilde{\boldsymbol{w}}(n)$ where $\tilde{\boldsymbol{z}}(n) = [\tilde{\boldsymbol{z}}_a^T(n), \tilde{\boldsymbol{z}}_{L_m}^T(n), \tilde{\boldsymbol{z}}_{H_m}^T(n)]^T$, $\tilde{\boldsymbol{w}}(n) = [\tilde{\boldsymbol{w}}_a^T(n), \tilde{\boldsymbol{w}}_{L_m}^T(n), \tilde{\boldsymbol{w}}_{H_m}^T(n)]^T$, and the error is given by $e(n) = y(n) - \tilde{y}(n)$. The weight updating equations are [5]:

$\tilde{\boldsymbol{w}}_a(n+1) = \tilde{\boldsymbol{w}}_a(n) + 2\bar{\lambda}\Lambda_a^{-2}(n)e(n)\tilde{\boldsymbol{z}}_a(n)$
$\tilde{\boldsymbol{w}}_{L_m}(n+1) = \tilde{\boldsymbol{w}}_{L_m}(n) + 2\bar{\lambda}\lambda_{L_m}^{-2}(n)e(n)\tilde{\boldsymbol{z}}_{L_m}(n)$
$\tilde{\boldsymbol{w}}_{H_m}(n+1) = \tilde{\boldsymbol{w}}_{H_m}(n) + 2\bar{\lambda}\lambda_{H_m}^{-2}(n)e(n)\tilde{\boldsymbol{z}}_{H_m}(n)$

where $\Lambda_a^2(n)$ is a diagonal matrix whose elements (k,k) are estimates of the power of the transformed input signal $\boldsymbol{z}_a$. The diagonal element $\lambda_{a_m}^2(n)$ is determined through the exponential averaging [5] $\lambda_{a_m}^2(n) = (1-\lambda)\lambda_{a_m}^2(n-1) + \lambda z_{a_{m,1}}^2(n)$, $0 < \lambda < 1$, where $m = 1, 2, \text{—}, (M'+1)$ with $M' < M$ being the level corresponding to the present adaptive interval. λ is a smoothing factor that depends of the stationarity of the input signal, and $z_{a_{m,1}}(n)$ is the first element of the transformed input vector for level m [5].

In Phase 2, the adaptive weight vector $\tilde{\boldsymbol{w}}_a(n)$ contains the active coefficients of all the levels, as illustrated in Fig. 4.

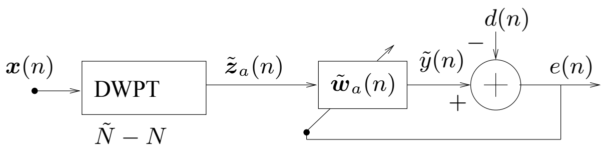

Fig. 4. Block diagram for the 2nd phase of WPB.

Using NLMS, the desired and the error signals are given by $\tilde{y}(n) = \tilde{\boldsymbol{z}}_a^T(n)\tilde{\boldsymbol{w}}_a(n)$ and $e(n) = y(n) - \tilde{y}(n)$, respectively. The weight update equation is

$$\tilde{\boldsymbol{w}}_a(n+1) = \tilde{\boldsymbol{w}}_a(n) + 2\bar{\lambda}\Lambda_a^{-2}(n)e(n)\tilde{\boldsymbol{z}}_a(n),$$

where $\Lambda_a^2(n)$ is a diagonal matrix whose elements are estimates of the input signal power.

3. COMPUTATIONAL COMPLEXITY

After each AI, a variable number of operations is performed to decide the new subband splitting and to determine the significant coefficients. This number is very small, as compared to the computational complexity of the complete identification, and can be neglected in the analysis of the computational complexity [5]. Considering a filter bank with $N = 2^M$ coefficients implementing low-pass and high-pass Haar filters, defining an operation as a multiplication or an addition, and defining $\tilde{N}$ as the number of weights effectively adapted, the computational complexity of Phase 1 of the WPB algorithm as a function of the number of levels is given by:

1 level: $(4\tilde{N}+13)$ operations and 2 divisions;
2 levels: $(4\tilde{N}+21)$ operations and 3 divisions;
$(M-1)$ levels: $(4\tilde{N}+8M-3)$ operations and M divisions.

In Phase 2, the complete DWPT must be considered. For M levels, the DWPT-Haar requires $4M$ operations, the normalization for M levels requires $4(M+1)$ operations and the adaptive weight updating requires $(4\tilde{N}+1)$ operations and $(M+1)$ divisions. Thus, the computational complexity of Phase 2 of the WPB-Haar algorithm is $(4\tilde{N}+8M+5)$ operations and $(M+1)$ divisions. Equivalent processing using the technique in [5] requires $(4\tilde{N}+8M)$ operations and M divisions (including the filtering operation by the adaptive filter).

The calculations above did not include the comparisons with the threshold whenever a decision to deactivate coefficients is required. The number of such comparisons is variable, depending on the number of active coefficients in each level and at each AI.

4. SIMULATION

This section compares results obtained using the proposed WPB algorithm with results obtained using the Haar-Basis (HB) [5], Haar TD-LMS (wavelet transformed NLMS) and NLMS (time-domain) algorithms. The simulations use sparse echo channel models from [6]. The detection threshold used was $TH = \lambda_{fa}\sqrt{\frac{\bar{\lambda}\tilde{\lambda}(k)}{\tilde{\lambda}_\delta^2(k)}}$ [5], where $\tilde{\lambda}(k) = (1-\bar{\lambda})\tilde{\lambda}(k-1) + \bar{\lambda}e^2(k)$ is an estimate of the current value of $E[e^2(k)]$. $\lambda_{fa} = 3,86$ and $\lambda_{fa} = 0,77$ were used in Phase 1 and Phase 2 of WPB, respectively. The algorithm HB uses the probability of false alarm $P_{fa} = 0,01$ (1%), which corresponds to $\lambda_{fa} = 2,57$ for a Gaussian distribution about the mean.

In Phase 1 of WPB, the AI's were defined to facilitate the construction of the DWPT. The AI_1has to different adaptation intervals. In the first band splitting, the AI was set to 6000 iterations. The selection interval within AI_1had 2000 iterations. All remaining AI's intervals of Phase 1 were set to $\frac{1}{16\bar{\lambda}}$ iterations. In Phase 2 of WPB, the AI was the same used for HB in [5], given by $AI = 1/4\bar{\lambda}$, where $\bar{\lambda}$ is the NLMS step size.

Because the value of the smoothing factor λ was not specified for algorithm HB in [5], $\lambda = 0,001$ was used in all HB algorithm implementations. This value led to the simulation results shown in [5]. The control set used for the HB algorithm corresponded to level 3 (parameter $\lambda = 7$ in [5]).

The input $\boldsymbol{x}(n)$ was generated using [5, Eq. (29)]. The interference noise power was -40dB. The step sizes used were $\bar{\lambda}_{NLMS} = \frac{1}{8}$ for the NLMS algorithm, $\bar{\lambda}_{TD-LMS} = \frac{1}{8N}$ ($N = 512$ is the number of adaptive weights) for the Haar TD-LMS algorithm, $\bar{\lambda}_{HB} = \frac{1}{8\hat{N}}$ for the HB algorithm and $\bar{\lambda}_{WPB} = \frac{1}{10\tilde{N}}$ for the WPB algorithm, where $\hat{N}$ and $\tilde{N}$ are the numbers of coefficients effectively adapted by the HB and WPB algorithms, respectively. Fig. 5 and

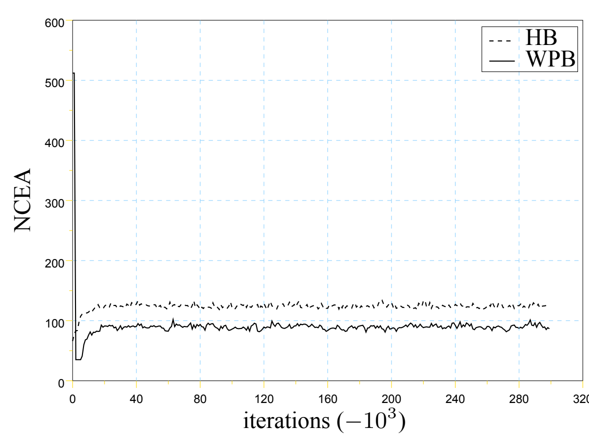

Fig. 5. NCEA of the models gm_1.

Fig. 6 show the number of coefficients effectively adapted (NCEA) by the HB and WPB algorithms for sparse responses gm_1 and gm_7 from [6]. It can be verified that WPB adapts less coefficients for gm_1 and about the same number of coefficients for gm_7. Figs. 7 and Fig. 8 show the corresponding mean-square deviations (MSD), normalized with respect to the squared norm of the response to be identified. Both algorithms present comparable performances for gm_1 (with a slightly better performance the HB algorithm). For

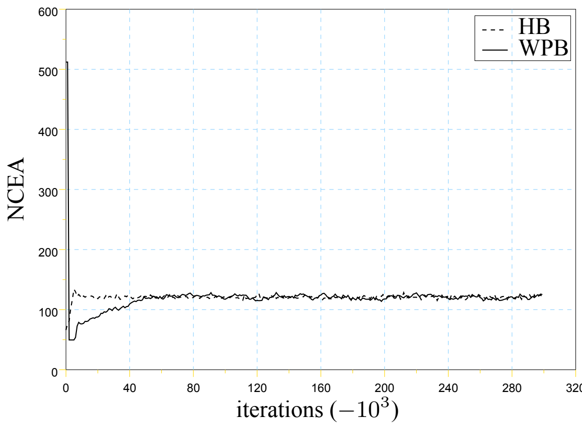

Fig. 6. NCEA of the models gm_7.

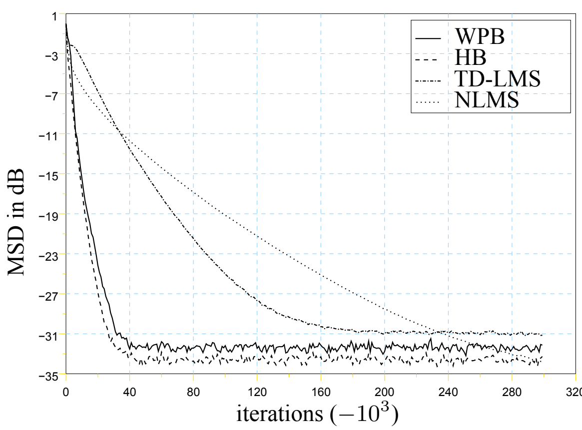

Fig. 7. Normalized MSD of the model gm_1.

response gm_7, however, the HB algorithm leads to a poor result, even though gm_7 satisfies the requirement of rich frequency content. The results clearly show the robustness o the WPB algorithm to the spectral properties of the sparse response to be identified. This is a consequence of the adaptive construction of the DWPT.

5. CONCLUSION

This paper has presented a wavelet-packet-based (WPB) algorithm for identification of sparse impulse responses with arbitrary spectral composition. The proposed strategy adaptively constructs a wavelet-packet structure tailored to the spectrum of the unknown sparse response. The computational complexity of the new technique is comparable to the best existing wavelet-domain solution. The resulting number of effectively adapted coefficients is very close to the minimum. Simulation results have illustrated the robustness of the WPB algorithm to the spectral content of the response to be identified.

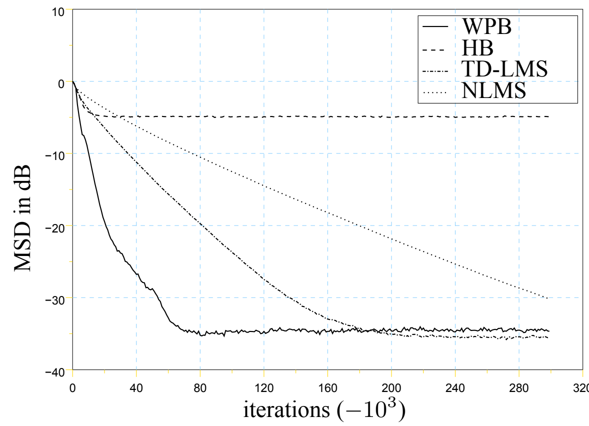

Fig. 8. Normalized MSD of the model gm_7.

6. REFERENCES

[1] Nurgun Erdol and Filiz Basbug, "Wavelet transform based adaptive filters: Analysis and new results," *IEEE Transactions on Signal Processing*, vol. 44, no. 9, pp. 2163 – 2171, Sept 1996.

[2] M. R. Petraglia and J. C. B. Torres, "Performance analysis of adaptive filter structure employing wavelet and sparse subfilters," *IEE Proceedings-Vision, Image and Signal Processing*, vol. 149, no. 2, pp. 115 – 119, April 2002.

[3] M. I. Doroslovacki and Howard Fan, "Wavelet-based linear system modeling and adaptive filtering," *IEEE Trans. on Signal Proc.*, vol. 44, no. 5, pp. 1156 – 1167, May 1996.

[4] N. J. Bershad and A. Bist, "Fast coupled adaptation for sparse impulse responses using a partial Haar transform," *IEEE Trans. on Signal Proc.*, vol. 53, no. 3, pp. 966 – 976, March 2005.

[5] K. C. Ho and S. D. Blunt, "Rapid identification of a sparse impulse response using an adaptive algorithm in the Haar domain," *IEEE Trans. on Signal Proc.*, vol. 51, no. 3, pp. 628–638, March 2003.

[6] *Digital Network Echo Cancellers*, ITU-T Recommendation G. 168, 2004.

[7] R. R. Coifman, Y. Meyer, and M. V. Wickerhauser, *Wavelet analysis and signal processing*, Jones and Barlett, Boston, 1992.

[8] Stphane Mallat, *A wavelet tour of signal processing*, Academic Press, 1998.