ADAPTIVE REDUCED-RANK MMSE PARAMETER ESTIMATION BASED ON AN ADAPTIVE DIVERSITY-COMBINED DECIMATION AND INTERPOLATION SCHEME

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ABSTRACT

This work proposes a low-complexity reduced-rank method for general parameter estimation using an adaptive decimation and interpolation scheme based on diversity-combining. The new approach employs an iterative procedure to jointly optimize the interpolation, decimation and estimation tasks for reduced-rank parameter estimation. We describe joint iterative minimum mean-squared error (MMSE) design filters, propose alternative decimation structures, including the optimal decimation scheme, and develop low-complexity adaptive algorithms for the proposed structure. Simulations for a block equalization application in doubly-selective channels show the remarkable potential of the proposed scheme.

Index Terms— Adaptive estimation, reduced-rank techniques, iterative methods.

1. INTRODUCTION

Reduced-rank estimation is a very powerful technique that has gained considerable attention in the last few years due to its effectiveness in low-sample support situations where it can offer improved convergence performance at an affordable complexity [1]-[7]. The origins of reduced-rank parameter estimation lie in the problem of feature selection encountered in statistical signal processing, which refers to a process whereby a data space is transformed into a feature space, that theoretically has the same dimension of the original data space. It is, however, desirable to devise a transformation in such a way that the data vector can be represented by a reduced number of effective features and yet retain most of the intrinsic information content of the input data [1]. In this context, the existing reduced-rank methods attempt to obtain a low-rank approximation of an observation data vector $\mathbf{r}(i)$ with dimension M, that provides faster acquisition of the signal statistics usually leading to superior convergence and better tracking performance. Amongst the available reducedrank methods, the designer may resort to techniques such as the early eigen-decomposition approaches, the promising multistage Wiener filter (MWF) [4],[5], the auxiliary-vector filtering (AVF) [6] and the flexible adaptive interpolated FIR filters with time-varying interpolators [7].

In this work, we propose a reduced-rank parameter estimator based on a novel adaptive diversity combined interpolation and decimation scheme that is simple, flexible, and provides a remarkable performance advantage over existing techniques. The novel approach consists of an iterative procedure where the interpolation, decimation and estimation tasks

are jointly optimized. In the novel scheme, the number of elements for estimation is substantially reduced, resulting in considerable computational savings and very fast convergence performance for tracking dynamic signals. A unique feature of the proposed method is that, unlike existing schemes, it does not rely on the full-rank covariance matrix \mathbf{R} (that may require a considerable amount of data to be estimated) before projecting the received data onto a reduced-rank subspace. The proposed reduced-rank approach skips the processing stage with \mathbf{R} and directly obtains the subspace of interest through a set of simple interpolation and decimation operations, while the estimator order does not scale with system size. In order to compute the reduced set of parameter estimators of the resulting method, we describe joint iterative MMSE design filters for both interpolator and reduced-rank estimators and propose alternative decimation structures for the proposed scheme. We also develop low-complexity LMS reduced-rank adaptive algorithms for the proposed structure.

This paper is organized as follows. The proposed reducedrank MMSE parameter estimation scheme is described in Section 2. Sections 3 is dedicated to the derivation of the MMSE filter expressions, whereas Section 4 is devoted to adaptive LMS algorithms. Section 5 presents and discusses the simulation results and Section 6 gives the concluding remarks.

2. PROPOSED ADAPTIVE REDUCED-RANK MMSE ESTIMATION SCHEME

The framework of the proposed adaptive reduced-rank MMSE parameter estimation scheme is detailed in this section. Fig. 1 shows the structure of the system, where an interpolator, a decimator unit with several decimation branches and a reduced-rank filter that are time-varying are employed.



Fig. 1. Proposed adaptive reduced-rank filter structure.

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The $M \times 1$ received vector $\mathbf{r}(i) = [r_0^{(i)} \dots r_{M-1}^{(i)}]^T$, where $(\cdot)^T$ denotes transpose, is filtered by the interpolator filter $\mathbf{v}(i) = [v_0^{(i)} \dots v_{N_{I-1}}^{(i)}]^T$, yielding the interpolated received vector $\mathbf{r}_{I}(i)$, which is then decimated by B decimation patterns in parallel, leading to B different $M/L \times 1$ dimensional vectors $\mathbf{\bar{r}}_b(i)$, where L is the decimation factor. The proposed architecture, that employs several decimation branches in parallel to improve parameter estimation, is inspired by the use of receive diversity to improve the reliability of wireless communications links [8]. The novel decimation procedure corresponds to discarding M - M/L samples of $\mathbf{r}_I(i)$ of each set of M received samples with different patterns, resulting in B different decimated vectors $\mathbf{\bar{r}}_b(i)$ with reduced dimension M/L and then computing the inner product of $\mathbf{\bar{r}}_b(i)$ with the $M/L \times 1$ vector of the reduced-rank filter coefficients $\mathbf{\bar{w}}(i) = [\mathbf{\bar{w}}_0^{(i)} \dots \mathbf{\bar{w}}_{M/L-1}^{(i)}]^T$ that minimizes the squared norm of the error signal.

2.1. Adaptive Interpolation and Decimation Structure

The front-end adaptive filtering is carried out by the interpolator filter $\mathbf{v}(i)$ on the received vector $\mathbf{r}(i)$ and yields the interpolated received vector $\mathbf{r}_{\mathrm{I}}(i) = \mathbf{V}^{H}(i)\mathbf{r}(i)$, where $(\cdot)^{H}$ denotes Hermitian transpose and the $M \times M$ convolution matrix $\mathbf{V}^{H}(i)$ with the coefficients of the interpolator is given by

$$\mathbf{V}^{H}(i) = \begin{bmatrix} v_{0}^{(i)} & \dots & v_{N_{\mathrm{I}}-1}^{(i)} & \dots & 0 & 0 & 0\\ 0 & v_{0}^{(i)} & \dots & v_{N_{\mathrm{I}}-1}^{(i)} & \dots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \dots & 0 & \dots & v_{0}^{(i)} \end{bmatrix}.$$
(1)

Let us introduce an alternative way of expressing the vector $\mathbf{r}_{I}(i)$, that will be useful when dealing with the different decimation patterns, through the following equivalence:

$$\mathbf{r}_{\mathrm{I}}(i) = \mathbf{V}^{H}(i)\mathbf{r}(i) = \boldsymbol{\Re}_{\mathrm{o}}(i)\mathbf{v}^{*}(i), \qquad (2)$$

where the $M \times N_I$ matrix with the received samples of $\mathbf{r}(i)$ and that implements convolution is described by

$$\boldsymbol{\Re}_{\mathrm{o}}(i) = \begin{bmatrix} r_{0}^{(i)} & r_{1}^{(i)} & \dots & r_{N_{\mathrm{I}}-1}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{M-1}^{(i)} & r_{M}^{(i)} & \dots & r_{M+N_{\mathrm{I}}-2}^{(i)} \end{bmatrix}.$$
(3)

The decimated interpolated observation vector $\bar{\mathbf{r}}_b(i) = \mathbf{D}_b \mathbf{r}_1(i)$ for branch *b* is obtained with the aid of the $M/L \times M$ decimation matrix \mathbf{D}_b that adaptively minimizes the squared norm of the error at time instant *i*. The matrix \mathbf{D}_b is mathematically equivalent to signal decimation with a chosen pattern on the $M \times 1$ vector $\mathbf{r}_1(i)$, which corresponds to the removal of M - M/L samples of $\mathbf{r}_1(i)$ of each set of M observed samples. In what follows, we present alternative decimation schemes.

2.2. Adaptive Decimation Schemes

Here, we propose an optimal approach and three alternative procedures for designing the decimation unit of the novel reducedrank scheme, where the common framework is the use of

parallel branches with decimation patterns that yield *B* decimated vectors $\bar{\mathbf{r}}_b(i)$ as candidates. Mathematically, the signal selection scheme chooses the decimation pattern \mathbf{D}_b and consequently the decimated interpolated observation vector $\bar{\mathbf{r}}_b(i)$ that minimizes $|e_b(i)|^2$, where $e_b(i) = d(i) - \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}_b(i)$ is the error signal at branch *b*. Once the decimation pattern is selected for the time instant *i*, the decimated interpolated vector is computed as follows $\bar{\mathbf{r}}(i) = \mathbf{D}(i)\mathbf{r}_1(i)$. The decimation pattern $\mathbf{D}(i)$ is selected on the basis of the following criterion:

$$\mathbf{D}(i) = \mathbf{D}_b \text{ when } \mathbf{D}_b(i) = \arg\min_{1 \le b \le B} |e_b(i)|^2, \quad (4)$$

where the optimal decimation pattern \mathbf{D}_{opt} for the proposed scheme with decimation factor L is derived through the counting principle, where we consider a procedure that has M samples as possible candidates for the first row of \mathbf{D}_{opt} and M - m samples as candidates for the following M/L - 1 rows of \mathbf{D}_{opt} , where m denotes the mth row of the matrix \mathbf{D}_{opt} , resulting in a number of candidates equal to

$$\mathbf{B} = \underbrace{M \cdot (M-1) \dots (M-M/L+1)}_{M/L \text{ terms}} = \frac{M!}{(M-M/L)!}.$$
 (5)

The optimal decimation scheme described in (4)-(5) is, however, very complex for practical use because it requires M/Lpermutations of M samples for each symbol interval and carries out an extensive search over all possible patterns. Therefore, a decimation scheme that renders itself to practical and low-complexity implementations is of great interest.

In order to consider a general framework for alternative sub-optimal decimation schemes with decimation factor L and using a finite number of B parallel branches let us describe the following structure:

where m (m = 1, 2, ..., M/L) denotes the *m*-th row and r_m is the number of zeros chosen according to the following proposed alternative decimation patterns:

- A. Uniform (U) Decimation with B = 1. We make $r_m = (m-1)L$ and this corresponds to the use of a single branch on the decimation unit.
- **B.** Pre-Stored (PS) Decimation. We select $r_m = (m-1)L + (b-1)$ which corresponds to the utilization of uniform decimation for each branch *b* out of *B* branches and the different patterns are obtained by picking up adjacent samples with respect to the previous and succeeding decimation patterns.
- C. Random (R) Decimation. We choose r_m as a discrete uniform random variable, which is independent for each row m out of B branches and whose values range between 0 and M 1.

The uniform approach of case A. corresponds to a single branch on the decimation unit, however, one can exploit the processed samples through a more elegant and effective way with the deployment of several branches in parallel. In this regard, the pre-stored decimation of case B. in which the designer utilizes uniform decimation for each branch b and the different patterns are obtained by choosing adjacent samples with respect to the previous and succeeding decimation patterns. This is particular advantageous since it is very simple, consists of sliding patterns in parallel and can be easily implemented by digital signal processors. The random decimation scheme of case C. requires the use of a discrete uniform random generator for producing the B decimation patterns which are employed in parallel. In this regard, r_m does not have to be necessarily changed for each interval, but it can be used for the whole set of data.

3. REDUCED-RANK JOINT ITERATIVE MMSE FILTER DESIGN

Let us describe the MMSE filter design of the proposed reducedrank structure. The strategy, which allows us to devise solutions for both interpolator and receiver, is to express the estimated symbol $x(i) = \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}(i)$ as a function of $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$:

$$\begin{aligned} x(i) &= \bar{\mathbf{w}}^{H}(i)\bar{\mathbf{r}}(i) = \bar{\mathbf{w}}^{H}(i) \big(\mathbf{D}(i)\mathbf{V}^{H}(i)\mathbf{r}(i) \big) \\ &= \bar{\mathbf{w}}^{H}(i) \big(\mathbf{D}(i)\boldsymbol{\Re}_{\mathrm{o}}(i) \big) \mathbf{v}^{*}(i) = \bar{\mathbf{w}}^{H}(i)\boldsymbol{\Re}(i)\mathbf{v}^{*}(i) \quad (7) \\ &= \mathbf{v}^{H}(i) \big(\boldsymbol{\Re}^{T}(i)\bar{\mathbf{w}}^{*}(i) \big) = \mathbf{v}^{H}(i)\mathbf{u}(i), \end{aligned}$$

where $\mathbf{u}(i) = \Re^T(i)\bar{\mathbf{w}}^*(i)$ is an $N_{\mathrm{I}} \times 1$ vector, the M/L coefficients of $\bar{\mathbf{w}}(i)$ and the N_{I} elements of $\mathbf{v}(i)$ are assumed complex and the $M/L \times N_{\mathrm{I}}$ matrix $\Re(i)$ is $\Re(i) = \mathbf{D}(i)\Re_{\mathrm{o}}(i)$.

The MMSE solutions for $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$ can be computed through the optimization problem whose cost function is

$$J_{\text{MSE}}(\bar{\mathbf{w}}(i), \mathbf{v}(i)) = E\Big[|d(i) - \mathbf{v}^{H}(i)\Re^{T}(i)\bar{\mathbf{w}}^{*}(i)|^{2}\Big], \quad (8)$$

where d(i) is the desired symbol at time index (i) and $E[\cdot]$ stands for expected value. By fixing the interpolator $\mathbf{v}(i)$ and minimizing (8) with respect to $\bar{\mathbf{w}}(i)$ the interpolated Wiener filter weight vector is

$$\bar{\mathbf{w}}(i) = \boldsymbol{\alpha}(\mathbf{v}) = \bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i), \qquad (9)$$

where $\mathbf{\bar{R}}(i) = E[\mathbf{\bar{r}}(i)\mathbf{\bar{r}}^{H}(i)]$, $\mathbf{\bar{p}}(i) = E[d^{*}(i)\mathbf{\bar{r}}(i)]$, $\mathbf{\bar{r}}(i) = \Re(i)\mathbf{v}^{*}(i)$ and by fixing $\mathbf{\bar{w}}(i)$ and minimizing (8) with respect to $\mathbf{v}(i)$ the interpolator weight vector is

$$\mathbf{v}(i) = \boldsymbol{\beta}(\bar{\mathbf{w}}) = \mathbf{R}_u^{-1}(i)\mathbf{p}_u(i), \tag{10}$$

where $\mathbf{R}_{u}(i) = E[\mathbf{u}(i)\mathbf{u}^{H}(i)]$, $\mathbf{p}_{u}(i) = E[d^{*}(i)\mathbf{u}(i)]$ and $\mathbf{u}(i) = \mathbf{\Re}^{T}(i)\bar{\mathbf{w}}^{*}(i)$. The associated MSE expressions are

$$J(\mathbf{v}) = J_{\text{MSE}}(\boldsymbol{\alpha}(\mathbf{v}), \mathbf{v}) = \sigma_d^2 - \bar{\mathbf{p}}^H(i)\bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i), \quad (11)$$

$$J_{\text{MSE}}(\bar{\mathbf{w}}, \boldsymbol{\beta}(\bar{\mathbf{w}})) = \sigma_d^2 - \mathbf{p}_u^H(i) \mathbf{R}_u^{-1}(i) \mathbf{p}_u(i), \qquad (12)$$

where $\sigma_d^2 = E[|d(i)|^2]$. Note that points of global minimum of (8) can be obtained by $\mathbf{v}_{opt} = \arg\min_{\mathbf{v}} J(\mathbf{v})$ and $\bar{\mathbf{w}}_{opt} = \boldsymbol{\alpha}(\mathbf{v}_{opt})$ or $\bar{\mathbf{w}}_{opt} = \arg\min_{\bar{\mathbf{w}}} J_{MSE}(\bar{\mathbf{w}}, \boldsymbol{\beta}(\bar{\mathbf{w}}))$ and $\mathbf{v}_{opt} = \boldsymbol{\beta}(\bar{\mathbf{w}}_{opt})$. At the minimum point (11) equals (12) and the MMSE for the proposed structure is achieved. We remark that (9) and (10) are not closed-form solutions for $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$ since (9) is a function of $\mathbf{v}(i)$ and (10) depends on $\bar{\mathbf{w}}(i)$ and thus it is necessary to iterate (9) and (10) with an initial guess to obtain a solution. An alternative iterative MMSE solution is sought via adaptive algorithms in the next section.

4. ADAPTIVE ALGORITHMS

We describe LMS algorithms [9] to estimate the parameters of the reduced-rank filter, the decimation and the interpolator filter. Consider $\mathbf{r}(i)$ and the adaptive processing of the proposed scheme, as in Fig. 1. With the aid of the convolution matrix in (1), we compute $\mathbf{r}_{I}(i)$ and then compute the decimated interpolated observation vectors $\mathbf{r}_{b}(i)$ for the *B* branches with the aid of the decimation patterns \mathbf{D}_{b} , where $1 \leq b \leq B$. Once the *B* candidate vectors $\bar{\mathbf{r}}_{b}(i)$ are computed, we select the vector $\bar{\mathbf{r}}_{b}(i)$ that minimizes the squared norm of

$$e_b(i) = d(i) - \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}_b(i).$$
(13)

Based on the signal selection that minimizes $|e_b(i)|^2$, we choose the corresponding reduced-rank observation vector $\mathbf{\bar{r}}(i)$ and select the error of the proposed LMS algorithm e(i) as the error $e_b(i)$ with smallest squared magnitude of the *B* branches

$$\mathbf{r}(i) = \mathbf{r}_b(i) \text{ and } e(i) = e_b(i)$$

when $b = \arg\min_{1 \le b \le B} |e_b(i)|^2.$ (14)

Given the reduced-rank observation vector $\bar{\mathbf{r}}(i)$ and the desired signal d(i), we consider the following cost function:

$$J_{\rm MSE} = |d(i) - \mathbf{v}^H(i) \mathbf{\Re}^T(i) \bar{\mathbf{w}}^*(i)|^2.$$
(15)

Taking the gradient terms of (15) with respect to $\mathbf{v}(i)$, $\bar{\mathbf{w}}(i)$ and using the gradient descent rules [9] for the interpolator $\mathbf{v}(i+1) = \mathbf{v}(i) - \eta \nabla_{\mathbf{v}} J_{\text{MSE}}(\bar{\mathbf{w}}(i), \mathbf{v}(i))$ and the reducedrank filter $\bar{\mathbf{w}}(i+1) = \bar{\mathbf{w}}(i) - \mu \nabla_{\mathbf{w}} J_{\text{MSE}}(\bar{\mathbf{w}}(i), \mathbf{v}(i))$ yields:

$$\mathbf{v}(i+1) = \mathbf{v}(i) + \eta e^*(i)\mathbf{u}(i), \tag{16}$$

$$\bar{\mathbf{w}}(i+1) = \bar{\mathbf{w}}(i) + \mu e^*(i)\bar{\mathbf{r}}(i), \qquad (17)$$

where $e(i) = d(i) - \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}(i)$, $\mathbf{u}(i) = \Re^T(i)\bar{\mathbf{w}}^*(i)$, μ and η are the step sizes of the algorithm for $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$. The LMS algorithm for the proposed structure described in this section has a computational complexity $O(M/L + N_I)$. In fact, the proposed structure trades off one LMS algorithm with complexity O(M) against two LMS algorithms with complexity O(M/L) and $O(N_I)$, operating simultaneously.

5. SIMULATIONS

In this section we analyze the proposed reduced-rank scheme and LMS algorithms in a block equalization application with doubly-selective channels. We consider the insertion of a cyclic prefix with Q symbols at the transmitter and its removal at the receiver. Let us define the $M \times 1$ QPSK symbol block $\mathbf{s}(i) = [s_0(i) \dots s_{M-1}(i)]^T$, with $s_m(i) \in \{\pm 1, \pm j\}$ and $m = 1, \dots, M$. The $M \times 1$ received block after the removal of the cyclic prefix can be written as

$$\mathbf{r}(i) = \mathbf{H}(i)\mathbf{s}(i) + \mathbf{n}(i) \tag{18}$$

where $\mathbf{n}(i) = [n_1(i) \dots n_M(i)]^T$ is the complex Gaussian noise vector with $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$, and $\mathbf{H}(i)$ is a block convolution matrix that cannot be diagonalized by the Discrete Fourier Transform (DFT) unless the channel can be considered constant over one block interval. In doubly-selective channels, the designer has to consider other alternatives to equalization rather than DFT-based, as pointed out in [10]. We consider linear equalizers with decisions given by $\hat{s}_m =$ $\operatorname{sgn}\{\Re[\bar{\mathbf{w}}_m^H(i)\bar{\mathbf{r}}(i)] + j\Im[\bar{\mathbf{w}}_m^H(i)\bar{\mathbf{r}}(i)]\}, \text{ where } m = 1, \dots, M,$ $\mathrm{sgn}(\cdot)$ is the slicer and the operators $\Re(\cdot)$ and $\Im(\cdot)$ take the real and imaginary parts, respectively. The simulations use Q = 8 with M = 32 or Q = 16 with M = 64, 3-path channels with coefficients obtained with Clarke's model [8] and relative powers given by 0, -3 and -6 dB, where in each run the spacing of paths is obtained from a discrete uniform random variable between 1 and 4 symbols for Q = 16 and between 1 and 2 symbols for Q = 8, and curves are averaged over 100 runs.



Fig. 2. BER performance versus number of decimation branches.

We first evaluate the MSE performance for a scenario with data support of 500 blocks of the proposed scheme for various ranks and number of branches B, the existing MWF and fullrank methods. The MWF uses a rank with D = M/L = 4, i.e. a filter with only 4 taps, whereas the proposed scheme employs an interpolator with only 3 taps and the pre-stored decimation (PS-DEC) scheme, which was the best value obtained in our studies for its length. The results in Fig. 2 show that the proposed system is able to approach the optimal MMSE performance (which assumes known channels and noise variance) as the number of branches is increased.

The bit error rate (BER) performance versus the number of blocks and $E_{\rm b}/N_0$ was evaluated with the proposed decimation schemes, as shown in Fig. 3. The algorithms are trained with 200 blocks and then switch to decision-directed mode. The results indicate that the proposed scheme with the optimal decimation (OPT-DEC) achieves the best performance, followed by the proposed method with the PS-DEC, the random decimation system (R-DEC), the MWF and the full-rank approach. The performance of the uniform decimation (U-DEC) scheme with B = 1 is significantly inferior to



Fig. 3. BER versus (a) number of blocks (b) $E_{\rm b}/N_0$ for 1000 blocks.

those that employ several branches and is slightly inferior to the full-rank method.

6. CONCLUSIONS

We have proposed a low-complexity reduced-rank method for general parameter estimation using an adaptive decimation and interpolation scheme based on diversity-combining. The proposed approach was analyzed for block equalization in doubly-selective channels, and was shown to outperform the best known methods and approach the optimal MMSE estimator.

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