A Weighted Element-wise Block Adaptive Frequency-Domain Equalization in Frequency-selective Time-varying Channels

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***Abstract*— In this paper, a weighted element-wise block adaptive frequency-domain equalization (WEB-FDE) is proposed for a single-carrier system with the cyclic-prefix. In the WEB-FDE, the one-tap equalizer corresponding to a frequency-bin first preserves input DFT elements (element-wise block). Its coefficient in each block is then calculated by minimizing a weighted squared norm of the *a posteriori error*. Simulation results in a time-varying typical urban (TU) channel show that the bit-error-rate (BER) performances of the WEB-FDE outperform those of the normalized least-mean-square (NLMS)-FDE and recursive-least-square (RLS)-FDE.**

***Index Terms-A posteriori error*, frequency-domain equalization (FDE).**

I. Introduction

Minimum-mean-square-error frequency-domain equalization (MMSE-FDE) of the cyclic-prefix-based single-carrier (SC) systems has been expected to be an alternative scheme to the time-domain equalization (TDE) and orthogonal frequency division multiplexing (OFDM) systems [1]. On the other hand, it is noticed that the MMSE-FDE requires an estimation of the channel state information (CSI), which would not be accomplished by a simple technique such as an interpolation technique employed in the OFDM systems. Therefore, further researches on the adaptive scheme to estimate the CSI or an adaptive equalization [3] are required.

In this paper, we present an adaptive FDE which does not require the CSI. In order to improve the adaptive FDE efficiently, we first decompose the transmission of N-symbol blocks of the SC-FDE into N parallel symbol level transmissions in a frequency-domain [1], [3]. This decomposition allows us to calculate independently the one-tap equalizer over a frequency-bin with its input discrete Fourier Transform (DFT) element. Furthermore, the element-wise block adaptive FDE could be achieved with respect to the one-tap. The block adaptive filtering schemes in TDEs have been studied [4], [5]. However, to the best of our knowledge, no previous work has been reported on the block adaptive scheme for the SC-FDE.

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Moreover, the approach of [5] may not be suitable over time-varying channels because an equal weighting of all the past information to calculate the coefficient degrades the tracking ability of channel variations. Motivated by the above facts, we propose a weighted element-wise block adaptive FDE (WEB-FDE). In the WEB-FDE, the correction term of the equalizer coefficient in each block is calculated in a weighted least-square (LS) sense that the weighted squared norm of the *a posteriori error* vector is minimized. Finally, we evaluate the performances of the WEB-FDE in a time-varying typical urban (TU) channel.

In this paper, $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ represent the transpose, complex conjugate, and Hermitian transpose operators, respectively.

II. System and Channel Models

The input symbols of PSK or QAM constellation with symbol duration of T_s are partitioned into blocks of length N. Each block is then preceded by a cyclic-prefix of length N_G to prevent interblock interference (IBI) and makes the linear convolution of the symbols with the channel to a circular convolution. Let us define the k-th N-dimentional input block $\boldsymbol{x}^{(k)} \overset{def}{=} [x^{(k)}(0), x^{(k)}(1), \ldots, x^{(k)}(N-1)]$, where $x^{(k)}(n)$ is the n-th symbol of the k-th block. Then the resultant cyclically extended block is $x^{(k)}(n) = x^{(k)}(N+n)$ for $-N_G \leq n \leq -1$. In order to initialize the equalizer coefficients, training sequence (TS) blocks are also inserted every several blocks. At the transmitter side, the discrete-time block is shaped with a pulse-shaping filter

$$s(t) = \sum_{k=-\infty}^{\infty} \sum_{n=-N_G}^{N-1} x^{(k)}(n) g_T(t - nT_s - kT_B) \qquad (1)$$

where $s(t)$ represents the transmitted continuous-time signal, $g_T(t)$ is a square-root-raised cosine (SRC) filter having a roll-off factor α_f and filter duration T_f, and $T_B = (N + N_G)T_s$ denotes the total duration of the cyclically extended data block. The radio channel is modeled as a frequency-selective time-varying finite channel impulse response (CIR) [6]

$$\hat{h}^{(k)}(\tau) = \sum_{p=1}^{N_p} \alpha_p^{(k)} \delta(\tau - \tau_p) \qquad (2)$$

where $\hat{h}^{(k)}(\tau)$ indicates that the time-varying channel is assumed quasi-static, i.e., constant during the transmission of a block symbol and variable on block-by-block basis. $\delta(t)$ is the Dirac delta function, and N_p denotes the number of resolvable propagation path. $\alpha_p^{(k)}$ and τ_p denote the time-varying complex-valued channel gain corresponding to the k-th transmitted block and a relative delay of the p-th path, respectively. The additive noise $\tilde{n}^{(k)}(t)$ is assumed to be white. At the receiver, assuming perfect frequency and block-timing synchronization, the signal is matched filtered by $g_R(t)$ and sampled at $1/T_s$. After the cyclic-prefix is discarded, the orthonormal DFT matrix $\mathcal{F} = (1/\sqrt{N})e^{-j2\pi il/N}$ (for $0 \le i, l \le N-1$) generates the received k-th N-dimensional signal block, which can be expressed in terms of a frequency-bin as

$$Y^{(k)}(l) = X^{(k)}(l)H^{(k)}(l) + \tilde{N}^{(k)}(l), 0 \le l \le N-1 \quad (3)$$

where $Y^{(k)}(l)$ and $X^{(k)}(l)$ denote the l-th DFT element of the received and transmitted blocks, respectively, and $\tilde{N}^{(k)}(l)$ denotes corresponding noise. $H^{(k)}(l)$ is the l-th DFT coefficient of the channel response. In (3), it is assumed that N_G is larger than or equal to practical channel memory $N_C = \text{int}[(\tau_{max} + T_f)/T_s]$, where τ_{max} is the maximum delay spread of a total CIR, i.e., $h^{(k)}(\tau) = \hat{h}^{(k)}(\tau) \otimes g(\tau)$ ($g(\tau) = g_R(\tau) \otimes g_T(\tau)$). $\otimes$ is a linear convolution operator, and int$[a]$ denotes the maximum integer that is not exceeding a.

III. The Structure and Algorithm of WEB-FDE

The development of the WEB-FDE is accomplished with respect to the l-th tap corresponding to the l-th frequency-bin. The derived algorithm is then equivalently applied for $0 \le l \le N-1$. Let us define that the element-wise block length is B. After the k-th block is received, the input B-dimensional element-wise block $\boldsymbol{Y}^{(k)}(l)$ is defined as

$$\boldsymbol{Y}^{(k)}(l) = [Y^{(k-B+1)}(l), \ldots, Y^{(k-1)}(l), Y^{(k)}(l)] \quad (4)$$

where $Y^{(m)}(l)$ represents the l-th element of the received m-th block, and the elements, $Y^{(m)}(l)$ for $k-B+1 \le m \le k-1$, prior to $Y^{(k)}(l)$ represent the past received elements. Then, the equalizer output vector for (4) is

$$\hat{\boldsymbol{X}}^{(k)}(l) = C^{*(k)}(l)\boldsymbol{Y}^{(k)}(l) \quad (5)$$

where $C^{(k)}(l)$ represents the l-th tap coefficient, and the estimated B-dimensional vector $\hat{\boldsymbol{X}}^{(k)}(l) \overset{def}{=} [\hat{X}^{(k-B+1)}(l), \ldots, \hat{X}^{(k-1)}(l), \hat{X}^{(k)}(l)]$, each of which corresponds to the l-th DFT element of the transmitted k-th block [see (8)-(9)].

Fig. 1 depicts the conceptual structure of the element-wise block adaptive FDE. The principle of the FDE can be explained with respect to filtering, detection and update parts. In the filtering part, the l-th DFT element of each m-th (for $k-B+1 \le m \le k$) receive block is first fed to the l-th tap, and $\hat{\boldsymbol{X}}^{(k)}(l)$ is calculated. The detection part performs an exact inverse function of the filtering part. The elements of $\hat{\boldsymbol{X}}^{(k)}(l)$ for $0 \le l \le N-1$ are rearranged to estimate the transmitted m-th block prior to the corresponding inverse DFT (IDFT). Through the IDFTs and decision devices, all of the B-blocks are estimated. For an algorithm update, these blocks are transformed back to the frequency-domain using corresponding DFTs, where decision blocks are rearranged with the form of (4) to generate error vectors. These error vectors are used to update the tap coefficient according to the following proposed algorithm. After the $(k+1)$-th DFT output is received, $\boldsymbol{Y}^{(k+1)}(l)$ is formulated as (6) by applying a shifting property

$$\boldsymbol{Y}^{(k+1)}(l) = [Y^{(k-B+2)}(l), \ldots, Y^{(k)}(l), Y^{(k+1)}(l)] \quad (6)$$

From this shifting property, it is noticed that only the k-th block out of all of the estimated B-blocks is transferred to the channel decoding block as shown in Fig. 1.

In the next, we describe the coefficient update algorithm of the element-wise block adaptive FDE, which can be formulated as

$$C^{(k+1)}(l) = C^{(k)}(l) + \Delta C^{(k)}(l) \quad (7)$$

where $\Delta C^{(k)}(l)$ represents the correction vector of $C^{(k)}(l)$. Let us define $B \times 1$ *a priori error* vector $\boldsymbol{e}^{(k)}(l)$ and *a posteriori error* vector $\overline{\boldsymbol{e}}^{(k)}(l)$, respectively, as follows

$$\boldsymbol{e}^{(k)}(l) = \boldsymbol{X}^{(k)}(l) - \boldsymbol{Y}^{(k)H}(l)C^{(k)}(l), \quad (8)$$

$$\overline{\boldsymbol{e}}^{(k)}(l) = \boldsymbol{X}^{(k)}(l) - \boldsymbol{Y}^{(k)H}(l)C^{(k+1)}(l) \quad (9)$$

where $\boldsymbol{X}^{(k)}(l) \overset{def}{=} [X^{(k-B+1)}(l), \ldots, X^{(k-1)}(l), X^{(k)}(l)]^H$. By pre-multiplying both sides of (7) with $\boldsymbol{Y}^{(k)H}(l)$ and subtracting $\boldsymbol{X}^{(k)}(l)$ from the both sides, (9) can be rewritten in terms of $\boldsymbol{e}^{(k)}(l)$ and $\Delta C^{(k)}(l)$ as

$$\overline{\boldsymbol{e}}^{(k)}(l) = \boldsymbol{e}^{(k)}(l) - \boldsymbol{Y}^{(k)H}(l)\Delta C^{(k)}(l). \quad (10)$$

The $\Delta C^{(k)}(l)$ in (7) is calculated by applying a weighted least square (LS) criterion defined as

$$J^{(k)}(l) = \overline{\boldsymbol{e}}^{(k)H}(l)\boldsymbol{W}\overline{\boldsymbol{e}}^{(k)}(l) \quad (11)$$

where a squared norm of $\overline{\boldsymbol{e}}^{(k)}(l)$ is attenuated geometrically by $B \times B$ diagonal matrix $\boldsymbol{W} = \text{diag}(\lambda^{B-1}, \ldots, \lambda^0)$ with the forgetting factor λ that satisfies $0 < \lambda \le 1$. By substituting (10) into (11), and then minimizing $J^{(k)}(l)$ with respect to $\Delta C^{*(k)}(l)$, the correction vector $\Delta C^{(k)}(l)$ can be calculated as

$$\Delta C^{(k)}(l) = \frac{\boldsymbol{Y}^{(k)}(l)}{\boldsymbol{Y}^{(k)}(l)\boldsymbol{W}\boldsymbol{Y}^{(k)H}(l)}\boldsymbol{W}\boldsymbol{e}^{(k)}(l). \quad (12)$$

where scalar term $R^{(k)}(l) \overset{def}{=} \boldsymbol{Y}^{(k)}(l)\boldsymbol{W}\boldsymbol{Y}^{(k)H}(l)$, and it can be updated recursively from (4) and (6)

$$R^{(k)}(l) = \lambda R^{(k-1)}(l) + |Y^{(k)}(l)|^2 \quad (13)$$

with $R^{(0)}(l) = \epsilon$ (ϵ is a large constant value). Substituting (12) and (13) into (7), the WEB-FDE algorithm is

$$C^{(k+1)}(l) = C^{(k)}(l) + [R^{(k)}(l)]^{-1}\boldsymbol{Y}^{(k)}(l)\boldsymbol{W}\boldsymbol{e}^{(k)}(l). \quad (14)$$

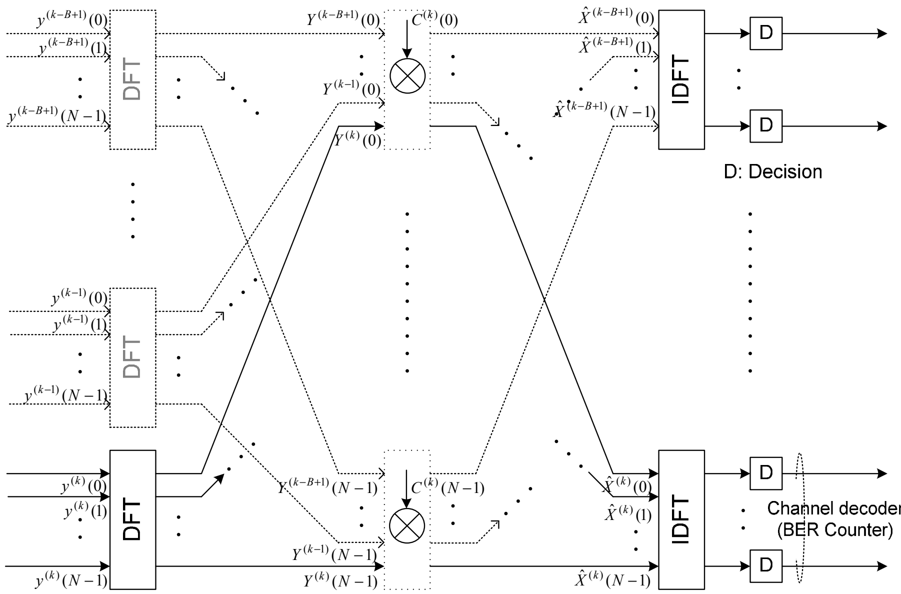

Fig. 1. The conceptual structure of the element-wise block adaptive frequency-domain equalization.

The WEB-FDE is initialized in a training mode by using the desired signal vectors $\boldsymbol{X}^{(k)}(l)$. Then, it switches to a decision-directed mode for tracking, where the temporary desired signal vectors is the DFT elements of decision vectors for $\hat{\boldsymbol{X}}^{(k)}(l)$. From (12), it is found that if $B = 1$, the WEB-FDE can be regarded as a normalized least-mean-square (NLMS)-FDE with a step-size of one [4].

A. Numerical Stability Problem of WEB-FDE

The computational complexity of the WEB-FDE is assessed by considering a fast Fourier transform (FFT) and inverse FFT (IFFT) operators instead of DFT and IDFT operators. It is mainly related to the added blocks in order to minimize the numerical stability problem caused by an error propagation. It is well-known that an error propagation is often generated in a decision feedback equalizer (DFE) due to the incorrect decision symbols in the feedback filter. On the other hand, the error propagation would occur in the WEB-FDE. That is, incorrect past decision vectors to calculate $\boldsymbol{e}^{(k)}(l)$ in (14) cause the corresponding equalizer coefficient to be far from optimum coefficient, which affects the stability of the WEB-FDE. Accordingly, in order to minimize this phenomenon, we renovate the past decision blocks every coefficient update time. This approach increases an hardware and computational complexity, i.e., an additional $(B-1)$-IFFTs and $(B-1)$-FFTs are required to perform the detection and update parts for the past received blocks in Fig. 1. However, it may be preferable to minimize the phenomenon without the help of the channel coding and decoding scheme. On the other hand, it is noticed in Fig. 1 that the outputs of $(B-1)$-dotted blocks practically correspond to those of buffers preserving the past FFT outputs. Therefore, in the filtering part, the required computational complexity for complex multiplications and additions of the FFT at the k-th block are $(N/2)\log_2 N$ and $N\log_2 N$, respectively. Table I shows the computational complexities of the WEB-FDE, RLS-FDE [4] and NLMS-FDE.

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON (COMPLEX OPERATION)

Algorithm	Computational Complexity
NLMS-FDE	$(3N/2)\log_2 N + 18N$ (Mul.)
	$3N\log_2 N + 13N$ (Add.)
RLS-FDE	$(3N/2)\log_2 N + 32N$ (Mul.)
	$3N\log_2 N + 25N$ (Add.)
WEB-FDE	$\frac{(2B+1)N}{2}\log_2 N + 14BN + 6N$ (Mul.)
	$(2B+1)N\log_2 N + 11BN + 5N$ (Add.)

IV. PERFORMANCE EVALUATION

We evaluate the performance of the WEB-FDE for uncoded QPSK system in a time-varying typical urban (TU) channel [7]. In the simulations, one block consists of 64 symbols, i.e., $N = 64$, and the symbol period is $T_s = 3.69\mu s$ as for GSM and EDGE [7] providing a normalized rms delay spread $\tau = 0.2886$ and radio channel memory of 3 (sample delay) for TU channel model. The roll-off factor and filter duration of a raised cosine filter $g(t)$ were set to be $\alpha_f = 0.35$ as IS-136 TDMA [7] and $T_f = 4T_s$, respectively. Therefore, a total channel memory corresponds to $N_C = 7$, and $N_G = 7$ was set in order to eliminate the IBI. To initialize the equalizer coefficient, one block of TS of $N = 64$ is inserted every 25 data blocks.

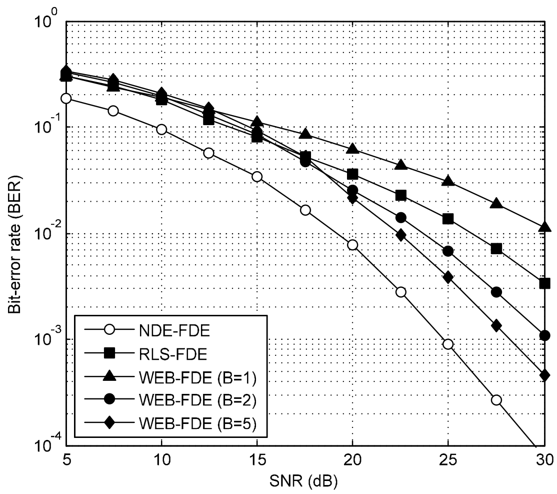

Fig. 2. BER performances of WEB-FDE ($B = 1, 2$ and 5) and RLS-FDE

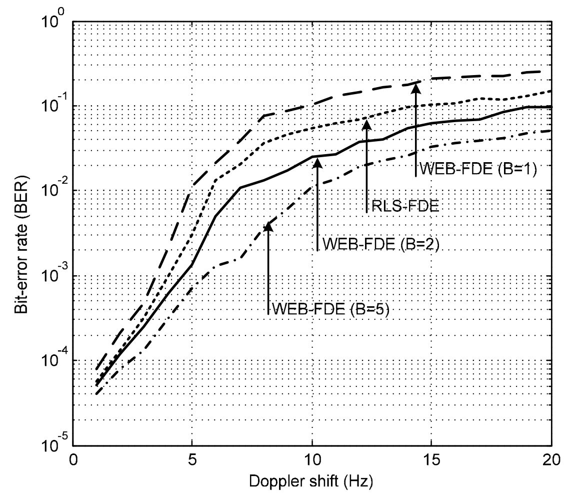

Fig. 3. BER performances according to Doppler shift.

Though we focus on the bit-error-rate (BER) performances in simulation evaluations, it is needed to make mention of the convergence speed of the WEB-FDE. In the WEB-FDE, the convergence speed property is generally determined according to the initial equalizer operation during $1 \leq k < B$. If an equalizer is valid only for $k \geq B$, its convergence speed would be degraded as compared with a non-block equalizer ($B = 1$). Else if, according to an equalizer operation during $1 \leq k < B$, the convergence speed is faster than or equal to the case of $B = 1$. For the case of the latter, we can consider two methods in order to update the equalizer coefficient. The first method is to update the equalizer coefficient like the non-block equalizer until B-blocks fill-up, and then update by using (14). Therefore, convergence speed is equal to the case of $B = 1$. The second method is to update the equalizer coefficient by using (14) during $1 \leq k < B$. For example, given $B = 5$ and two received blocks ($k = 1, 2$), (14) is updated by assuming virtually $\tilde{B} = 2$. The operation like this is accomplished until $\tilde{B} = B - 1$. Obviously, the second method provides a faster convergence speed than the first method because one TS block is used B times, which can be easily shown from the shifting property of (6). From the above descriptions, the second method has been adopted in the following simulations. The BER performance evaluation of WEB-FDE is analyzed and compared with RLS-FDE [3], when various input SNRs and Doppler shift $f_d = 5$ Hz are given. Furthermore, a sub-optimum BER bound is provided by considering the no decision error FDE (NDE-FDE). The parameters $B = 1, 2$ and 5 for the WEB-FDE were selected. $\lambda = 0.8$ and $\lambda_{\text{RLS}} = 0.75$ are chosen to allow fast tracking for the WEB-FDE and RLS-FDE, respectively. From Fig. 2, it is shown that the WEB-FDE exhibits a significant improvement of performance in the $B = 2$, while the gain generated by $B = 5$ is reduced. Moreover, the WEB-FDE ($B \geq 2$) have better BER performance than the RLS-FDE as SNR increases.

Fig. 3 shows simulation results in Doppler shift-varying channel condition. Let us assume that Doppler shift varies from 1 Hz to 20 Hz with input SNR of 30 dB. Parameters were set up identically with Fig. 2. It is also shown that the WEB-FDE ($B = 2$ and 5) have better BER performances than the NLMS-FDE (WEB-FDE of $B = 1$) and RLS-FDE even in the presence of a relatively fast fading.

V. Conclusions

In this paper, we have proposed a novel block adaptive FDE named as WEB-FDE, in which the correction term of the equalizer coefficient was calculated by minimizing the weighted *a posteriori error* vector-based LS criterion at each block iteration. From the simulation results in a time-varying TU channel, we could verify the superiority of the WEB-FDE ($B \geq 2$) as compared to the NLMS-FDE and RLS-FDE. Furthermore, it is expected that the proposed block adaptive scheme can be also applied to estimate the CIR for the MMSE-FDE and zero-forcing FDE.

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